Style over Substance?
Advertising, Innovation and Endogenous Market Structure *

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BSE Summer Forum

Rise of Intangibles

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* The views expressed are the authors' and may not represent those of Banco de España or the Eurosystem.

Motivation

- Firms use two key intangibles to increase sales, markups and profits:
  1. R&D ($\approx$ 2.4% of GDP) $\Rightarrow$ Increases *intrinsic* quality of products ["substance"].
  2. Advertising ($\approx$ 2.2% of GDP) $\Rightarrow$ Increases *extrinsic* quality of products ["style"].
- Both are similar from the firm's perspective (attract demand to $\uparrow$ profits)...
- ...but potentially very different aggregate implications:
  - R&D $\rightarrow$ Direct engine of economic growth via positive externalities.
  - ADV $\rightarrow$ Does not contribute to growth (diverts demand away from competitors).
- How do R&D and ADV interact? How do they shape competition? Can ADV be socially useful?

This paper:

Study implications of R&D-ADV interaction for (i) markups and market structure, (ii) firm and industry dynamics, (iii) economic growth and (iv) welfare.

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Outline

1. Endogenous Growth Model in GE:

- Key features:
  - (i) Oligopolistic competition within industries.
  - (ii) Endogenous market structure $\rightarrow$ Endogenous number of small & large firms + entry/exit.
  - (iii) Competition in ADV (rat-race) and R&D (step-by-step innovations).
- Remain agnostic about whether R&D and ADV are substitutes or complements.
- R&D and ADV are demand shifters $\rightarrow$ Used strategically to increase markups and profits.

2. Calibration:

- Target two inverse-U relationships in data: (i) R&D vs. market share; (ii) ADV vs. market share.
- Calibration delivers R&D and ADV as substitutes.

3. Two Quantitative Exercises:

1. Prohibit the use of ADV altogether $\rightarrow$ Study macro consequences.
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Preview of Results

- Shutting down ADV $\rightarrow$ Positive and negative effects:
  - **(+)** Innovation and economic growth go up, via substitution effect with R&D.
  - **(+)** Markups decrease and competition improves (more small and large firms).
  - **(–)** **...but** resources are reallocated away from most productive firms (initial output $\downarrow$).
- In net terms $\rightarrow$ ADV shutdown yields welfare loss of **0.86%** $\Rightarrow$ ADV socially useful!.
- However, optimal ADV tax is positive and high! ($= 62.9\%$):
  - An ADV tax increases dynamic efficiency by diverting resources toward innovation.
  - The decrease in static efficiency (from misallocation) is rather limited.
  - Revenue is raised by reducing excessive ADV spending due to the “rat race”.

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Related Literature

- **Role of Intengibles and Demand-Driven Dynamics**:

  Drozd and Nosal (2012), Gourio and Rudanko (2014), Fitzgerald, Haller and Yedid-Levi (2017), De Ridder (2020), Morlacco and Zeke (2020), Weiss (2020), Ignaszak and Sedláček (2021), Roldan-Blanco and Gilbukh (2021).

- **Firm Heterogeneity and Markups**:

  Edmond, Midrigan and Xu (2018), Akcigit and Ates (2019), Peters and Walsh (2019), Liu, Mian and Sufi (2019), Aghion, Bergeaud, Boppart, Klenow and Li (2019), Peters (2020), Burstein, Carvalho and Grassi (2020), Baqaee and Farhi (2020), Cavenaile, Celik and Tian (2021), De Loecker, Eeckhout and Mongey (2021).

- **Advertising in Economics**:

  - *Classics:*

    Dorfman and Steiner (1954), Butters (1977), Becker and Murphy (1993), Bagwell (2007).

  - *Recent advances:*

    Dinlersoz and Yorukoglu (2012), Perla (2019), Greenwood, Ma and Yorukoglu (2021), Rachel (2021), Cavenaile and Roldan-Blanco (2021), Argente, Fitzgerald, Moreira and Priolo (2021), Klein and Şener (2022).

Model

Environment: Preferences and Technology

- Continuous and infinite time, $t \in \mathbb{R}_+$.
- **Preferences:** [Inelastic labor supply, $L = 1$]

$$\max \int_0^{+\infty} e^{-\rho t} \ln(C_t) \mathrm{d}t \qquad \text{s.t. } \dot{\mathcal{A}}_t \leq r_t \mathcal{A}_t + w_t - C_t$$

- **Final good:** [Used for consumption, R&D, ADV, and new business entry]

$$Y_t = \exp\left(\int_0^1 \ln\left(y_{jt}\right) \mathrm{d}j \right)$$

- **Industry** j**:**

$$y_{jt} = \left(\tilde{y}_{cjt}^{\frac{\gamma-1}{\gamma}} + \tilde{y}_{sjt}^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}}, \qquad \gamma \geq 1$$

  - Small Firms ($\tilde{y}_{cjt}$) $\rightarrow$ *Competitive fringe:* Homogenous good in perfect competition (zero markups).
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$$\tilde{y}_{cjt} = \int_{F_{jt}} y_{ckjt} \mathrm{d}k, \quad \text{with} \quad y_{ckjt} = q_{ckjt} l_{ckjt}$$

- **Large firms:** Endogenous number $N_{jt} \in \{1, \ldots, \overline{N}\}$ of large firms. Technology: (where $\eta > \gamma$)

$$\tilde{y}_{sjt} = \left(\sum_{i=1}^{N_{jt}} \underbrace{\hat{\omega}_{ijt}}_{\substack{\text{Extrinsic}\\ \text{quality}}} y_{ijt}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}, \quad \text{where} \quad y_{ijt} = \underbrace{q_{ijt}}_{\substack{\text{Intrinsic}\\ \text{quality}}} l_{ijt} \ \text{ and } \ \hat{\omega}_{ijt} \equiv \frac{1+\omega_{ijt}}{\frac{1}{N_{jt}} \sum_{h=1}^{N_{jt}} (1+\omega_{hjt})}$$

- Large firms compete (*simultaneous Cournot*) in quantities and advertising:
  - Choose y_{ijt} and ω_{ijt} taking $\left\{ y_{hjt}, \omega_{hjt} : \forall h \neq i \right\}$ as given.
  - Convex ADV costs:

$$A_{ijt} = \chi_a \, \omega_{ijt}^{\phi_a} Y_t, \qquad \chi_a > 0, \ \phi_a > 1$$

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Environment: R&D

- To advance $q_{ijt} \rightarrow q_{ijt}(1+\lambda)$ at Poisson rate z_{ijt}, pay:

$$R_{ijt} = \chi {z_{ijt}}^{\phi} Y_t, \qquad \chi > 0,\ \phi > 1$$

- *Technology gaps*: (between large firms i and $h \neq i$)

$$\frac{q_{ijt}}{q_{hjt}} = (1+\lambda)^{n_{ijt}^{h}}, \qquad \text{with } n_{ijt}^{h} \in \Big\{ \underbrace{-\bar{n},\dots,-1}_{\text{i is behind h}},\ \underset{\substack{\uparrow \\ \text{Neck to neck}}}{0},\ \underbrace{1,\dots,\bar{n}}_{\text{i is ahead of h}} \Big\}$$

  - Small firms keep a *constant* gap $\zeta \equiv \frac{q_{jt}^{leader}}{q_{cjt}} > 0$ with leader, where $q_{jt}^{leader} \equiv \max\{q_{hjt}\}_{h=1}^{N_{jt}}$.

- **Entry and exit:** ▸ Details
  - Small firms $\rightarrow$ Become large firms (*entry*) via innovation, *exit* exogenously at rate $\tau > 0$.
  - Large firms $\rightarrow$ Become small firms (*exit*) if they lag by more than $\bar{n}$ steps.
  - Entrepreneurs $\rightarrow$ Measure-one, they create and sell off new small firms.

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Equilibrium: Static Block

- Every firm $i \in \{1, \ldots, N_j\}$ plays a *simultaneous Cournot game* with all $h \neq i$:
  1. **Output choice:** Pre-ADV profits (π) and markups (M):

$$\pi_{ij} = \sigma_{ij}\left(1 - \frac{1}{M_{ij}}\right) Y, \quad \text{with } M_{ij} = \left[\left(\frac{\eta - 1}{\eta}\right) - \left(\frac{\gamma - 1}{\gamma}\right)\sigma_{ij} - \left(\frac{\eta - \gamma}{\eta\gamma}\right)\tilde{\sigma}_{ij}\right]^{-1}$$

where

$$\sigma_{ij} \equiv \frac{p_{ij}y_{ij}}{p_{cj}y_{cj} + \sum_{k=1}^{N_j} p_{kj}y_{kj}} \quad \text{and} \quad \tilde{\sigma}_{ij} \equiv \frac{p_{ij}y_{ij}}{\sum_{k=1}^{N_j} p_{kj}y_{kj}}$$

  2. **ADV choice:** Optimal ω is only a function of σ_{ij} and $\tilde{\sigma}_{ij}$ as well. ▸ Details

- Solving the static part:
  1. It can be shown that $\{\sigma_{ij}, \tilde{\sigma}_{ij}\}_{i=1}^{N_j}$ is only a function of $\mathbf{n}_{ij} \equiv \{n_{ij}^k\}_{k \neq i}$ (set of technology gaps).
  2. For each $(\mathbf{n}, N)$, system of 2N equations in 2N unknowns $\Rightarrow$ Get $\left\{\pi(\mathbf{n}_i, N), \omega(\mathbf{n}_i, N)\right\}_{i=1}^{N}$

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Equilibrium: Value Function

- HJB equation for large firm $i \in \{1, \ldots, N\}$, with set of technology gaps $\mathbf{n}_i \equiv \{n_i^k\}_{k \neq i}$:

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\begin{aligned}
rV(\mathbf{n}_i, N) = \max_{z_i} \Bigg\{ & \underbrace{\pi(\mathbf{n}_i, N)}_{\text{Pre-ADV profits}} - \underbrace{\chi_a \Big(\omega(\mathbf{n}_i, N)\Big)^{\phi_a} Y}_{\text{ADV costs}} - \underbrace{\chi z_i^{\phi} Y}_{\text{R\&D costs}} \\
& + \underbrace{z_i \Big[V\Big(\mathbf{n}_i \setminus \{n_i^k = \bar{n}\} + 1, N - |\{n_i^k = \bar{n}\}|\Big) - V(\mathbf{n}_i, N) \Big]}_{\text{Successful innovation}} + \sum_{\{k : n_i^k = -\bar{n}\}} \underbrace{z_k \Big(0 - V(\mathbf{n}_i, N)\Big)}_{\text{Exit}} \\
& + \sum_{\{k : n_i^k > -\bar{n}\}} \underbrace{z_k \Big[V\Big(\mathbf{n}_i \setminus \{n_i^k\} \cup \{n_i^k - 1\} \setminus \{n_i^l = \bar{n} + n_i^k\}, N - |\{n_i^l = \bar{n} + n_i^k\}|\Big) - V(\mathbf{n}_i, N) \Big]}_{\text{Large firm } k \neq i \text{ innovates}} \\
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More HJBs → Small Firms and Entrepreneurs

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$$+ \sum_{\{k: n_i^k > -\bar{n}\}} \underbrace{z_k \Bigg[V\Big(\mathbf{n}_i \backslash \{n_i^k\} \cup \{n_i^k - 1\} \backslash \{n_i^l = \bar{n} + n_i^k\}, N - |\{n_i^l = \bar{n} + n_i^k\}|\Big) - V(\mathbf{n}_i, N) \Bigg]}_{\text{Large firm } k \neq i \text{ innovates}}$$

$$+ \underbrace{X \Bigg[V\Big(\mathbf{n}_i \cup \Big\{ \min\{\bar{n}, \bar{n} + \min(\mathbf{n}_i)\} \Big\}, \min(N+1, \bar{N})\Big) - V(\mathbf{n}_i, N) \Bigg]}_{\text{Entry}} \Bigg\} + \underbrace{\dot{V}(\mathbf{n}_i, N)}_{\text{Drift}}$$

▸ More HJBs → Small Firms and Entrepreneurs

Equilibrium: Value Function

- HJB equation for large firm $i \in \{1, \ldots, N\}$, with set of technology gaps $\mathbf{n}_i \equiv \{n_i^k\}_{k \neq i}$:

$$
\begin{aligned}
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▸ More HJBs → Small Firms and Entrepreneurs

Equilibrium: Market Clearing and BGP

- **Aggregate Markup:**
  - Labor market clearing gives aggregate markup (M_t) as the inverse of the labor share:

$$M_t \equiv \left[\int_0^1 \left(\sigma_{cjt} + \sum_{i=1}^{N_{jt}} \sigma_{ijt} M_{ijt}^{-1} \right) \mathrm{d}j \right]^{-1} = \left(\frac{w_t}{Y_t} \right)^{-1}$$

- **BGP:**
  - Identify each industry with state $\Theta \equiv (N, \vec{n})$, where $\vec{n} \equiv$ Set of technology gaps w.r.t. leader.
  - Distribution over industry states is time-invariant, $\mu(\Theta)$.
  - Growth rate:

$$g = \ln(1+\lambda) \left[\sum_{\Theta} z_L(\Theta) \mu(\Theta) \right], \qquad z_L(\Theta) \equiv \text{ Arrival rate of } \textit{leader} \text{ innovation in industry } \Theta$$

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Calibration

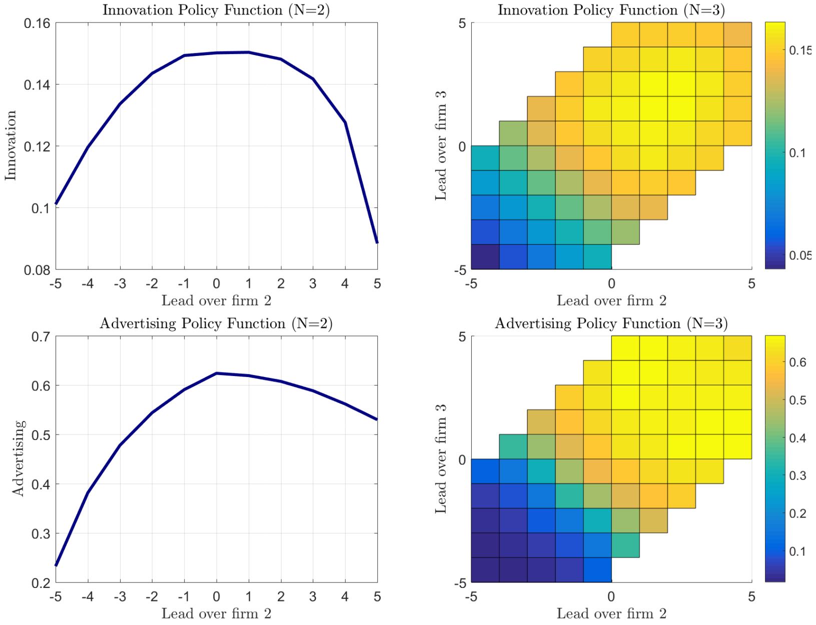
SMM Estimation: U.S. Data, 1976-2004

▸ Moments Details

- Proxy innovation with stock of patent citations (from NBER Patent Database Project).
- Target two inverse-U relationships: (i) R&D and market share (σ); (ii) ADV and market share (σ).

Parameter		Value	Target	Model	Data	Data Source
Innovation step	λ	0.166	Growth rate	2.2%	2.2%	US BEA
Rel. quality (small firms)	ζ	0.708	Labor share	63.8%	65.2%	Karabarbounis-Neiman (2013)
			Average markup	34.2%	35.0%	De Loecker et al. (2020)
EoS (large vs small)	γ	2.964	SD(markups)	0.44	0.35	De Loecker et al. (2020)
EoS (between large firms)	η	11.67	Average profitability	13.6%	14.4%	Compustat
R&D scale (large firms)	χ	77.48	R&D share of GDP	2.5%	2.4%	NSF
R&D scale (small firms)	ν	3.163	SD(leader rel. quality)	0.16	0.22	NBER Patent + Compustat
R&D curvature (large)	ϕ	4.485	Top point(Innov., σ)	0.48	0.51	NBER Patent + Compustat
R&D curvature (small)	ϵ	4.551	Intercept(Innov., σ)	0.98	0.63	NBER Patent + Compustat
			Avg. leader rel. quality	0.51	0.75	NBER Patent + Compustat
ADV cost scale	χ_a	0.066	ADV share of GDP	2.2%	2.2%	Coen Structured ADV Dataset
			Intercept(ADV, σ)	7.61	6.26	Compustat
ADV cost curvature	ϕ_a	3.365	Top point(ADV, σ)	0.52	0.53	Compustat
Exit rate (small firms)	τ	0.115	Firm entry rate	11.5%	11.5%	BDS (US Census)
Cost of small firm creation	ψ	0.060	Set to obtain $m = 1$ in eq'm			

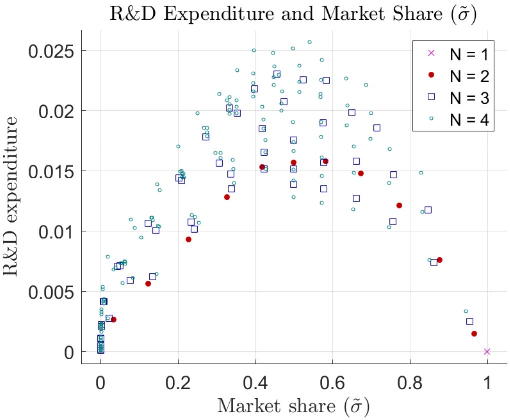
Behavior of R&D and ADV: Firm Level

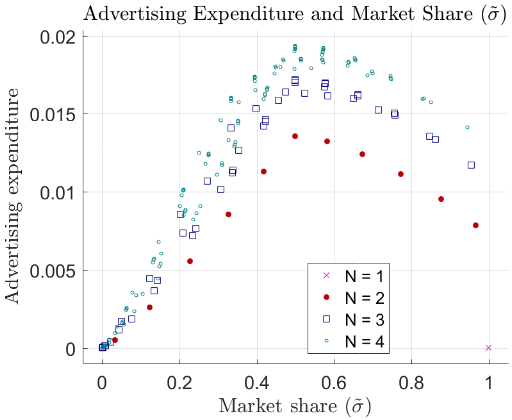


- **R&D:** (top panels)
  - Highest around neck-to-neck states.
  - This is where firms have highest marginal gains in profits.
- **ADV:** (bottom panels)
  - Leaders advertise more than followers.
  - This shifts demand toward more productive firms.

More: ▸ Correlation Matrix

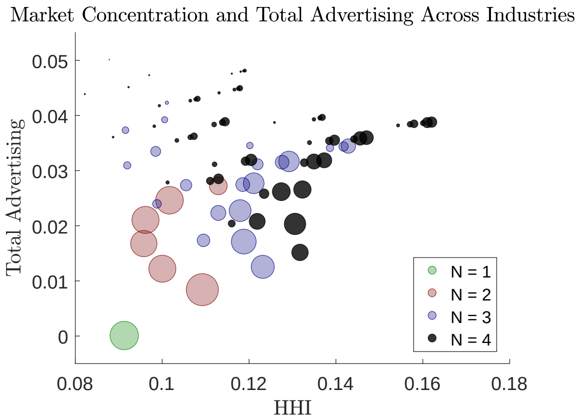
Behavior of R&D and ADV: Within Industries





- Within industries:
  - R&D and ADV are inverse-U shaped in market share relative to other large firms ($\tilde{\sigma}$).
  - This is true even when we hold the number of large firms (N) fixed.
  - *Recall:* Intercept and top points are calibration targets.

Behavior of R&D and ADV: Across Industries



- Across industries:
  1. More concentrated industries have higher ADV expenditures.
  2. Industries with more large firms have higher investment in ADV (and R&D).

     $\Rightarrow$ The presence of more large firms intensifies the “rat race”.

Aggregate Effects of Advertising

Advertising Shutdown

	Benchmark	ADV Shutdown	% change
Growth rate	2.201%	2.273%	3.26%
R&D/GDP	2.467%	2.613%	5.92%
Large firm innovation	0.339	0.394	16.17%
Small firm innovation	0.096	0.112	16.44%
Average markup	34.2%	25.4%	-25.73%
Std. dev. markup	0.442	0.340	-23.12%
Labor share	0.638	0.663	3.84%
Average # large firms per industry	2.864	3.264	13.99%
Mass of small firms	1.000	1.328	32.84%
Initial output	1.159	1.105	-4.63%

- **Experiment:** Prohibit ADV expenditures.
  1
  2
  3
  4

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- **Experiment:** Prohibit ADV expenditures.
  1. Economic growth goes up $\Rightarrow$ R&D and ADV are substitutes (Cavenaile and Roldan-Blanco (2021)).
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- **Experiment:** Prohibit ADV expenditures.
  1. Economic growth goes up
  2. Markups and dispersion go down $\Rightarrow$ ADV responsible for ≈26% of avg. and ≈23% of dispersion.
  3.
  4.

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  3. More competition $\Rightarrow$ More large firms and more small firms.
  4.

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- **Experiment:** Prohibit ADV expenditures.
  1. Economic growth goes up
  2. Markups and dispersion go down
  3. More competition
  4. **But...** Initial output goes down! *Why?*

Advertising Shutdown

- Effects on initial output are a priori ambiguous. ⇒ In net terms, initial output ↓ (by 4.63%)
  - **(+)** ADV shutdown decreases markups and their dispersion...
  - **(–)** ... but also reallocates resources away from productive large firms (misallocation ↑)...
  - **(–)** ... thereby decreasing extrinsic quality of cheaper-to-produce varieties.

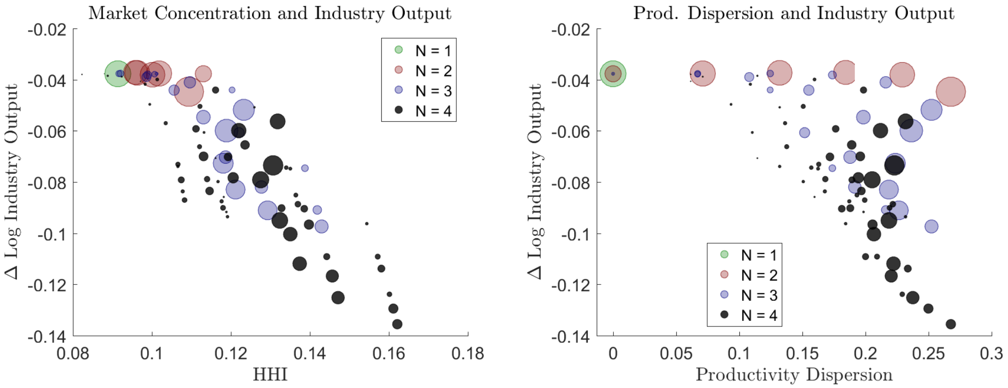

Figure: Industries with higher concentration/dispersion see larger decreases in output from ADV shutdown.

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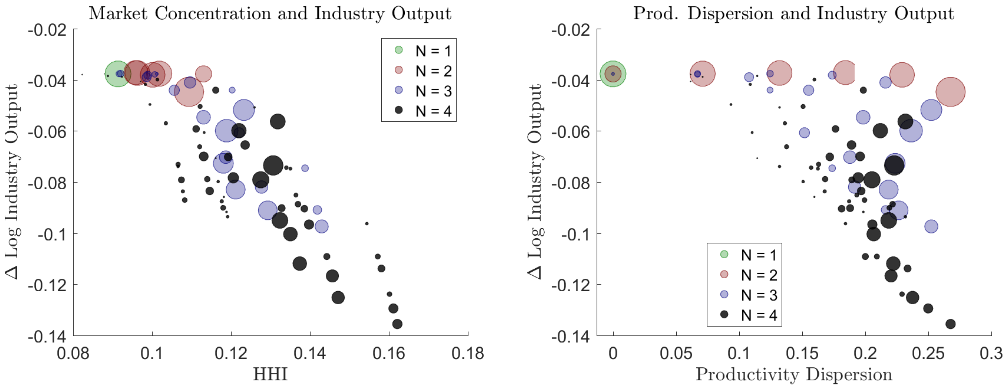

Figure: Industries with higher concentration/dispersion see larger decreases in output from ADV shutdown.

Advertising Shutdown

- ADV is good for static efficiency but bad for innovation and growth $\rightarrow$ Net welfare effect?
- Change in welfare between BGPs: ▸ Derivation

$$\Delta \text{Welfare} = \frac{1}{\rho}\left[\Delta\begin{pmatrix}\text{Consumption}\\ \text{share of initial output}\end{pmatrix} + \underbrace{\Delta\begin{pmatrix}\text{Market share}\\ \text{of large firms}\end{pmatrix} - \Delta\begin{pmatrix}\text{Labor}\\ \text{share}\end{pmatrix}}_{\Delta\text{ (Initial output)}}\right] + \frac{1}{\rho^2}\Delta\begin{pmatrix}\text{Output}\\ \text{growth}\end{pmatrix}$$

- Further decompose Consumption-Equivalent Welfare (CEW) into: ▸ CEW Formula
  - **Static CEW:** Counterfactual keeping distribution $\mu(\Theta)$ fixed.
  - **Dynamic CEW:** Full model letting innovation respond (i.e. $\mu(\Theta)$ adjusts).

	Static CEW	**Dynamic CEW**
(1) Consumption share	2.32%	2.10%
(2) Market share of large firms	-2.44%	-0.96%
(3) Labor share	-3.37%	-3.70%
(4) Output growth	0.00%	1.81%
Total	**-3.64%**	**-0.84%**

ADV is socially desirable, particularly in the short-run (before distribution over industry states adjusts)

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ADV is socially desirable, particularly in the short-run (before distribution over industry states adjusts)

Should We Tax or Subsidize Advertising?

- Despite ADV being socially useful $\Rightarrow$ Optimal (linear) ADV tax is positive and high! (= 62.9%).

	Benchmark	Optimal Tax (64.6%)	% change
Growth rate	2.201%	2.215%	0.64%
R&D/GDP	2.467%	2.466%	-0.08%
Large firm innovation	0.339	0.346	2.22%
Small firm innovation	0.096	0.099	3.07%
Average markup	34.2%	32.0%	-1.66%
Std. dev. markup	0.442	0.418	-5.51%
Labor share	0.638	0.644	0.95%
Average # large firms per industry	2.864	2.917	1.87%
Mass of small firms	1.000	1.042	4.15%
Initial output	1.159	1.142	-1.44%
Overall CEW change		**0.515%**	

- Even with high rates $\rightarrow$ Highly productive firms still invest more in ADV compared to others.
- Loss in output (from increased static misallocation) is dominated by:

  (i) higher g; (ii) more business dynamism; (iii) less resources wasted on "rat race".

Conclusion

- We study the interaction between R&D, advertising and market structure:
  - Fit inverse-U shaped relationships: (i) R&D and market share; (ii) ADV and market share.
  - In the model, ADV has positive and negative effects:
    - **(+)** Reallocates resources toward more efficient firms, decreasing misallocation.
    - **(–)** Decreases economic growth, increases markups, and hurts competition.
  - In net terms $\rightarrow$ ADV is socially useful (banning it implies 0.84% loss in CEW).
  - Yet, ADV should be taxed (optimal linear tax $\approx$ 63%):
    - Increases dynamic efficiency and reduces excessive spending in ADV "rat race"...
    - ...while still preserving the allocative benefits of ADV (more resources to top firms).

Thank you!

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    - ...while still preserving the allocative benefits of ADV (more resources to top firms).

Thank you!

Appendix

Appendix: Entry and Exit

Back to Environment

1. **Entry/Exit of Large Firms:**
   - Small firm k can become a large firm at rate X_{kjt} by paying cost:

$$R^e_{kjt} = \nu {X_{kjt}}^{\epsilon} Y_t, \qquad \nu > 0, \ \epsilon > 1$$

   - If successful, enter as most laggard ($\bar{n}$ steps behind leader) $\Rightarrow$ # large firms: $N_{jt} + 1$.
   - If large firm lags by more than $\bar{n}$, it becomes small firm (exit) $\Rightarrow$ # large firms: $N_{jt} - 1$.

2. **Entry/Exit of Small Firms:**
   - Measure-one of entrepreneurs, can create a small firm at rate e_t by paying cost:

$$E_t = \psi {e_t}^2 Y_t, \qquad \psi > 0$$

   - If successful, new small firm enters a random industry $j \in [0, 1]$.
   - Small firms exit at exogenous rate $\tau > 0$:

$$\dot{m}_t = e_t - \tau m_t$$

Appendix: Optimal ADV spending

Back to Static Part

- **ADV choice:**

$$\frac{\sigma_{ijt}}{1+\omega_{ijt}}\left[1-\frac{\hat{\omega}_{ijt}}{N_{jt}}+\frac{\gamma-\eta}{(\eta-1)\gamma}\left(\tilde{\sigma}_{ijt}-\frac{\hat{\omega}_{ijt}}{N_{jt}}\right)+\frac{\eta}{\eta-1}\frac{\gamma-1}{\gamma}\sigma_{ijt}\left(\frac{\hat{\omega}_{ijt}}{N_{jt}\tilde{\sigma}_{ijt}}-1\right)\right]=\chi_a\phi_a\left(\omega^*_{ijt}\right)^{\phi_a-1}$$

where recall

$$\sigma_{ijt} \equiv \frac{p_{ijt}y_{ijt}}{p_{cjt}y_{cjt}+\sum_{k=1}^{N_{jt}} p_{kjt}y_{kjt}} \quad \text{and} \quad \tilde{\sigma}_{ijt} \equiv \frac{p_{ijt}y_{ijt}}{\sum_{k=1}^{N_{jt}} p_{kjt}y_{kjt}}$$

Appendix: Small Firms and Entrepreneurs HJBs

Back to Equilibrium

- **Industry state:** $\Theta \equiv (N, \vec{n})$, where $\vec{n} \equiv \{0, \ldots, \overline{n}\}^{N-1}$ is set of gaps of followers wrt industry leader ("L").
- **Small firm's problem:**

$$rV^e(\Theta) = \max_X \Bigg\{ - \underbrace{\nu X^\epsilon Y}_{\text{R\&D costs}} + \underbrace{XV\Big(\{\mathbf{n}_L - \overline{n}\} \cup \{-\overline{n}\}, N+1\Big)}_{\text{Become a large firm}} - \underbrace{\tau V^e(\Theta)}_{\text{Exit}}$$

$$+ \underbrace{\sum_{\Theta'} p(\Theta, \Theta')\Big(V^e(\Theta') - V^e(\Theta)\Big)}_{\text{Industry transitions}} \Bigg\} + \underbrace{\dot{V}^e(\Theta)}_{\text{Drift}}$$

- **Entrepreneur's problem:**

$$\rho S = \max_e \Bigg\{ \underbrace{-\psi e^2 Y}_{\text{Creation cost}} + \underbrace{e \sum_{\Theta} V^e(\Theta)\mu(\Theta)}_{\text{Value of small firm}} \Bigg\}, \text{ where } \mu(\Theta) \equiv \text{ Mass of industries in state } \Theta$$

Appendix: Moments Details

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1 **Inverse-U between R&D, ADV and relative sales:**

- Innovation in the data measured as average patent citations for each firm.
- Regress: (firm i, SIC-4 industry j, year t)

$$AvgCitations_{ijt} = \alpha_1 \sigma_{ijt} + \alpha_2 \sigma_{ijt}^2 + \boldsymbol{X}_{ijt}^{\top} \gamma + u_{ijt}$$
$$\ln\left(1 + xad_{ijt}\right) = \beta_1 \sigma_{ijt} + \beta_2 \sigma_{ijt}^2 + \boldsymbol{X}_{ijt}^{\top} \boldsymbol{\delta} + u_{ijt}$$

where $\boldsymbol{X}_{ijt} \in \{$profitability, leverage, log R&D stock, firm age, time and industry FE, $\ldots\}$.

- Target $\alpha_1, \alpha_2 > 0$ and $\beta_1, \beta_2 > 0$ in the calibration.

2 **Relative quality of the leader:**

- Proxy quality with the stock of past patent citations.
- Define relative quality of the leader as the quality of the top firm divided by the sum of the qualities of the top four firms in its SIC4 industry.

Appendix: Firm-Level Correlations

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	Markup	R&D	Advertising	SG&A	Profitability
Markup	1.000				
R&D	0.359	1.000			
Advertising	0.698	0.853	1.000		
SG&A	0.555	0.960	0.965	1.000	
Profitability	0.603	0.617	0.643	0.655	1.000

Notes: Correlation coefficients in the estimated model between markups, R&D, advertising, SG&A and profitability at the firm level. SG&A is computed as the sum of advertising and R&D.

- Within firms:
  - R&D, ADV, profitability and markups are all positively correlated.
  - Consistent with correlation patterns in De Loecker, Eeckhout and Unger (2020).

Appendix: Welfare Decomposition

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- Welfare in BPG:

$$W = \frac{1}{\rho}\left(\ln\left(C_0\right) + \frac{g}{\rho}\right) = \frac{1}{\rho}\left(\ln\left(Y_0\right) + \ln\left(\frac{C_0}{Y_0}\right) + \frac{g}{\rho}\right)$$

with

$$\ln\left(Y_0\right) = \underbrace{\int_0^1 \ln\left(q_{cj0}\right)dj}_{\substack{\text{Fringe quality} \\ \textbf{[constant across BGPs]}}} + \underbrace{\sum_{\Theta} f(\Theta)\mu(\Theta)}_{\text{Average output large firms}} - \underbrace{\ln\left(\frac{w_0}{Y_0}\right)}_{\text{Labor share}}$$

where

$$f(\Theta) \equiv \frac{1}{\gamma - 1}\ln\underbrace{\left\{1 + \left(\sum_{i=1}^{N(\Theta)} \hat{\omega}_{i0}(\Theta)\left(\frac{y_{i0}}{\tilde{y}_{c0}}(\Theta)\right)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta(\gamma-1)}{\gamma(\eta-1)}}\right\}}_{=1/\sigma_{c0}(\Theta)} = -\frac{1}{\gamma - 1}\ln\left(1 - \sum_{i=1}^{N(\Theta)} \sigma_{i0}(\Theta)\right)$$

- Change in welfare between BGPs:

$$\Delta W = \frac{1}{\rho}\left(\Delta \ln\left(\frac{C_0}{Y_0}\right) - \Delta\sum_{\Theta}\frac{\mu(\Theta)}{\gamma - 1}\ln\left(1 - \sum_{i=1}^{N(\Theta)}\sigma_{i0}(\Theta)\right) - \Delta\ln\left(\frac{w_0}{Y_0}\right)\right) + \frac{1}{\rho^2}\Delta g$$

Appendix: Consumption-Equivalent Welfare

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- Welfare in BPG:

$$W = \frac{1}{\rho}\left(\ln\left(C_0\right) + \frac{g}{\rho}\right)$$

- **Consumption-equivalent welfare (CEW):**

$$\ln\left(C_0^B\right) + \frac{g^B}{\rho} = \ln\left(C_0^A(1+\iota)\right) + \frac{g^A}{\rho} \quad \Leftrightarrow \quad \boxed{\iota = \frac{C_0^B}{C_0^A}\exp\left\{\frac{g^B - g^A}{\rho}\right\} - 1}$$

$\iota \equiv$ % change in lifetime consumption that the representative household of economy A requires to remain indifferent with respect to economy B.