# *Style over Substance?*
# *Advertising, Innovation and Endogenous Market Structure*[*]


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[*] The views expressed herein are the authors' and may not represent those of Banco de España or the Eurosystem.

# Motivation

- Large firms compete to gain market power.
- Two key tools to increase sales, markups and profits:
  - R&D ($\approx$ 2.4% of GDP) $\Rightarrow$ Increases *intrinsic* quality of products.
  - Advertising ($\approx$ 2.2% of GDP) $\Rightarrow$ Increases *extrinsic* quality of products.
- While similar from firm's perspective, very different aggregate implications:
  - Competition in R&D $\rightarrow$ Direct engine of economic growth.
  - Competition in ADV $\rightarrow$ May lead to excessive spending from "rat race" competition.
- ***Questions:*** How do R&D and ADV shape market structure? Is ADV socially useful?

**This paper:**

Study implications of R&D-ADV interaction for (i) markups, (ii) market structure, (iii) firm and industry dynamics, (iv) economic growth and (v) welfare.

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# Outline

**1. GE Endogenous Growth Model:**

- (i) R&D and ADV decisions; (ii) oligopolistic competition; (iii) endogenous market structure.
- R&D and ADV are demand shifters → Used strategically to increase markups and profits.

**2. Calibration:**

- Calibrate model to US data at the firm-, industry-, and aggregate-level.
- Target two inverse-U relationships: (i) R&D vs. market share; (ii) ADV vs. market share.

**3. Two Quantitative Exercises:**

1. Prohibit the use of ADV and study the welfare consequences.
2. Explore optimal taxes/subsidies on ADV.

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# Preview of Results

- Shutting down ADV $\rightarrow$ Positive and negative effects:
  - **(+)** Innovation and economic growth go up, via substitution effect with R&D.
  - **(+)** Markups decrease and competition improves (more small and large firms).
  - **(–)** **...but** resources are reallocated away from more efficient firms (initial output $\downarrow$).
- In net terms $\rightarrow$ ADV shutdown yields welfare loss of 0.54%.
- However, optimal ADV tax is positive and high! ($\approx$ 64%):
  - An ADV tax both increases dynamic efficiency and raises revenue.
  - It reduces the excessive spending on ADV due to the “rat race”...
  - ...while still preserving the benefits of ADV in the form of reduced misallocation.

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# Related Literature

- **Role of Intengibles**:

  Drozd and Nosal (2012), Belo, Lin and Vitorino (2014), Gourio and Rudanko (2014), Fitzgerald, Haller and Yedid-Levi (2017), De Ridder (2020), Morlacco and Zeke (2020).

- **Firm Heterogeneity and Markups**:

  Edmond, Midrigan and Xu (2018), Akcigit and Ates (2019), Peters and Walsh (2019), Liu, Mian and Sufi (2019), Aghion, Bergeaud, Boppart, Klenow and Li (2019), Peters (2020), Burstein, Carvalho and Grassi (2020), Baqaee and Farhi (2020), Cavenaile, Celik and Tian (2020), De Loecker, Eeckhout and Mongey (2021).

- **Advertising in Economics**:

  - *Classics:*

    Dorfman and Steiner (1954), Butters (1977), Becker and Murphy (1993), Benhabib and Bisin (2002), Bagwell (2007).

  - *Recent advances:*

    Dinlersoz and Yorukoglu (2012), Molinari and Turino (2017), Perla (2019), Greenwood, Ma and Yorukoglu (2021), Rachel (2021), Cavenaile and Roldan-Blanco (2021).

# Model

# **Environment:** Preferences and Technology

- Continuous and infinite time, $t \in \mathbb{R}_+$.
- **Preferences:**

$$\max \int_0^{+\infty} e^{-\rho t} \ln(C_t) \mathrm{d}t \qquad \text{s.t. } \dot{\mathcal{A}}_t \leq r_t \mathcal{A}_t + w_t - C_t$$

- **Final good:**

$$Y_t = \exp\left(\int_0^1 \ln\left(y_{jt}\right) \mathrm{d}j\right)$$

- **Industry *j*:**

$$y_{jt} = \left(\tilde{y}_{cjt}^{\frac{\gamma-1}{\gamma}} + \tilde{y}_{sjt}^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}}, \qquad \gamma \geq 1$$

  - Small Firms $\rightarrow$ Produce homogenous good, perfect competition (*competitive fringe*).
  - Large Firms $\rightarrow$ Produce differentiated goods, interact strategically within *j*, competitively across.

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# **Environment:** Preferences and Technology

- **Small firms:** Endogenous measure $F_{jt} > 0$ of small firms.

$$\tilde{y}_{cjt} = \int_{F_{jt}} y_{ckjt} \mathrm{d}k, \quad \text{with} \;\; y_{ckjt} = q_{ckjt} \cdot l_{ckjt}$$

- **Large firms:** Endogenous number $N_{jt} \in \{1, \ldots, \overline{N}\}$ of large firms.

$$\tilde{y}_{sjt} = \left( \sum_{i=1}^{N_{jt}} \underbrace{\hat{\omega}_{ijt}}_{\substack{\text{Extrinsic} \\ \text{quality}}} \cdot y_{ijt}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}, \quad \text{where} \;\; \hat{\omega}_{ijt} \equiv \frac{1+\omega_{ijt}}{\frac{1}{N_{jt}} \sum_{h=1}^{N_{jt}} (1+\omega_{hjt})} \;\; \text{and} \;\; y_{ijt} = \underbrace{q_{ijt}}_{\substack{\text{Intrinsic} \\ \text{quality}}} \cdot l_{ijt}$$

- Large firms compete strategically (*simultaneous Cournot*):
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# **Environment:** ADV and R&D by Large Firms

1. **Advertising:**
   - To get $\omega_{ijt} > 0$, pay:

$$A_{ijt} = \chi_a \, {\omega_{ijt}}^{\phi_a} Y_t, \qquad \chi_a > 0, \; \phi_a > 1$$

2. **R&D:**
   - To advance $q_{ijt} \rightarrow q_{ijt}(1+\lambda)$ at Poisson rate $z_{ijt}$, pay:

$$R_{ijt} = \chi {z_{ijt}}^{\phi} Y_t, \qquad \chi > 0, \; \phi > 1$$

   - *Technology gaps:* (relative to large firm $h \neq i$)

$$\frac{q_{ijt}}{q_{hjt}} = (1+\lambda)^{n^h_{ijt}}, \qquad \text{with } n^h_{ijt} \in \Big\{ \underbrace{-\bar{n}, \dots, -1}_{i \text{ is behind h}}, \; \underset{\substack{\uparrow \\ \text{Neck} \\ \text{to neck}}}{0}, \; \underbrace{1, \dots, \bar{n}}_{i \text{ is ahead of h}} \Big\}$$

   - Small firms keep a fixed gap $\zeta = \frac{q^{leader}_{jt}}{q_{cjt}} > 0$, where $q^{leader}_{jt} = \max_{h=1,\dots,N_{jt}} \{q_{hjt}\}$.

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# **Environment:** Entry and Exit

1. **Entry/Exit of Large Firms:**

   - Small firm $k$ can become a large firm at rate $X_{kjt}$ by paying cost:

   $$R^e_{kjt} = \nu {X_{kjt}}^{\epsilon} Y_t, \qquad \nu > 0, \ \epsilon > 1$$

   - If successful, enter $\overline{n}$ steps behind industry leader $\Rightarrow$ # large firms: $N_{jt} + 1$.
   - If large firm lags by more than $\overline{n}$, it becomes small firm $\Rightarrow$ # large firms: $N_{jt} - 1$.

2. **Entry/Exit of Small Firms:**

   - Measure-one of entrepreneurs, can create a small firm at rate $e_t$ by paying cost:

   $$E_t = \psi {e_t}^2 Y_t, \qquad \psi > 0$$

   - If successful, entrepreneur sells firm on a competitive market.
   - Small firms exit at exogenous rate $\tau > 0$.

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# **Equilibrium:** Static Block

- Every large firm $i \in \{1, \ldots, N_{jt}\}$ plays **simultaneous Cournot game** with each $k \neq i$:

  1. **Output choice:** Markups ($M$) and profits before advertising ($\pi$), taking $\{y_{hjt}\}_{h \neq i}$ as given:

$$\pi_{ijt} = \sigma_{ijt}\left(1 - M_{ijt}{}^{-1}\right) Y_t$$

$$M_{ijt} \equiv \frac{p_{ijt}}{w_t / q_{ijt}} = \left\{\left(\frac{\eta - 1}{\eta}\right) - \left(\frac{\gamma - 1}{\gamma}\right) \sigma_{ijt} - \left(\frac{\eta - \gamma}{\eta\gamma}\right) \tilde{\sigma}_{ijt}\right\}^{-1}$$

  where

$$\sigma_{ijt} \equiv \frac{p_{ijt} y_{ijt}}{p_{cjt} y_{cjt} + \sum_{k=1}^{N_{jt}} p_{kjt} y_{kjt}} \quad \text{and} \quad \tilde{\sigma}_{ijt} \equiv \frac{p_{ijt} y_{ijt}}{\sum_{k=1}^{N_{jt}} p_{kjt} y_{kjt}}$$

  2. **ADV choice:** Internalize effect on markups ($M$) and sales ($\sigma$), taking $\{\omega_{hjt}\}_{h \neq i}$ as given:

$$\left.\frac{\partial \sigma_{ijt}}{\partial \omega}\right|_{\omega = \omega^*_{ijt}} \left(1 - M_{ijt}{}^{-1}\right) + \sigma_{ijt}\left(1 - \left.\frac{\partial M_{ijt}{}^{-1}}{\partial \omega}\right|_{\omega = \omega^*_{ijt}}\right) = \chi_a \phi_a \left(\omega^*_{ijt}\right)^{\phi_a - 1}$$

# **Equilibrium:** Dynamic Block (1/2)

- **Firm states:** $\mathbf{n}_i \equiv \{n_i^k\}_{k \neq i}$ (technology gaps) and $N$ (# large firms in the industry).

$$
\begin{aligned}
rV(\mathbf{n}_i, N) = \max_{z_i} \Bigg\{ & \underbrace{\pi(\mathbf{n}_i, N)}_{\text{Pre-ADV profits}} - \underbrace{\chi_a \left(\omega_i^*\right)^{\phi_a} Y}_{\text{ADV costs}} - \underbrace{\chi z_i^{\phi} Y}_{\text{R\&D costs}} \\
& + \underbrace{z_i \left[ V\left( \mathbf{n}_i \backslash \{n_i^k = \bar{n}\} + \mathbf{1}, N - |\{n_i^k = \bar{n}\}| \right) - V(\mathbf{n}_i, N) \right]}_{\text{Successful innovation}} + \sum_{\{k : n_i^k = -\bar{n}\}} \underbrace{z_k \Big( 0 - V(\mathbf{n}_i, N) \Big)}_{\text{Exit}} \\
& + \sum_{\{k : n_i^k > -\bar{n}\}} \underbrace{z_k \left[ V\left( \mathbf{n}_i \backslash \{n_i^k\} \cup \{n_i^k - 1\} \backslash \{n_i^l = \bar{n} + n_i^k\}, N - |\{n_i^l = \bar{n} + n_i^k\}| \right) - V(\mathbf{n}_i, N) \right]}_{\text{Large firm } k \neq i \text{ innovates}} \\
& + \underbrace{X \left[ V\left( \mathbf{n}_i \cup \Big\{ \min\left\{\bar{n}, \bar{n} + \min(\mathbf{n}_i)\right\} \Big\}, \min(N+1, \bar{N}) \right) - V(\mathbf{n}_i, N) \right]}_{\text{Entry}} \Bigg\} + \underbrace{\dot{V}(\mathbf{n}_i, N)}_{\text{Drift}}
\end{aligned}
$$

# **Equilibrium:** Dynamic Block (2/2)

- **Industry state:** $\Theta \equiv (N, \vec{n})$, where $\vec{n} \equiv \{0, \ldots, \overline{n}\}^{N-1}$ is set of gaps of followers wrt industry leader ("$L$").
- **Small firm's problem:**

$$rV^e(\Theta) = \max_X \Big\{ - \underbrace{\nu X^\epsilon Y}_{\text{R\&D costs}} + \underbrace{XV\Big(\{\mathbf{n}_L - \overline{n}\} \cup \{-\overline{n}\}, N+1\Big)}_{\text{Become a large firm}} - \underbrace{\tau V^e(\Theta)}_{\text{Exit}}$$

$$+ \underbrace{\sum_{\Theta'} p(\Theta, \Theta')\Big(V^e(\Theta') - V^e(\Theta)\Big)}_{\text{Industry transitions}} \Big\} + \underbrace{\dot{V}^e(\Theta)}_{\text{Drift}}$$

- **Entrepreneur's problem:**

$$\rho S = \max_e \Big\{ \underbrace{-\psi e^2 Y}_{\text{Creation cost}} + \underbrace{e \sum_{\Theta} V^e(\Theta)\mu(\Theta)}_{\text{Value of small firm}} \Big\}, \text{ where } \mu(\Theta) \equiv \text{ Mass of industries in state } \Theta$$

- **BGP growth rate:**

$$g = \ln(1+\lambda)\left[\sum_{\Theta} z_L(\Theta)\mu(\Theta)\right], \qquad z_L(\Theta) \equiv \text{ Arrival rate of leader innovation in industry } \Theta$$

# **Equilibrium:** Market Clearing

- **Labor Market Clearing (LMC):**
  - Labor input choices:

$$l_{ijt} = \sigma_{ijt} \left( \frac{M_{ijt}}{M_t} \right)^{-1} \quad \text{[Large firms]} \qquad \text{and} \qquad l_{ckt} = \sigma_{cjt} \left( \frac{1}{M_t} \right)^{-1} \quad \text{[Small firms]}$$

  - LMC gives aggregate markup ($M_t$) as the inverse of the labor share:

$$M_t \equiv \left[ \int_0^1 \left( \sigma_{cjt} + \sum_{i=1}^{N_{jt}} \sigma_{ijt} M_{ijt}^{-1} \right) \mathrm{d}j \right]^{-1} = \left( \frac{w_t}{Y_t} \right)^{-1}$$

- **Product Market Clearing:**

$$Y_t = C_t + \underbrace{\int_0^1 \sum_{i=1}^{N_{jt}} \chi z_{ijt}^{\phi} Y_t dj}_{\text{R\&D by large firms}} + \underbrace{\int_0^1 \sum_{i=1}^{N_{jt}} \chi_a \omega_{ijt}^{\phi_a} Y_t dj}_{\text{ADV by large firms}} + \underbrace{\int_0^1 F_{jt} \nu X_{jt}^{\epsilon} Y_t dj}_{\text{R\&D by small firms}} + \underbrace{\psi e_t^2 Y_t dj}_{\text{Entrepreneur costs}}$$

# Calibration

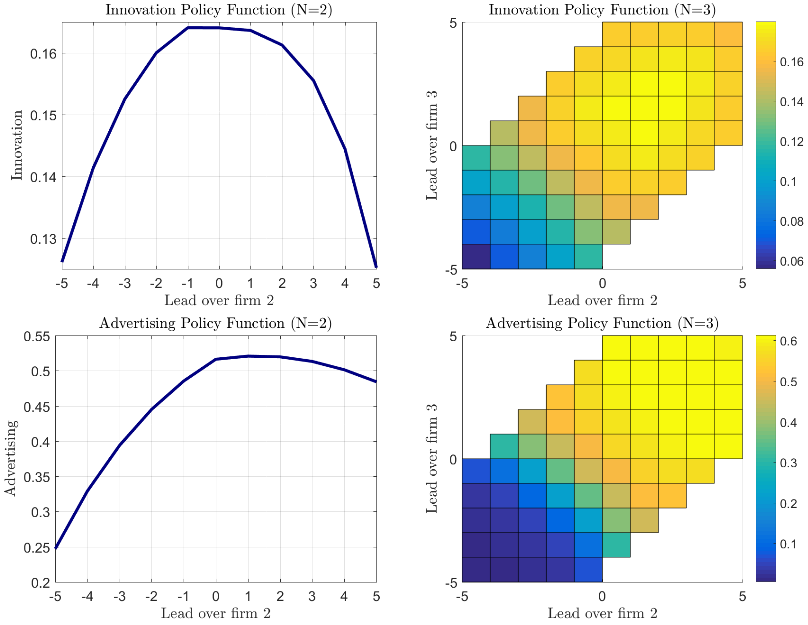
# Calibration

▸ Moments Details ▸ Jacobian Matrix

- Proxy innovation with average patent citations (from NBER Patent Database Project).
- Target two inverse-U relationships: (i) R&D and market share ($\sigma$); (ii) ADV and market share ($\sigma$).

| Parameter | | Value | Target | Model | Data | Data Source |
|---|---|---|---|---|---|---|
| Innovation step | $\lambda$ | 0.143 | Growth rate | 2.2% | 2.2% | US BEA |
| Rel. quality (small firms) | $\zeta$ | 0.770 | Labor share | 63.9% | 65.2% | Karabarbounis-Neiman (2013) |
| | | | Average markup | 34.7% | 35.0% | De Loecker et al. (2020) |
| EoS (large vs small) | $\gamma$ | 2.312 | SD(markups) | 0.46 | 0.35 | De Loecker et al. (2020) |
| EoS (between large firms) | $\eta$ | 11.13 | Average profitability | 13.2% | 14.4% | Compustat |
| R&D scale (large firms) | $\chi$ | 59.42 | R&D share of GDP | 2.7% | 2.4% | NSF |
| R&D scale (small firms) | $\nu$ | 2.665 | SD(leader rel. quality) | 0.17 | 0.22 | NBER Patent + Compustat |
| R&D curvature (large) | $\phi$ | 4.604 | Top point(Innov., $\sigma$) | 0.46 | 0.51 | NBER Patent + Compustat |
| R&D curvature (small) | $\epsilon$ | 4.526 | Intercept(Innov., $\sigma$) | 1.03 | 0.63 | NBER Patent + Compustat |
| | | | Avg. leader rel. quality | 0.50 | 0.75 | NBER Patent + Compustat |
| ADV cost scale | $\chi_a$ | 0.052 | ADV share of GDP | 2.3% | 2.2% | Coen Structured ADV Dataset |
| | | | Intercept(ADV, $\sigma$) | 8.93 | 6.26 | Compustat |
| ADV cost curvature | $\phi_a$ | 2.429 | Top point(ADV, $\sigma$) | 0.48 | 0.53 | Compustat |
| Exit rate (small firms) | $\tau$ | 0.115 | Firm entry rate | 11.5% | 11.5% | BDS (US Census) |
| Cost of small firm creation | $\psi$ | 0.093 | Set to obtain $F = 1$ in eq'm | | | |

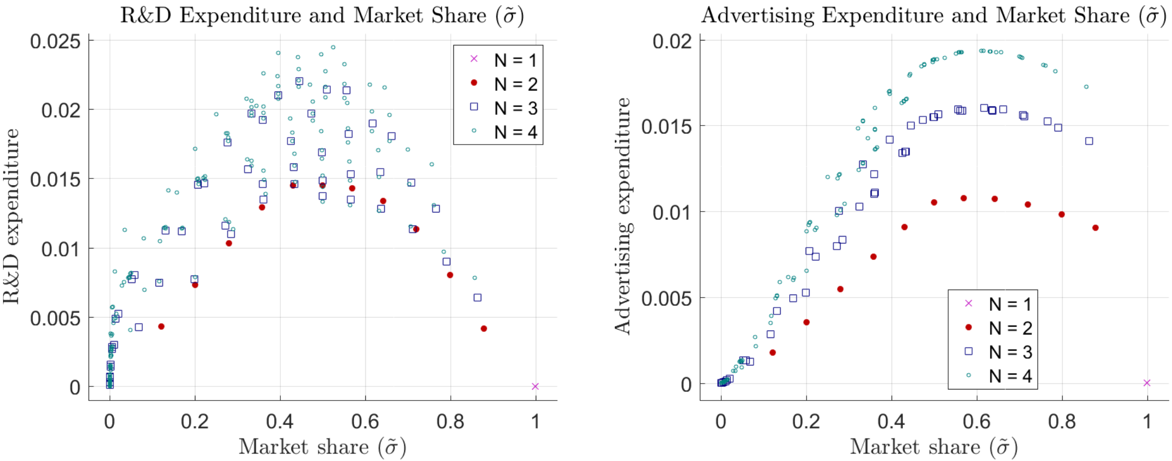
# **Behavior of R&D and ADV:** Firm Level




- **R&D:** (top panels)
  - Highest around neck-to-neck states.
  - This is where firms have highest marginal gains in profits.
- **ADV:** (bottom panels)
  - Leaders advertise more than followers.
  - This shifts demand toward more productive firms.

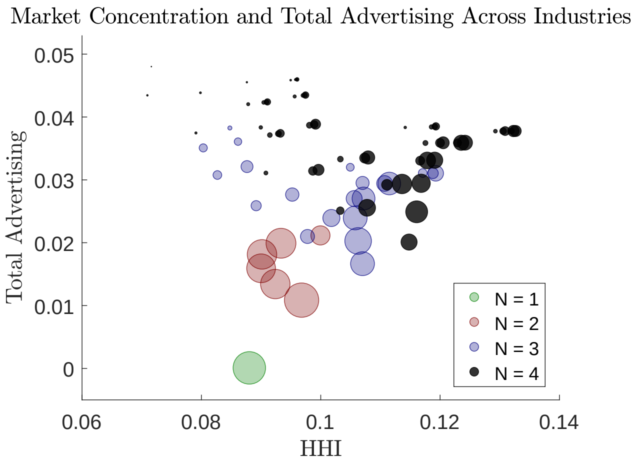
More: ▸ Correlation Matrix

# **Behavior of R&D and ADV:** Within Industries




- Within industries:
  - R&D and ADV are inverse-U shaped in market share relative to other large firms ($\tilde{\sigma}$).
  - *Recall:* Intercept and top points are calibration targets.

# **Behavior of R&D and ADV:** Across Industries




- Across industries:
  1. Industries with more large firms have higher investment in ADV (and R&D).
  2. More concentrated industries have higher ADV expenditures.

# Aggregate Effects of Advertising

# Advertising Shutdown

| | Benchmark | ADV Shutdown | % change |
|---|---|---|---|
| Growth rate | 2.200% | 2.246% | 2.08% |
| R&D/GDP | 2.704% | 2.825% | 4.46% |
| Large firm innovation | 0.392 | 0.461 | 17.45% |
| Small firm innovation | 0.109 | 0.125 | 15.06% |
| Average markup | 34.7% | 25.0% | -27.95% |
| Std. dev. markup | 0.465 | 0.345 | -25.82% |
| Labor share | 0.639 | 0.665 | 3.95% |
| Average # large firms per industry | 2.864 | 3.338 | 16.57% |
| Mass of small firms | 1.000 | 1.367 | 36.71% |
| Initial output | 1.432 | 1.378 | -3.75% |

- **Experiment:** Prohibit ADV expenditures. ▸ Other Variables of Interest

  1. Economic growth goes up ⇒ R&D and ADV are substitutes (Cavenaile and Roldan-Blanco (2021)).
  2.
  3.
  4.

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| Average markup | 34.7% | 25.0% | -27.95% |
| Std. dev. markup | 0.465 | 0.345 | -25.82% |
| Labor share | 0.639 | 0.665 | 3.95% |
| Average # large firms per industry | 2.864 | 3.338 | 16.57% |
| Mass of small firms | 1.000 | 1.367 | 36.71% |
| Initial output | 1.432 | 1.378 | -3.75% |

- **Experiment:** Prohibit ADV expenditures. Other Variables of Interest

1. Economic growth goes up
2. Markups and dispersion go down $\Rightarrow$ ADV responsible for $\approx$28% of avg. and $\approx$26% of dispersion.
3. 
4. 

# Advertising Shutdown

| | Benchmark | ADV Shutdown | % change |
|---|---|---|---|
| Growth rate | 2.200% | 2.246% | 2.08% |
| R&D/GDP | 2.704% | 2.825% | 4.46% |
| Large firm innovation | 0.392 | 0.461 | 17.45% |
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- **Experiment:** Prohibit ADV expenditures. Other Variables of Interest
  1. Economic growth goes up
  2. Markups and dispersion go down
  3. More competition ⇒ More large firms and more small firms.
  4.

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- **Experiment:** Prohibit ADV expenditures. ▸ Other Variables of Interest
  1. Economic growth goes up
  2. Markups and dispersion go down
  3. More competition
  4. **But...** Initial output goes down! *Why?*

# Advertising Shutdown

- Effects on initial output are a priori ambiguous. ⇒ In net terms, initial output ↓
  - (+) ADV shutdown decreases markups and their dispersion...
  - (–) ... but also reallocates resources away from productive large firms...
  - (–) ... and decreases extrinsic quality of cheaper-to-produce varieties.

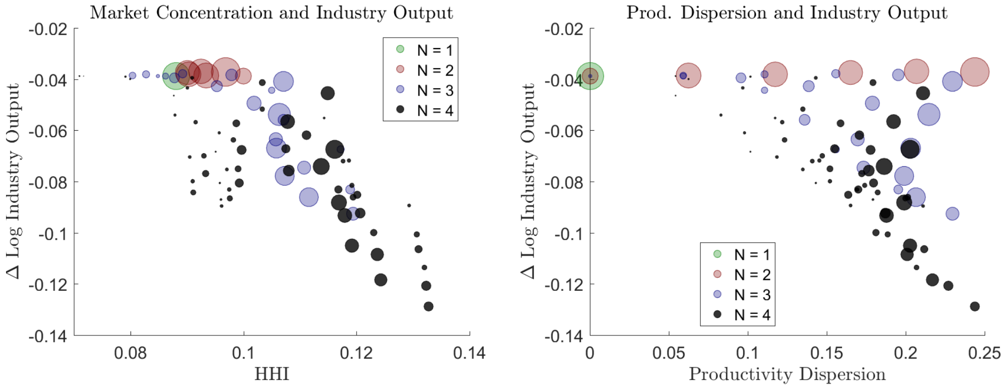


**Figure:** Industries with higher concentration/dispersion see larger decreases in output from ADV shutdown.

# Advertising Shutdown

- Effects on initial output are a priori ambiguous. $\Rightarrow$ **In net terms, initial output $\downarrow$**
  - **(+)** ADV shutdown decreases markups and their dispersion...
  - **(–)** ... but also reallocates resources away from productive large firms...
  - **(–)** ... and decreases extrinsic quality of cheaper-to-produce varieties.

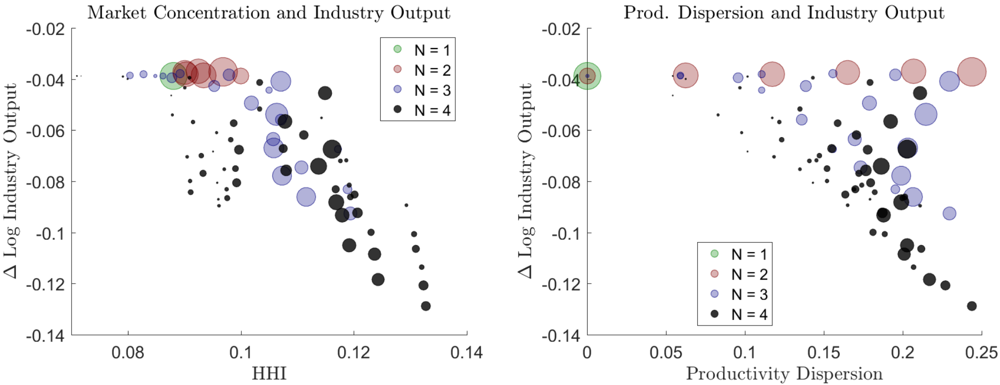


**Figure:** Industries with higher concentration/dispersion see larger decreases in output from ADV shutdown.

# Advertising Shutdown

- Change in welfare between BGPs: ▸ Derivation

$$\Delta W = \frac{1}{\rho}\Bigg[ \underbrace{\underbrace{\Delta \sum_{\Theta} f(\Theta)\mu(\Theta)}_{\text{(1)} \propto \text{ Avg. market share of large firms}} - \Delta \underbrace{\ln\left(\frac{w_0}{Y_0}\right)}_{\text{(2) Labor share}}}_{=\Delta \ln(Y_0)} + \Delta \underbrace{\ln\left(\frac{C_0}{Y_0}\right)}_{\text{(3) Consumption share}} \Bigg] + \frac{1}{\rho^2}\Delta \underbrace{g}_{\text{(4) Output growth}}$$

- Further decompose Consumption-Equivalent Welfare (CEW) in: ▸ CEW Formula ▸ Insights from Planner
  - **Static CEW:** Counterfactual keeping distribution $\mu(\Theta)$ fixed.
  - **Dynamic CEW:** Full model letting innovation respond (i.e. $\mu(\Theta)$ adjusts).

| | Static CEW | Dynamic CEW |
|---|---|---|
| (1) Market share of large firms | -1.98% | 0.05% |
| (2) Labor share | -3.19% | -3.80% |
| (3) Consumption share | 2.40% | 2.16% |
| (4) Output growth | 0.00% | 1.15% |
| **Total** | **-2.83%** | **-0.54%** |

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# Should We Tax or Subsidize Advertising?

- Despite ADV is socially useful $\Rightarrow$ Optimal ADV tax is positive and high! ($\approx$ 64%).

| | Benchmark | Optimal Tax (64.6%) | % change |
|---|---|---|---|
| Growth rate | 2.200% | 2.220% | 0.92% |
| R&D/GDP | 2.704% | 2.736% | 1.16% |
| Large firm innovation | 0.392 | 0.411 | 4.83% |
| Small firm innovation | 0.109 | 0.115 | 5.83% |
| Average markup | 34.7% | 31.3% | -9.80% |
| Std. dev. markup | 0.465 | 0.425 | -8.64% |
| Labor share | 0.639 | 0.648 | 1.31% |
| Average # large firms per industry | 2.864 | 2.995 | 4.57% |
| Mass of small firms | 1.000 | 1.086 | 8.65% |
| Initial output | 1.432 | 1.409 | -1.61% |
| **Overall CEW change** | | **0.612%** | |

- Even with high rates $\rightarrow$ Highly productive firms still invest more in ADV compared to others.
- Loss in output dominated by:

  (i) higher $g$; (ii) more business dynamism; (iii) less resources wasted on "rat race".

# Conclusion

- We study the interaction between R&D, advertising and market structure:
  - Fit inverse-U shaped relationships: (i) R&D and market share; (ii) ADV and market share.
  - In the model, ADV has positive and negative effects:
    - **(+)** Reallocates resources toward more efficient firms, decreasing misallocation.
    - **(–)** Decreases economic growth, increases markups, and hurts competition.
  - In net terms $\rightarrow$ ADV is socially useful (banning it implies 0.54% loss in CEW).
  - Yet, ADV should be taxed (optimal $\approx 64\%$):
    - Increases dynamic efficiency and reduces excessive spending due to “rat race”...
    - ...while still preserving the allocative benefits of ADV.

**Thank you!**

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  - Yet, ADV should be taxed (optimal $\approx 64\%$):
    - Increases dynamic efficiency and reduces excessive spending due to “rat race”...
    - ...while still preserving the allocative benefits of ADV.

**Thank you!**

# Appendix

# Appendix: Moments Details

Back to Calibration

1 **Inverse-U between R&D, ADV and relative sales:**

- Innovation in the data measured as average patent citations for each firm.
- Regress: (firm *i*, SIC-4 industry *j*, year *t*)

$$AvgCitations_{ijt} = \alpha_1 \sigma_{ijt} + \alpha_2 \sigma_{ijt}^2 + \boldsymbol{X}_{ijt}^\top \gamma + u_{ijt}$$
$$\ln\left(1 + xad_{ijt}\right) = \beta_1 \sigma_{ijt} + \beta_2 \sigma_{ijt}^2 + \boldsymbol{X}_{ijt}^\top \delta + u_{ijt}$$

where $\boldsymbol{X}_{ijt} \in \{$profitability, leverage, log R&D stock, firm age, time and industry FE, $\ldots\}$.

- Target $\alpha_1, \alpha_2 > 0$ and $\beta_1, \beta_2 > 0$ in the calibration.

2 **Relative quality of the leader:**

- Proxy quality with the stock of past patent citations.
- Define relative quality of the leader as the quality of the top firm divided by the sum of the qualities of the top four firms in its SIC4 industry.

# **Appendix:** Jacobian Matrix (1/2)

Back to Calibration

| | $\lambda$ | $\eta$ | $\chi$ | $\nu$ | $\zeta$ | $\phi$ |
|---|---|---|---|---|---|---|
| Growth rate | 0.866 | -0.052 | -0.248 | -0.031 | -0.309 | 2.029 |
| R&D/GDP | 0.021 | -0.121 | -0.191 | -0.054 | -1.119 | 1.021 |
| Advertising/GDP | -0.784 | -0.304 | 0.041 | -0.091 | -1.100 | 0.197 |
| Average markup | 0.115 | 0.002 | -0.001 | 0.002 | -0.210 | -0.009 |
| Std. dev. markup | 0.675 | 0.100 | -0.014 | 0.027 | -0.237 | -0.052 |
| Labor share | -0.023 | 0.010 | -0.001 | 0.002 | 0.174 | 0.001 |
| Entry rate | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| Avg profitability | 0.266 | 0.012 | 0.036 | 0.014 | -0.672 | -0.230 |
| Avg leader rel. quality | 0.497 | 0.139 | -0.033 | 0.072 | 0.260 | -0.089 |
| Std. dev. leader rel. quality | 0.371 | 0.181 | -0.058 | 0.106 | 0.367 | 0.233 |
| Intercept(innovation, rel. sales) | 0.288 | 0.007 | -0.157 | 0.036 | 0.109 | -0.028 |
| Top point(innovation, rel. sales) | 0.295 | 0.079 | 0.000 | 0.003 | 0.130 | -0.207 |
| Intercept(advertising, rel. sales) | -0.553 | -0.161 | 0.003 | -0.003 | -0.195 | -0.037 |
| Top point(advertising, rel. sales) | 0.336 | 0.152 | 0.011 | -0.021 | 0.086 | -0.075 |

**Notes:** This table reports reports the Jacobian matrix associated with the estimation of the baseline model. Each entry of the matrix reports the percentage change in each moment given one percent increase in each parameter.

# **Appendix:** Jacobian Matrix (2/2)

Back to Calibration

| | $\epsilon$ | $\chi_a$ | $\phi_a$ | $\gamma$ | $\psi$ | $\tau$ |
|---|---|---|---|---|---|---|
| Growth rate | 0.198 | 0.009 | 0.037 | -0.229 | -0.055 | -0.096 |
| R&D/GDP | 0.344 | 0.004 | 0.066 | -0.620 | -0.095 | -0.167 |
| Advertising/GDP | 0.636 | -0.283 | -0.558 | -0.331 | -0.158 | -0.279 |
| Average markup | -0.005 | -0.026 | -0.058 | -0.101 | 0.003 | 0.005 |
| Std. dev. markup | -0.165 | -0.089 | -0.262 | -0.331 | 0.048 | 0.084 |
| Labor share | -0.016 | 0.013 | 0.022 | 0.063 | 0.003 | 0.006 |
| Entry rate | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 1.000 |
| Avg profitability | -0.086 | -0.034 | -0.050 | -0.216 | 0.025 | 0.043 |
| Avg leader rel. quality | -0.485 | -0.036 | -0.124 | 0.264 | 0.125 | 0.220 |
| Std. dev. leader rel. quality | -0.579 | -0.039 | -0.163 | 0.324 | 0.184 | 0.324 |
| Intercept(innovation, rel. sales) | -0.181 | 0.036 | -0.077 | 0.039 | 0.062 | 0.109 |
| Top point(innovation, rel. sales) | -0.022 | -0.021 | -0.100 | 0.181 | 0.006 | 0.010 |
| Intercept(advertising, rel. sales) | -0.007 | 0.075 | 0.127 | -0.568 | -0.005 | -0.010 |
| Top point(advertising, rel. sales) | 0.119 | -0.055 | -0.149 | 0.318 | -0.037 | -0.064 |

**Notes:** This table reports reports the Jacobian matrix associated with the estimation of the baseline model. Each entry of the matrix reports the percentage change in each moment given one percent increase in each parameter.

# **Appendix:** Firm-Level Correlations

Back to Results

| | Markup | R&D | Advertising | SG&A | Profitability |
|---|---|---|---|---|---|
| Markup | 1.000 | | | | |
| R&D | 0.368 | 1.000 | | | |
| Advertising | 0.667 | 0.850 | 1.000 | | |
| SG&A | 0.548 | 0.956 | 0.967 | 1.000 | |
| Profitability | 0.737 | 0.671 | 0.719 | 0.724 | 1.000 |

**Notes:** Correlation coefficients in the estimated model between markups, R&D, advertising, SG&A and profitability at the firm level. SG&A is computed as the sum of advertising and R&D.

- Within firms:
  - R&D, ADV, profitability and markups are all positively correlated.

# **Appendix:** Advertising Shutdown (full table)

Back to ADV Shutdown

| | Benchmark | ADV Shutdown | % change |
|---|---|---|---|
| Growth rate | 2.200% | 2.246% | 2.08% |
| R&D/GDP | 2.704% | 2.825% | 4.46% |
| Advertising/GDP | 2.277% | 0.000% | -100.00% |
| Average markup | 34.7% | 25.0% | -27.95% |
| Std. dev. markup | 0.465 | 0.345 | -25.82% |
| Labor share | 0.639 | 0.665 | 3.95% |
| Average profitability | 0.132 | 0.122 | -7.52% |
| Large firm innovation | 0.392 | 0.461 | 17.45% |
| Small firm innovation | 0.109 | 0.125 | 15.06% |
| Output share of large firms | 0.422 | 0.423 | 0.35% |
| Average # large firms per industry | 2.864 | 3.338 | 16.57% |
| Mass of small firms | 1.000 | 1.367 | 36.71% |
| Initial output | 1.432 | 1.378 | -3.75% |

**Notes:** BGP values in benchmark calibration and counterfactual economy in which advertising is shut down.

# Appendix: Welfare Decomposition

Back to ADV Shutdown

- Welfare in BPG:

$$W = \frac{1}{\rho}\left(\ln\left(Y_0\right) + \ln\left(\frac{C_0}{Y_0}\right) + \frac{g}{\rho}\right)$$

with

$$\ln\left(Y_0\right) = \underbrace{\int_0^1 \ln\left(q_{cj0}\right) dj}_{\substack{\text{Fringe quality} \\ \textbf{[constant across BGPs]}}} + \underbrace{\sum_{\Theta} f(\Theta)\mu(\Theta)}_{\text{Average output large firms}} - \underbrace{\ln\left(\frac{w_0}{Y_0}\right)}_{\text{Labor share}}$$

where

$$f(\Theta) \equiv \frac{1}{\gamma - 1} \ln\left\{1 + \left(\sum_{i=1}^{N(\Theta)} \hat{\omega}_{i0}(\Theta)\left(\frac{y_{i0}}{\widetilde{y}_{c0}}(\Theta)\right)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta(\gamma-1)}{\gamma(\eta-1)}}\right\}$$

- Change in welfare between BGPs:

$$\Delta W = \frac{1}{\rho}\left(\Delta \sum_{\Theta} f(\Theta)\mu(\Theta) - \Delta \ln\left(\frac{w_0}{Y_0}\right) + \Delta \ln\left(\frac{C_0}{Y_0}\right)\right) + \frac{1}{\rho^2}\Delta g$$

# **Appendix:** Consumption-Equivalent Welfare

Back to ADV Shutdown

- Welfare in BPG:

$$W = \frac{1}{\rho}\left(\ln\left(C_0\right) + \frac{g}{\rho}\right)$$

- **Consumption-equivalent welfare (CEW):**

$$\ln\left(C_0^B\right) + \frac{g^B}{\rho} = \ln\left(C_0^A(1+\iota)\right) + \frac{g^A}{\rho} \quad \Leftrightarrow \quad \boxed{\iota = \frac{C_0^B}{C_0^A}\exp\left\{\frac{g^B - g^A}{\rho}\right\} - 1}$$

$\iota \equiv$ % change in lifetime consumption that the representative household of economy $A$ requires to remain indifferent with respect to economy $B$.

# Appendix: Insights from Planner's Solution

Back to ADV Shutdown

- Labor allocation $\Rightarrow$ **[SP]:** $l^*_{ijt} = \sigma^*_{ijt}$ and **[DE]:** $l_{ijt} = \sigma_{ijt} M_t / M_{ijt}$, with:

$$\textbf{[SP]:} \quad \sigma^*_{ijt} = \frac{\left(\hat{\omega}^*_{ijt}\right)^{\eta} \left(\sum_{k=1}^{N_{jt}} \left(\hat{\omega}^*_{kjt}\right)^{\eta} \left(\frac{q_{kjt}}{q_{ijt}}\right)^{\eta-1}\right)^{\frac{\gamma-\eta}{\eta-1}}}{\left(\frac{q_{cjt}}{q_{ijt}}\right)^{\gamma-1} + \left(\sum_{k=1}^{N_{jt}} \left(\hat{\omega}^*_{kjt}\right)^{\eta} \left(\frac{q_{kjt}}{q_{ijt}}\right)^{\eta-1}\right)^{\frac{\gamma-1}{\eta-1}}}$$

$$\textbf{[DE]:} \quad \sigma_{ijt} = \frac{\left(\hat{\omega}_{ijt}\right)^{\eta} \left(\sum_{k=1}^{N_{jt}} \left(\hat{\omega}_{kjt}\right)^{\eta} \left(\frac{q_{kjt}}{q_{ijt}}\right)^{\eta-1} \left(\frac{M_{ijt}}{M_{kjt}}\right)^{\eta-1}\right)^{\frac{\gamma-\eta}{\eta-1}}}{\left(\frac{q_{cjt}}{q_{ijt}}\right)^{\gamma-1} \left(M_{ijt}\right)^{\gamma-1} + \left(\sum_{k=1}^{N_{jt}} \left(\hat{\omega}_{kjt}\right)^{\eta} \left(\frac{q_{kjt}}{q_{ijt}}\right)^{\eta-1} \left(\frac{M_{ijt}}{M_{kjt}}\right)^{\eta-1}\right)^{\frac{\gamma-1}{\eta-1}}}$$

- Allocation of ADV:

$$\textbf{[SP]:} \quad \left(\frac{\eta}{\eta-1}\right) \frac{\sigma^*_{ijt}}{\sum_{k=1}^{N_{jt}} (1+\omega^*_{kjt})} \left[\frac{\sum_{k\neq i}(1+\omega^*_{kjt})}{1+\omega^*_{ijt}} - \sum_{k\neq i} \frac{\sigma^*_{kjt}}{\sigma^*_{ijt}}\right] = \frac{\chi_a \phi_a \left(\omega^*_{ijt}\right)^{\phi_a - 1}}{1 - \int_0^1 \chi_a \sum_{k=1}^{N_{jt}} (\omega^*_{kjt})^{\phi_a} dj}$$

$$\textbf{[DE]:} \quad \left[1 - \left(\frac{\eta-\gamma}{\gamma(\eta-1)}\right)\tilde{\sigma}_{ijt} - \left(\frac{(\gamma-1)\eta}{\gamma(\eta-1)}\right)\sigma_{ijt}\right] \frac{\sigma_{ijt}}{\sum_{k=1}^{N_{jt}} (1+\omega_{kjt})} \left(\frac{\sum_{k\neq i}(1+\omega_{kjt})}{1+\omega_{ijt}}\right) = \chi_a \phi_a \omega_{ijt}^{\phi_a - 1}$$

- DE is "statically efficient" (i.e. maximizes $Y$) when **(i)** $M_{ijt} = M_t = 1$ and **(ii)** $\omega_{ijt} = \omega^*_{ijt}, \forall (i,j)$.