International Trade and Innovation Dynamics with Endogenous Markups*

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Abstract

Over the last decades, the United States has experienced a secular increase in market concentration and markups, as well as a doubling of the trade-to-GDP ratio. Our paper argues that these trends could be linked, pointing out an "innovation feedback effect" of trade. Lower trade costs increase innovation incentives for large global firms, and as the winners of the ensuing innovation races increase their technological advantage over global competitors and local firms, concentration and markups rise. To make this point formally, we develop a dynamic general equilibrium trade model with endogenous markups and endogenous innovation. We calibrate our model to US manufacturing data, and show that an increase in trade openness (consistent with the one observed between 1989 and 2007) increases the aggregate markup by 3.5 percentage points. This increase is entirely due to firms' innovation response: without this response, markups would have fallen by 4 percentage points.

Keywords: International Trade, Markups, Innovation, R&D, Productivity.

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1 Introduction

Since the 1970s, the US economy has experienced significant structural changes: industries have become increasingly concentrated (Covarrubias *et al.*, 2019; Autor *et al.*, 2020b), aggregate markups and profits have increased (De Loecker *et al.*, 2020; Barkai, 2020) and the labour share has declined (Karabarbounis and Neiman, 2014). At the same time, the US has also experienced a dramatic increase in trade openness. According to the World Bank, the trade-to-GDP ratio increased from 10.8% in 1970 to 26.3% in 2019. However, the academic literature has largely been silent on the possible connections between globalisation and the secular increase in concentration and market power.

Indeed, the potential effects of trade on competition are unclear. Traditionally, economists have emphasised the pro-competitive effects of trade, as import competition forces domestic firms to lower their markups. On the other hand, exporters might increase their markups and low-markup domestic firms could lose market share, leading to an ambiguous aggregate effect (Arkolakis *et al.*, 2019). In this paper, we propose a new dynamic link between trade and competition, which we call the "innovation feedback effect", and argue that it can account for some of the observed trends. Precisely, we argue that lower trade costs increase the stakes in innovation races between global firms. The winners of these innovation races increase their technological advantage over global and local rivals, enabling them to charge higher markups. As a result, lower trade costs lead to higher concentration and markups, and to a polarisation of the markup distribution.

We find suggestive evidence for this polarisation in the data: the distribution of industry-level markups in US manufacturing has become increasingly dispersed, and the industries with the most extreme changes in markups have seen the largest increases in trade openness.

To analyse these forces in greater detail, we develop a two-country general equilibrium trade model with endogenous markups and innovation. There is a continuum of industries. In each of them, three types of firms interact: a large Home firm (the "Home leader"), a large Foreign firm (the "Foreign leader"), and a competitive fringe of small domestic firms. Leaders face a variable cost of exporting, while the fringes do not participate in trade. The Home and Foreign leaders in each industry engage in Bertrand competition, as in Atkeson and Burstein (2008), while fringe firms price at marginal cost. As a result, each leader's markup is increasing in its relative productivity with respect to its rivals.

Leaders can also invest into innovation, which delivers stochastic productivity increases.[1]

[1] While fringe firms do not innovate, they may catch up to the leaders at an exogenous rate.

By increasing their productivity, successful innovators increase their market share, profits and markups. Crucially, leaders' profits functions are S-shaped in relative productivity. That is, the marginal gains from innovation are largest for firms which are technologically close to their competitors, and smallest for firms which are far behind or far ahead.

In this setup, lower trade costs have important implications for firm innovation and markups. In particular, lower trade costs raise the stakes in the innovation race between the Home and the Foreign leader (formally, they accentuate the S-shape of leader profit functions). Therefore, innovation incentives increase, and do so most strongly for leaders which are technologically close to their competitors. As these firms pull away from their rivals, the productivity distribution becomes more polarised, with a large share of industries in which one leader has a large advantage over all other firms. However, it is precisely in such concentrated industries that markups are highest. Thus, the innovation feedback effect explains that a fall in trade costs ultimately leads to an increase in both markup dispersion and in the aggregate markup, in line with the data.

To investigate the quantitative importance of the innovation feedback effect, we calibrate our model to reproduce the current state of the US manufacturing sector (targeting, in particular, the empirical distribution of industry-level markups). We then compare the resulting balanced growth path (BGP) equilibrium, with a trade-to-GDP ratio of 26% (its level in 2007) to an alternative BGP with higher trade costs, in which the trade-to-GDP ratio is only 15% (its value in 1989).

We find that the aggregate markup is 3.5 percentage points higher in the low-trade-cost BGP. To decompose the sources of this difference, we solve for the transition path from the high-trade-cost BGP to the low-trade-cost BGP, considering a one-time permanent surprise reduction in trade costs. We find that the innovation feedback effect is responsible for the entire markup increase. Indeed, on impact, when the productivity distribution is fixed, the fall in trade costs reduces the aggregate markup by 4.0 percentage points (mainly because domestic leaders lower their markups in the face of higher import competition). However, over time, the innovation response induces a polarisation of the productivity distribution that shifts the aggregate markup back up, raising it above its initial level.

We perform several robustness checks that confirm these results, and show that our model also matches several untargeted moments reasonably well (including the distribution of industry-level concentration ratios and import shares). Moreover, the model manages to reproduce the shift in the empirical industry-level markup distribution between 1989 and 2007. On the whole, these results suggest that trade might indeed have been one contributor to the rise of markups in the US manufacturing sector.

Finally, we analyse the welfare implications of our model. As trade increases innovation incentives, transitioning from the high-trade-cost BGP to the low-trade-cost BGP raises the long-run growth rate, by 0.14 percentage points per year. This explains that the consumption-equivalent welfare of the representative household rises substantially, by 15%. However, these gains are unequally shared, as profits increase more than wages. Moreover, higher markup dispersion is a source of greater misallocation of labour across firms.

Related literature A rapidly growing literature claims that the competitive landscape in the United States changed dramatically since the 1970s. For instance, influential papers have pointed out increases in markups (De Loecker *et al.*, 2020), markup dispersion (Edmond *et al.*, 2021), profits (Barkai, 2020), concentration rates (Covarrubias *et al.*, 2019), and a decline in the labour share (Autor *et al.*, 2020b; Kehrig and Vincent, 2021). While none of these trends is uncontroversial,[2] the bulk of the evidence does suggest that top firms have become more dominant in the US economy. These trends also hold for the manufacturing sector, which is the focus of our paper.[3]

There is a lively debate about the potential causes of these trends. Prominent explanations include changes in technology (De Ridder, 2019; Akcigit and Ates, 2022; Cavenaile *et al.*, 2021; De Loecker *et al.*, 2021; Ganapati, 2021) and changes in antitrust policy (Gutiérrez and Philippon, 2022). Globalisation has received less attention as a potential driving force. However, we claim that the dynamic effects of trade on innovation could have played a role in the increase in markups and markup dispersion. Furthermore, we find that this rise is due to within-industry forces, in line with the empirical literature (Baqaee and Farhi, 2020; De Loecker *et al.*, 2020; Autor *et al.*, 2020b; Kehrig and Vincent, 2021).

Besides the aforementioned studies on recent trends, our paper relates to a vast literature on the link between trade and competition. The theoretical literature, dating back at least to Krugman (1979), shows that trade lowers the markups of domestic firms, but increases the markups of exporters (Melitz and Ottaviano, 2008; Arkolakis *et al.*, 2019). The empirical literature finds mixed results (e.g. De Loecker and Warzynski, 2012; De Loecker *et al.*, 2016; Brandt *et al.*, 2017; Feenstra and Weinstein, 2017, De Loecker and Eeckhout, 2018).[4]

[2] Basu (2019) and Bond *et al.* (2021) critically discuss markup and profit measurement issues.

[3] Amiti and Heise (2021) recently pointed out that the sales concentration measures of the US Census Bureau only measure concentration among US firms (i.e., they include exports by US firms and do not include imports). Using micro-level trade data, they compute an alternative market-level concentration rate (excluding exports and including imports), and show that this rate did not increase since the 1990s.

[4] A smaller branch of this literature has studied the effect of trade on markup dispersion and misallocation

Our paper makes a contribution by focusing on the interaction between markups and innovation, while the literature has generally abstracted from innovation and considered fixed productivity distributions.

A small number of papers has explicitly considered the interaction between trade, innovation and competition (Impullitti and Licandro, 2016; Aghion *et al.*, 2022; Lim *et al.*, 2018; Impullitti *et al.*, 2022). Our model proposes a mechanism that is fundamentally different from the ones described in these papers, namely the fact that trade leads to a polarisation of the productivity distribution. We also show that this mechanism is quantitatively relevant for the US manufacturing sector.[5]

Finally, our paper is also related to an extensive literature on the effect of trade on innovation, dating back to Grossman and Helpman (1991) and Rivera-Batiz and Romer (1991). Important recent contributions include Baldwin and Robert-Nicoud (2008), Sampson (2016), Akcigit *et al.* (2021), Hsieh *et al.* (2019), Perla *et al.* (2021) and Bloom *et al.* (2020).[6] While the theoretical effect of trade on growth is ambiguous, the quantitative results of these papers suggest that positive effects prevail and that they are large. This is consistent with evidence from empirical studies, which generally find that exposure to trade increases innovation and technology adoption (Lileeva and Trefler, 2010; Bustos, 2011; Coelli *et al.*, 2022; Chen and Steinwender, 2021).[7] We contribute to this literature by exploring the feedback effect of changes in innovation on markups.

The remainder of the paper is structured as follows. Section 2 presents stylised facts on markups, markup dispersion and trade in the US manufacturing sector. Section 3 lays out our model and discusses its main features. Section 4 presents our calibration strategy, our quantitative results and robustness checks. Section 5 analyses the welfare consequences of changes in trade costs. Finally, Section 6 concludes.

(Epifani and Gancia, 2011; Edmond *et al.*, 2015; Asturias *et al.*, 2019; Weinberger, 2020).

[5]Impullitti and Licandro (2016) is probably closest to our work, combining an endogenous growth model with Cournot competition. In contrast to us, they find that markups decrease after a trade liberalisation: markups on domestic sales fall, markups on exports increase, and as domestic firms have higher market shares, the former effect dominates. In their model, trade also increases innovation incentives, leading to higher productivity growth. However, because all firms grow at the same rate, this change has no feedback effect on markups. Instead, in our model, the key feature is that changes in innovation incentives are heterogeneous across firms. This triggers a polarisation in the markup distribution that overturns the initial pro-competitive effect of trade. As we will show in Section 2, this polarisation is in line with the data.

[6]In particular, our model shares some features with Akcigit *et al.* (2021), who consider the interaction between trade policy and innovation in an economy with exogenous markups.

[7]There are important qualifications to this statement, especially for the effects of greater import competition, where the evidence is mixed. For instance, Bloom *et al.* (2016) find a positive effect of Chinese import competition on innovation for European textile firms, while Autor *et al.* (2020a) find a negative effect for US manufacturing firms. Shu and Steinwender (2018) provide an overview of this literature.

2 Markups and trade in US manufacturing

Our paper claims that trade has a dynamic effect on competition through innovation, which we call the "innovation feedback effect". This effect is based on the idea (formalised in Sections 3 and 4) that innovation incentives increase in a more open economy. Innovation has an element of randomness. In some industries, domestic firms will come up with decisive innovations, increasing their market shares and markups. In other industries, foreign firms are more successful innovators, and domestic firms will experience declining market shares and markups. Our model suggests that the net effect of these developments is an increase in markup dispersion and in the aggregate markup.

In this section, we present some motivating evidence for this narrative. First, we show that there has been a substantial increase in the dispersion of industry-level markups in the US manufacturing sector. Second, we present some suggestive evidence for this increase in dispersion being linked to an increase in trade openness. We will return to this evidence when we calibrate our model.

2.1 Data

To measure markups, we use the Compustat dataset, including all publicly listed firms in the United States. We compute firm-level markups as in De Loecker *et al.* (2020).[8] Our main focus, however, is on industry-level markups. Following Edmond *et al.* (2021), we define these as the cost-weighted average of firm-level markups within each NAICS 6-digit industry. We drop industries which have on average data for less than 3 firms per year.

To measure trade flows, we rely on industry-level import and export data constructed in Schott (2008).[9] Furthermore, to measure total sales by all domestic firms in an industry, we use the NBER-CES Manufacturing Industry Database.[10] With this data, we compute an index of trade openness as

$$\text{Trade openness}_{i,t} = \frac{\text{Exports}_{i,t} + \text{Imports}_{i,t}}{\text{Value of Shipments}_{i,t} - \text{Exports}_{i,t} + \text{Imports}_{i,t}}. \quad (1)$$

[8] Precisely, we use the production function elasticities estimated by De Loecker *et al.* (2020), published in the replication materials of their paper, and compute markups as the ratio of the output elasticity of variable inputs (cost of goods sold) to the sales share of variable inputs.

[9] The data can be downloaded at https://faculty.som.yale.edu/peterschott/international-trade-data/ for every year between 1989 and 2017.

[10] The database is accessible online at https://www.nber.org/research/data/nber-ces-manufacturing-industry-database.

That is, the trade openness of industry i in year t is given by the ratio of total trade to sales on the domestic market. Between 1989 and 2007, this index increased for more than 90% of the 96 manufacturing industries for which we have data in both years, showing that the US manufacturing sector has experienced a substantial increase in openness.

2.2 Markup dispersion and trade openness in US manufacturing

Figure 1 plots the distribution of markups across industries in 1989 and in 2007. The figure shows a striking shift of the cumulative distribution function: in 2007, there are both more low and more high-markup industries than there were in 1989. The coefficient of variation of the markup distribution (i.e., the standard deviation normalised by the mean) increased from 0.19 in 1989 to 0.28 in 2007.[11]

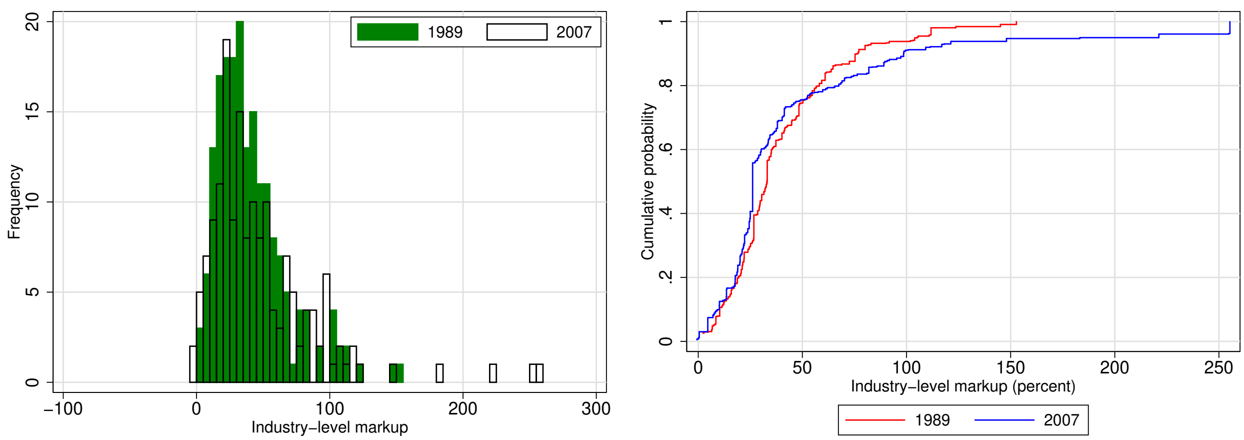

Figure 1: Distribution of industry-level markups in 1989 and 2007.

Notes: Industry-level markups are computed as indicated in the main text. Industries are defined at the NAICS 6-digit level, and excluded if they have on average less than 3 firms between 1989 and 2017. The left-hand side graph shows the raw distribution, while the right-hand side graph plots the cumulative distribution functions obtained when weighting by an industry's value of shipments.

Our theory of the innovation feedback effect provides one potential explanation for this increase in dispersion. The theory suggests that more trade leads to higher markups in some industries (the ones that are increasingly dominated by US firms) and lower markups in others (the ones that are increasingly dominated by foreign firms). To test this idea in the data, we divide industries into terciles according to the magnitude of their change in markups, measured by the difference between industry-level markups in 2007 and in 1989.

[11] These findings are related to Edmond *et al.* (2021), who document an increase in the dispersion of markups across all firms (while we focus here on the dispersion across industries).

Table 1 shows that trade openness has increased most in the lowest and highest tercile of markup increases. On the other hand, industries with intermediate markup increases have experienced much lower increases in trade openness.

Table 1: Changes in trade openness and changes in markups.

	Δ Markup < -0.02	$-0.02 < \Delta$ Markup < 0.08	Δ Markup > 0.08
Δ Trade openness	0.319	0.064	0.218
N. observations	32	32	32

Notes: For this table, we divide industries in three groups according to their changes in markups between 1989 and 2007, and compute the mean change in trade openness for each tercile. Means are weighted by industry value of shipments in 1989. Changes in trade openness are winsorized above at the 5% level.

Table 2 shows the same finding in regression form, regressing changes in trade openness on the level and the square of changes in industry markups. The coefficient of the square term is positive and statistically significant, pointing again to a higher increase in trade openness in industries with more extreme changes in markups. The result is robust to controlling for industry-level changes in sales.

Table 2: Changes in trade openness and changes in markups: regression evidence.

Dep. variable: Δ Trade openness		
	(1)	(2)
Δ Markups	-0.450^{**}	-0.367^{*}
	(0.210)	(0.194)
$(\Delta\text{Markups})^2$	0.256^{***}	0.248^{***}
	(0.090)	(0.085)
Δ ln Sales		-0.507^{***}
		(0.181)
R^2	0.11	0.16
N. observations	96	96

Notes: All variables refer to changes between 1989 and 2007, and are standardised to have mean 0 and standard deviation 1. Δ ln sales is the log change in an industry's value of shipments between 1989 and 2007. All regressions are weighted by industry value of shipments in 1989.

Obviously, the stylised facts presented in this section are just correlations, and do not prove any causal relationship between changes in trade openness and changes in markups. However, they provide suggestive evidence that there could be a link between both outcomes. In the next section, we present a model which formalises one particular innovation-driven channel that could explain this association.

3 Model

To analyse the interplay of trade, markups and innovation, we combine a model of trade between symmetric countries with two standard frameworks in the literature: the Atkeson and Burstein (2008) model of oligopolistic competition and a Schumpeterian endogenous growth model (for a review of this literature, see Aghion *et al.*, 2014).

3.1 Environment

Preferences Time is continuous, infinite, and indexed by $t \in \mathbb{R}_+$. The world consists of two large open economies, labeled Home (H) and Foreign (F). Each economy is populated by a representative household with discount rate $\rho > 0$. The representative household of country k is endowed with a fixed amount of time $\boldsymbol{L}^k$ each instant, which she supplies inelastically in her country's labour market. We assume throughout that both countries have the same labour endowment (i.e., $\boldsymbol{L}^H = \boldsymbol{L}^F = \boldsymbol{L}$).

The representative household's intertemporal utility function is

$$\boldsymbol{U}_0^k = \int_0^{+\infty} e^{-\rho t} \ln \boldsymbol{C}_t^k \mathrm{d}t, \tag{2}$$

where $\boldsymbol{C}_t^k$ stands for the consumption of a non-tradable final good. Consumption decisions are subject to the flow budget constraint $\dot{\boldsymbol{A}}_t^k \leq r_t^k \boldsymbol{A}_t^k + w_t^k \boldsymbol{L} - \boldsymbol{P}_t^k \boldsymbol{C}_t^k$, with $A_0^k > 0$ given. Here, w_t^k is the wage in country k, and $\boldsymbol{P}_t^k$ is the price of the final good. The household owns all domestic firms, and there are no international capital flows. Thus, the stock of wealth $\boldsymbol{A}_t^k$ equals the value of country-k assets, and the rate of return r_t^k is a priori country-specific.

Technology and competition The final good in each country is produced by a large number of firms operating under perfect competition. They produce the final good by assembling the output of a measure-one continuum of industries indexed by $j \in [0, 1]$, with a Cobb-Douglas technology:

$$\boldsymbol{Y}_t^k = \exp \left[\int_0^1 \ln Y_{j,t}^k \, \mathrm{d}j \right], \tag{3}$$

where $Y^k_{j,t}$ stands for the quantity of industry-j output used in country k. In each country k, the output of industry j is assembled using three intermediates. One intermediate is produced by a large Home firm (henceforth, the Home leader), and another one by a large Foreign firm (the Foreign leader). The third intermediate is produced by a country-specific competitive fringe. The fringe can be thought of as a large number of firms operating under perfect competition. The three intermediates are aggregated in a CES fashion, so that

$$Y^k_{j,t} = \left[(\omega_H)^{\frac{1}{\eta}} \left(y^k_{jH,t}\right)^{\frac{\eta-1}{\eta}} + (\omega_F)^{\frac{1}{\eta}} \left(y^k_{jF,t}\right)^{\frac{\eta-1}{\eta}} + (\omega_C)^{\frac{1}{\eta}} \left(y^k_{jC_k,t}\right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \text{with } \eta > 1. \quad (4)$$

Here, $y^k_{jH,t}$, $y^k_{jF,t}$ and $y^k_{jC_k,t}$ stand for the intermediates sold in country k by the Home leader, the Foreign leader and the domestic competitive fringe of industry j.[12] η is the elasticity of substitution between intermediates, and as it is larger than 1, it is easier to substitute between intermediates than between industries (where our Cobb-Douglas assumption imposes an elasticity of substitution of 1). Finally, the weights $\{\omega_c\}$ represent the quality of intermediates sold by producer c, holding $\sum_c \omega_c = 1$. We assume that qualities are fixed over time, and for symmetry, we impose throughout $\omega_H = \omega_F$.

The production of intermediates uses a simple linear technology:

$$y^k_{jc,t} = q_{jc,t}\ell^k_{jc,t}, \quad \text{for } c \in \{H, F, C_H, C_F\},$$

where $\ell^k_{jc,t}$ is labour used by producer c of industry j for its production in country k. $q_{jc,t}$ denotes the productivity of producer c. Home and Foreign leaders can increase their productivity through innovation, as we will describe below. When intermediates are exported, they are subject to an iceberg trade cost $\tau > 1$, so that τy units must be shipped to the other country for y units to arrive.

In each industry, the Home and Foreign leaders interact strategically in a static Bertrand game. That is, at every instant and for each market, each leader chooses a price that maximises its profits given the prices charged by all other firms. The fringes do not behave strategically, and charge a price equal to their marginal cost.

Innovation Leaders can increase their productivity by investing into R&D. We assume that by paying a flow cost equal to $\chi_i z^{\psi_i} \mathbf{Y}^k_t$ units of the final good (with $\chi_i > 0$ and

[12]We assume that fringe firms do not export (i.e., industry j in Home does not use the Foreign fringe's intermediate). This would be an equilibrium outcome if there were a small fixed cost of exporting. In the data, not all firms export and those that do are on average larger (e.g. Bernard *et al.*, 2018).

$\psi_i > 1$), a leader generates a Poisson arrival rate of innovations z. A successful innovation improves the leader's productivity by a factor $1+\lambda$, where $\lambda > 0$. By contrast, we assume that in every industry, the fringes in Home and Foreign have the same productivity (i.e., $q_{jC_H,t} = q_{jC_F,t} \equiv q_{jC,t}$), and that fringe firms do not innovate. However, both fringe firms and leaders whose productivity is below that of their foreign counterpart benefit from technological spillovers. Precisely, we assume that at an exogenous Poisson rate $\zeta > 0$, both fringes and the lagging leader catch up with the highest productivity leader.

Industry-level outcomes in our model crucially depend on firms' relative productivities. Given our assumptions, relative productivities can be summarised by two integers. First, we define the technology gap of the Home leader with respect to the Foreign leader, $n_{j,t} \in \mathbb{Z}$, as holding

$$\frac{q_{jH,t}}{q_{jF,t}} = (1+\lambda)^{n_{j,t}}. \tag{5}$$

We say that Home is leading in industry j if $n_{j,t} > 0$, lagging if $n_{j,t} < 0$, and neck-to-neck with Foreign if $n_{j,t} = 0$. Second, we define the technology gap of the Home leader with respect to the fringe, $n_{Cj,t} \in \mathbb{N}$, as holding

$$\frac{q_{jH,t}}{q_{jC,t}} = (1+\lambda)^{n_{Cj,t}}. \tag{6}$$

As the fringe can never become more productive than the least productive leader, $n_{Cj,t}$ is always non-negative, and holds $n_{Cj,t} \geq n_{j,t}$.

In our model's Balanced Growth Path (BGP) equilibrium, the innovation choices of Home and Foreign leaders generate an invariant distribution of technology gaps (n, n_C) over industries. Assuming that laggards are subject to technological spillovers (captured by the parameter ζ) is necessary to ensure the existence of such an invariant distribution, as it prevents relative productivities from diverging to infinitely large values.

Entry and exit At every instant, incumbent leaders can be displaced by entrants. For each country-industry pair (k, j), there is one potential entrant at each instant, which can invest $\chi_e x^{\psi_e} Y_t^k$ units of the final good to generate a Poisson arrival rate of innovation x.[13] An entrant that generates an innovation displaces the incumbent leader, who exits forever. An entrant that fails to innovate exits forever. Entrant innovations are equivalent to incumbents' innovations, i.e., they improve the productivity of the incumbent leader by a factor $1+\lambda$.

[13]The assumption that R&D costs of incumbents and potential entrants grow linearly with GDP is necessary to guarantee the existence of a Balanced Growth Path.

Market clearing The final good of the economy is used for consumption and for R&D. Thus, the aggregate resource constraint is

$$\boldsymbol{C}_t^k + \boldsymbol{R}_t^k = \boldsymbol{Y}_t^k, \tag{7}$$

where $\boldsymbol{R}_t^k$ stands for aggregate R&D at time t. Labour market clearing in turn requires that labour demand from domestic producers (fringe and domestic leaders) in each country k equals domestic labour supply:

$$\int_0^1 \left(\ell_{jk,t}^k + \ell_{jC_k,t}^k + \ell_{jk,t}^{k'} \right) \mathrm{d}j = \boldsymbol{L}^k, \tag{8}$$

for all $k, k' \in \{H, F\}$ with $k' \neq k$.

3.2 Equilibrium

Our analysis mostly focuses on a symmetric Balanced Growth Path (BGP) equilibrium, where both countries have the same wage and GDP, and aggregate variables grow at a constant rate. We thus analyse the BGP equilibrium first, and postpone the discussion of shocks and transitions between BGPs until Section 4.4.

3.2.1 Pricing decisions, market shares and profits

Demand functions The representative household in each country k maximises utility (2) subject to the flow budget constraint and a no-Ponzi condition, taking the initial wealth level as given.[14] This yields the standard Euler equation:

$$\frac{\dot{\boldsymbol{C}}_t^k}{\boldsymbol{C}_t^k} = r_t^k - \rho. \tag{9}$$

Final goods firms demand intermediate quantities $\left(y_{jH,t}^k, y_{jF,t}^k, y_{jC_k,t}^k\right)_{j\in[0,1]}$ from domestic and foreign firms. Their cost-minimisation problem implies the demand functions

$$y_{jc,t}^k = \omega_c \left(\frac{p_{jc,t}^k}{P_{j,t}^k} \right)^{-\eta} \frac{\boldsymbol{P}_t^k \boldsymbol{Y}_t^k}{P_{j,t}^k}, \quad \text{where } P_{j,t}^k = \left[\sum_{c=H,C_k,F} \omega_c \left(p_{jc,t}^k \right)^{1-\eta} \right]^{\frac{1}{1-\eta}}, \tag{10}$$

[14]Wealth in the economy is equal to the value of all domestic firms. In equilibrium, each country must hold a transversality condition ensuring that the present discounted value of wealth is zero in the limit of time.

where $p^k_{jc,t}$ is the price of the intermediate produced by producer c in industry j for country k, and $P^k_{j,t}$ is the ideal price index of industry j in country k. By symmetry, aggregate GDPs are equal across countries. Thus, from now on we omit country superscripts for aggregate GDP, $\boldsymbol{Y}_t$, and normalise the common price of the final good to $\boldsymbol{P}_t = 1$.

Pricing decisions In each industry j, the Home and Foreign leaders compete in a static Bertrand game. That is, they choose the optimal price for their good (on the Home and Foreign markets), taking the prices charged by the fringe and by the other leader as given. As each industry is small with respect to the aggregate economy, leaders also take the aggregate wage and price index as given. However, they do realise that they have market power in their industry, and that their decisions affect the industry price indices $P^H_{j,t}$ and $P^F_{j,t}$.

The pricing problem of leaders is similar to Atkeson and Burstein (2008), and its solution is discussed in greater detail in Appendix B.1. Throughout, we describe equilibrium conditions for the Home market, but the ones for the Foreign market are analogous (as there is no interaction between markets, the leader's problem is separable across markets).[15]

The Home leader's optimal price on the Home market is

$$p^H_{jH,t} = \mu^H_{jH,t} \frac{w_t}{q_{jH,t}}, \quad \text{where} \;\; \mu^H_{jH,t} \equiv \frac{\frac{\eta}{\eta-1} - \sigma^H_{jH,t}}{1 - \sigma^H_{jH,t}}. \tag{11}$$

$\sigma^H_{jH,t}$ stands for the market share of the Home leader on the Home market. That is, the Home leader charges a markup $\mu^H_{jH,t}$ over its marginal cost of production, and this markup is an increasing function of its market share. This is because leaders with higher market shares effectively face a less elastic demand curve, and therefore have more market power. Formally, the market share of producer c in industry j and country k is defined as

$$\sigma^k_{jc,t} \equiv \frac{p^k_{jc,t} y^k_{jc,t}}{P^k_{j,t} Y^k_{j,t}} = \omega_c \left(\frac{p^k_{jc,t}}{P^k_{j,t}} \right)^{1-\eta}. \tag{12}$$

The Foreign leader's optimal price on the Home market is

$$p^H_{jF,t} = \mu^H_{jF,t} \frac{\tau w_t}{q_{jF,t}}, \quad \text{where} \; \mu^H_{jF,t} \equiv \frac{\frac{\eta}{\eta-1} - \sigma^H_{jF,t}}{1 - \sigma^H_{jF,t}}. \tag{13}$$

This condition is analogous to the one of the Home leader, except for the fact that the

[15] Following Atkeson and Burstein (2008), we abstract from equilibria with dynamic collusion.

Foreign leader's marginal cost includes the variable trade cost τ.

Finally, as the fringe operates under perfect competition, its price holds

$$p_{jC_H,t} = \frac{w_t}{q_{jC,t}}. \tag{14}$$

Equations (11) to (14) pin down the equilibrium markups and market shares in every industry j as a function of firms' relative productivities. Indeed, equation (12) shows that market shares only depend on relative prices, and the pricing equations show that relative prices only depend on market shares and relative productivities. Appendix C provides further details for the solution of this system of equations.

Note that as relative productivities are fully characterised by the technology gap $\underline{n} \equiv (n, n_C)$, industry-level markups and market shares only depend on $\underline{n}$ and on the parameters η, τ and $\{\omega_c\}$. Thus, we henceforth identify an industry by its technology gap $\underline{n}$.

Profits From the above, it is easy to show that the profits of the Home and Foreign leaders in each market k are

$$\Pi^k_{c,t}(\underline{n}) = \pi^k_c(\underline{n})\boldsymbol{Y}_t, \text{ where } \pi^k_c(\underline{n}) = \left[\frac{\eta}{\sigma^k_c(\underline{n})} - (\eta - 1)\right]^{-1}, \tag{15}$$

and $\sigma^k_c(\underline{n})$ is the market share of firm c in country k in an industry characterised by a technology gap $\underline{n}$. Thus, profits are an increasing function of market shares and scale linearly with aggregate GDP.

Labour market clearing Home labour is employed by Home leaders (for domestic production and exports) and the Home competitive fringe. Using the demand equation (10), we can show that their respective labour demands are given by

$$\ell^H_{H,t}(\underline{n}) = \frac{\sigma^H_H(\underline{n})}{\mu^H_H(\underline{n})}\frac{\boldsymbol{Y}_t}{w_t}, \qquad \ell^F_{H,t}(\underline{n}) = \frac{\sigma^F_H(\underline{n})}{\mu^F_H(\underline{n})}\frac{\boldsymbol{Y}_t}{w_t}, \qquad \ell^H_{C_H,t}(\underline{n}) = \sigma^H_{C_H}(\underline{n})\frac{\boldsymbol{Y}_t}{w_t}. \tag{16}$$

Imposing labour market clearing then yields

$$\frac{w_t\boldsymbol{L}}{\boldsymbol{Y}_t} = \sum_{n=-\infty}^{+\infty}\sum_{n_C=0}^{+\infty}\varphi_t(\underline{n})\left(\sigma^H_{C_H}(\underline{n}) + \left(\frac{\eta-1}{\eta}\right)\sum_{k=H,F}\frac{\sigma^k_H(\underline{n})(1-\sigma^k_H(\underline{n}))}{\eta(1-\sigma^k_H(\underline{n})) + \sigma^k_H(\underline{n})}\right), \tag{17}$$

where $\varphi_t(\underline{n})$ stands for the mass of industries with technology gap $\underline{n}$ at time t. The technology gap distribution across industries is endogenous, and will be derived below. Given this distribution, equation (17) pins down the aggregate labour share.[16]

Aggregate markups We can now define a measure of aggregate markups, our main focus. As in Grassi (2018) and Burstein *et al.* (2020), we define industry-level markups as the inverse of the industry-level labour share.[17] Equation (16) implies

$$\mu(\underline{n}) \equiv \left(\sum_{c=H,C_H,F} \sigma_c^H(\underline{n}) \left(\mu_c^H(\underline{n})\right)^{-1} \right)^{-1}. \tag{18}$$

That is, the industry-level markup is a sales-weighted harmonic mean of firm markups (and, as shown in Edmond *et al.* (2021), this equals a cost-weighted arithmetic mean of markups). Likewise, we define the aggregate markup as the inverse of the aggregate labour share, $\boldsymbol{\mu}_t = \left(\frac{w_t \boldsymbol{L}}{\boldsymbol{Y}_t}\right)^{-1}$. This is a weighted harmonic mean of industry-level markups:

$$\boldsymbol{\mu}_t \equiv \left(\sum_{n=-\infty}^{+\infty} \sum_{n_C=0}^{+\infty} \varphi_t(\underline{n}) \left(\mu(\underline{n})\right)^{-1} \right)^{-1}. \tag{19}$$

Taking stock Conditional on the productivity of each firm at time t, the equilibrium conditions described so far fully pin down output, wages and markups. However, the productivity distribution is endogenous, shaped by the innovation choices of entrants and incumbents. We now turn to analysing these choices.

3.2.2 Dynamic R&D and entry problems

Choice problems As noted earlier, we focus on a symmetric BGP equilibrium, in which aggregate output in both countries grows at rate $g \equiv \frac{\dot{\boldsymbol{Y}}_t}{\boldsymbol{Y}_t}$. As aggregate R&D spending $\boldsymbol{R}_t^k$ grows at the same rate as aggregate output (a result that we verify later), consumption also grows at rate g. Using the Euler equation (9), this implies that $r_t^H = r_t^F = r = g + \rho$.

Our previous discussion shows that the dynamic problem of the Home leader in a given industry has only two state variables: the technology gap, $\underline{n}$, and aggregate GDP, $\boldsymbol{Y}_t$. Given these, the Home leader chooses an innovation rate $z_H(\underline{n})$ to maximise its value, taking as given the innovation policies of all other firms. We denote by $V_H(\underline{n}, \boldsymbol{Y}_t)$ the

[16] As noted earlier, we always have $n_{Cj,t} \geq n_{j,t}$ (as the fringe can never become more productive than the least productive leader). Thus, whenever $n_C < n$, $\varphi_t(n, n_C) = 0$.

[17] Note that at the firm-level, the markup is also the inverse of the (firm-level) labour share.

value function of the Home leader in an industry with technology gap $\underline{n}$ at time t. The Hamilton-Jacobi-Bellman (HJB) equation is

$$
\begin{aligned}
rV_H(\underline{n}, \boldsymbol{Y}_t) = \max_{z_H(\underline{n})} \Bigg\{ & \left(\pi_H^H(\underline{n}) + \pi_H^F(\underline{n})\right) \boldsymbol{Y}_t - \chi_i \left(z_H(\underline{n})\right)^{\psi_i} \boldsymbol{Y}_t - x_H(\underline{n}) V_H(\underline{n}, \boldsymbol{Y}_t) \\
& + z_H(\underline{n}) \Big(V_H(n+1, n_C+1, \boldsymbol{Y}_t) - V_H(\underline{n}, \boldsymbol{Y}_t) \Big) \\
& + \left(x_F(\underline{n}) + z_F(\underline{n})\right) \Big(V_H(n-1, n_C, \boldsymbol{Y}_t) - V_H(\underline{n}, \boldsymbol{Y}_t) \Big) \\
& + \zeta \Big(V_H(0, 0, \boldsymbol{Y}_t) - V_H(\underline{n}, \boldsymbol{Y}_t) \Big) \Bigg\} + \dot{V}_H(\underline{n}, \boldsymbol{Y}_t).
\end{aligned} \tag{20}
$$

The right-hand side of the HJB equation has the following parts. The first line lists flow profits from domestic sales and exports, flow expenditure on R&D, and the fact that at rate $x_H(\underline{n})$, the Home leader is displaced by entry. The second line shows that when the Home leader innovates (at rate $z_H(\underline{n})$), it increases its technology gap with respect to the Foreign leader and the fringe by one unit. The third line captures the arrival of Foreign innovations, at rate $x_F(\underline{n})$ (for Foreign entrants) and $z_F(\underline{n})$ (for Foreign incumbents). Both events reduce the technology gap between leaders by one unit, but leave the technology gap with respect to the fringe unchanged. Finally, the fourth line shows that at rate ζ, the leading technology diffuses to all firms in the industry and technology gaps are reset to zero.

Similarly, Home's potential entrants in an industry with technology gap $\underline{n}$ choose an arrival rate of innovations solving

$$
\max_{x_H(\underline{n})} \left\{ x_H(\underline{n}) V_H(n+1, n_C+1, \boldsymbol{Y}_t) - \chi_e \left(x_H(\underline{n})\right)^{\psi_e} \boldsymbol{Y}_t \right\}. \tag{21}
$$

Upon innovation, the potential entrant becomes the new incumbent, increasing the technology gap with respect to the Foreign leader and the fringe by one unit.

Dynamic solution Due to the symmetry of our model, we have

$$
V_H(n, n_C, \boldsymbol{Y}_t) = V_F(-n, n_C - n, \boldsymbol{Y}_t), \tag{22}
$$

for all t, and every technology gap (n, n_C) holding $n_C \geq \max(0, n)$. That is, the value functions of Home and Foreign leaders are symmetric. Equation (22) is important because it implies that in order to solve for the optimal R&D choices, we only need to focus on the dynamic problem of Home firms.[18]

We guess-and-verify that the value function of Home leaders is linear in aggregate GDP, so that $V_H(\underline{n}, \boldsymbol{Y}_t) = v_H(\underline{n})\boldsymbol{Y}_t$. After some straightforward algebra, we get

$$\begin{aligned}\Big(\rho + x_H(\underline{n})\Big) v_H(\underline{n}) = \max_{z_H(\underline{n})} \Bigg\{ & \pi_H^H(\underline{n}) + \pi_H^F(\underline{n}) - \chi_i \left(z_H(\underline{n})\right)^{\psi_i} \\ & + z_H(\underline{n}) \Big(v_H(n+1, n_C+1) - v_H(\underline{n}) \Big) \\ & + \left(x_F(\underline{n}) + z_F(\underline{n})\right) \Big(v_H(n-1, n_C) - v_H(\underline{n}) \Big) \\ & + \zeta \Big(v_H(0,0) - v_H(\underline{n}) \Big) \Bigg\},\end{aligned} \tag{23}$$

where we have used the fact that $\dot{V}_H(\underline{n}, \boldsymbol{Y}_t) = v_H(\underline{n}) g \boldsymbol{Y}_t$ and $\rho = r - g$. The first-order condition for the incumbent's problem yields:

$$z_H(\underline{n}) = \left(\frac{v_H(n+1, n_C+1) - v_H(\underline{n})}{\chi_i \psi_i} \right)^{\frac{1}{\psi_i - 1}}. \tag{24}$$

Thus, innovation choices depend on the difference between the Home leader's current value and its value in case of a successful innovation. Likewise, the first-order condition of the entrant's problem (21) yields:

$$x_H(\underline{n}) = \left(\frac{v_H(n+1, n_C+1)}{\chi_e \psi_e} \right)^{\frac{1}{\psi_e - 1}}. \tag{25}$$

Given the value function of the Home leader, equations (24) and (25) pin down the optimal R&D choices of Home leaders and entrants. Furthermore, using the symmetry described by equation (22), they can also be used to deduce the optimal R&D choices of Foreign leaders and entrants. To find these objects, we solve for the value function of the Home leader numerically. Appendix C contains further details on this.

The distribution of technology gaps Knowing firms' innovation choices, we can finally characterise the evolution of the equilibrium distribution of technology gaps, denoted $\varphi_t(\underline{n})$.

[18] For example, the value of a Home leader with a technology gap of 5 with respect to the Foreign leader and 6 with respect to the fringe is the same as the value of a Foreign leader with a technology gap of 5 with respect to the Home leader (implying a technology gap of -5 from the viewpoint of the Home leader) and 6 with respect to the fringe (implying a technology gap of $6 - 5 = 1$ between the Home leader and the fringe).

First, for all technology gaps $\underline{n} \neq (0,0)$, we have

$$\dot{\varphi}_t(\underline{n}) = \mathbf{1}_{(n_C>0)} i_H(n-1, n_C-1)\varphi_t(n-1, n_C-1) + i_F(n+1, n_C)\varphi_t(n+1, n_C) \qquad (26)$$
$$- \Big(i_H(n, n_C) + i_F(n, n_C) + \zeta\Big)\varphi_t(n, n_C),$$
$$\text{where} \quad i_H(n, n_C) \equiv z_H(n, n_C) + x_H(n, n_C), \quad i_F(n, n_C) \equiv z_F(n, n_C) + x_F(n, n_C),$$

are the total innovation rates in country H and F, and $\mathbf{1}_{(n_C>0)}$ is an indicator variable equal to one whenever $n_C > 0$ and zero otherwise. Inflows into non-zero states occur through Home innovation from state $(n-1, n_C-1)$ or Foreign innovation from state $(n+1, n_C)$.[19] Outflows from state $\underline{n} \neq (0,0)$ occur through innovation by any firm or catch-up by the fringe and lagging leader.

For the specific case of technology gap $\underline{n} = (0,0)$, we have

$$\dot{\varphi}_t(0,0) = i_F(1,0)\varphi_t(1,0) + \zeta\Big(1 - \varphi_t(0,0)\Big) - \Big(i_H(0,0) + i_F(0,0)\Big)\varphi_t(0,0). \qquad (27)$$

Equation (27) differs from equation (26) in two respects. On the inflows block, there is one more term, coming from the inflow of firms that catch up to the highest productivity firm, an event which resets all technology gaps to 0. On the outflows block, there is also one term less, as firms do not catch-up if they are already neck-to-neck with the Home leader.

On the BGP, the distribution of technology gaps is invariant over time, that is, we have $\dot{\varphi}_t(\underline{n}) = 0$ for all technology gaps $\underline{n}$. Together with the fact that the distribution sums to one, i.e. $\sum_{n=-\infty}^{+\infty} \sum_{n_C=0}^{+\infty} \varphi_t(n, n_C) = 1$, this condition yields a system of linear equations pinning down the invariant distribution.

Knowing the technology gap distribution, we can solve for all aggregate outcomes. In particular, we can derive an expression for the aggregate growth rate.

Lemma 3.1 *On the BGP, output in both countries grows at a constant rate given by*

$$g = \left(\sum_{n=-\infty}^{+\infty} \sum_{n_C=0}^{+\infty} \varphi(n, n_C) i_H(n, n_C) + \zeta \sum_{n=-\infty}^{-1} \sum_{n_C=0}^{+\infty} \varphi(n, n_C)|n| \right) \ln(1+\lambda).$$

Proof. See Appendix B.2.

[19] Note that inflows through Home innovation occur only when $n_C > 0$. This is because it is impossible to arrive into a state in which the Home leader is neck-to-neck with the fringe through Home innovation (which would put the Home leader at least one step ahead of the fringe).

Lemma 3.1 shows that output growth is proportional to the aggregate arrival rate of Home innovations. These are due to leaders' and entrants' own innovations (the first term in the parenthesis) as well as to Home leaders who are behind and jump $|n|$ steps to catch up to the frontier (the second term). Notice that Foreign innovations and catch-up by the fringes do not explicitly feature in the growth formula, but this does not mean that these forces do not contribute to growth. Indeed, in equilibrium, aggregate Home innovation is equal to aggregate Foreign innovation and to aggregate fringe catch-up (this is a necessary condition for the existence of a invariant technology gap distribution).

Finally, the R&D share of GDP is

$$\frac{\boldsymbol{R}_t}{\boldsymbol{Y}_t} = \sum_{n=-\infty}^{+\infty} \sum_{n_C=0}^{+\infty} \varphi(n, n_C) \left(\chi_i \left(z_H(n, n_C) \right)^{\psi_i} + \chi_e \left(x_H(n, n_C) \right)^{\psi_e} \right). \tag{28}$$

The R&D share is the same in both countries and constant over time. Thus, it is straightforward to see that aggregate consumption, which can be obtained residually from equation (7), indeed grows at rate g.

This completes the discussion of our model's equilibrium conditions. Before discussing our quantitative results, the next section builds some intuitions by describing important qualitative features of the BGP equilibrium.

3.3 Key properties of the model

3.3.1 Profits, market shares and markups

Figure 2 shows surface plots of the market shares of Home leaders, Foreign leaders and the Home fringe on the Home market, as a function of the industry's technology gap (n, n_C). Obviously, market shares are increasing in relative productivity. Thus, the Home leader has high market shares when it has high technology gaps n and n_C with respect to the Foreign leader and the fringe. Likewise, the Foreign leader and the fringe have high market shares if they enjoy a large advantage (respectively, for the fringe, a small disadvantage) with respect to the Home leader.

Figure 3 shows how market shares translate into profits and markups for the Home leader. Profits and markups are increasing in market share (recall equations (13) and (15)), and therefore also increasing in the leader's technology gap with respect to its competitors. Moreover, profits and markups are higher in the domestic market than in the export market for all technology gaps, as the trade cost τ increases the marginal cost of exports above the marginal cost of producing for the domestic market.

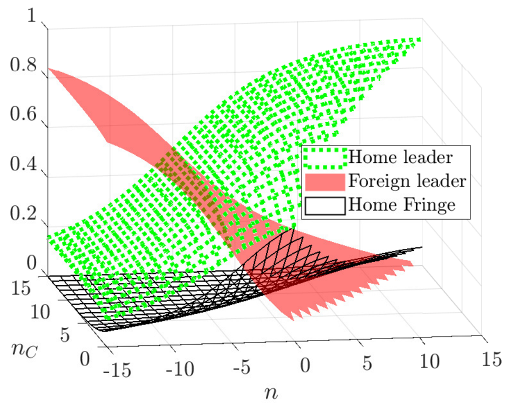

Figure 2: Market shares on the Home market.

Notes: This figure plots the market shares of the Home leader, Foreign leader and Home fringe on the Home market, as a function of the technology gap (n, n_C). The figure uses our baseline parameter values, listed in Table 3.

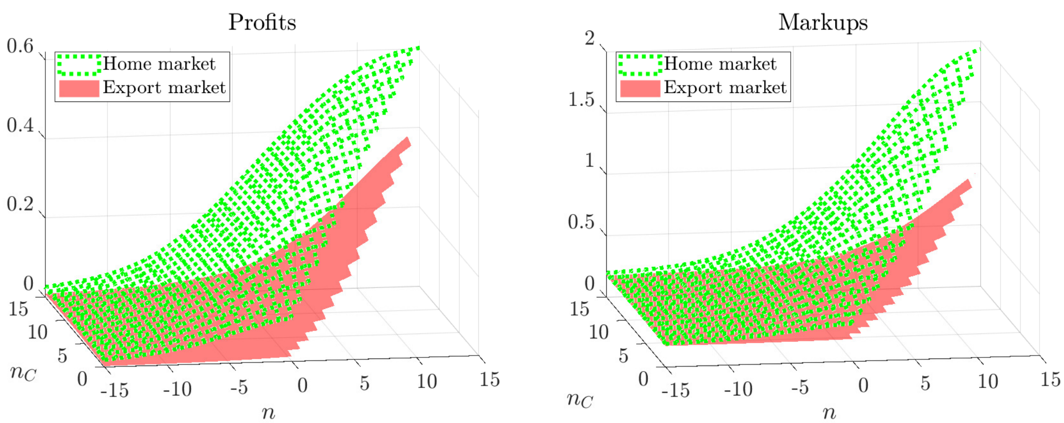

Figure 3: Profits and markups of the Home leader, in both markets.

Notes: Profits are normalised by GDP, and markups are net (i.e., we plot $\mu - 1$, where μ is the gross markup from equation (11)). The figure uses our baseline parameter values, listed in Table 3.

Crucially, market shares and profits are S-shaped in the technological gap. To show this more clearly, Figure 4 plots a series of two-dimensional cuts through the three-dimensional surfaces of the previous figures. Starting from a technology gap $(n_0, 0)$, and considering three different values for n_0, it shows how Home market shares and total profits (the sum of profits on both markets) evolve when gradually increasing the technology gap from this starting point. The x-axis in these figures lists the increase in the technology gap (i.e., point x corresponds to a technology gap $(n_0 + x, x)$, which would be reached if the Home leader were to make x innovations).

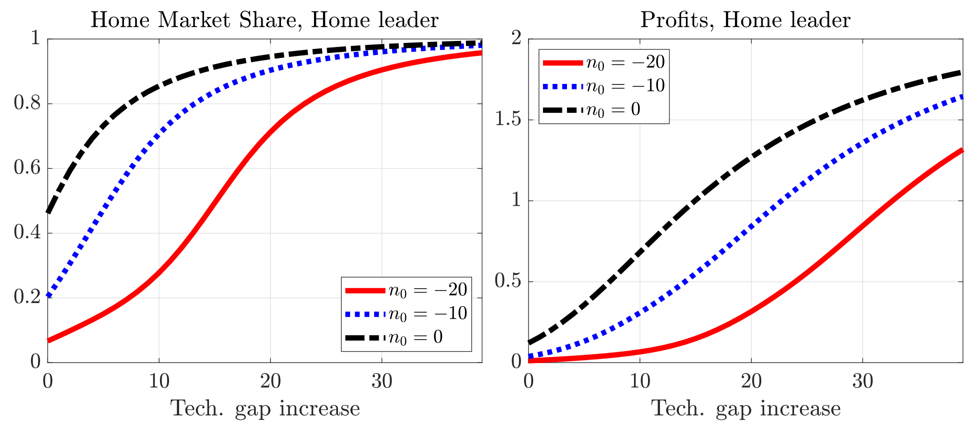

Figure 4: Home market shares and profits of Home leaders.

Notes: This figure plots the market shares and profits of the Home leader for technology gaps $(n_0 + x, x)$, where x is given on the horizontal axis. It uses our baseline parameter values, listed in Table 3.

The figure clearly illustrates the S-shape of market shares and profits.[20] When the Home leader has a high technology gap, its market share is close to 1, and it gains little by increasing its productivity even further. Likewise, when the Home leader is far behind its Foreign counterpart, it captures a negligible share of the market, and its profits would also not increase much if it were to increase its productivity. Thus, leaders which are far behind or far ahead have little incentive to innovate. However, when leaders are neck-to-neck, each innovation implies a large change in market shares, and innovation is strongly profitable.

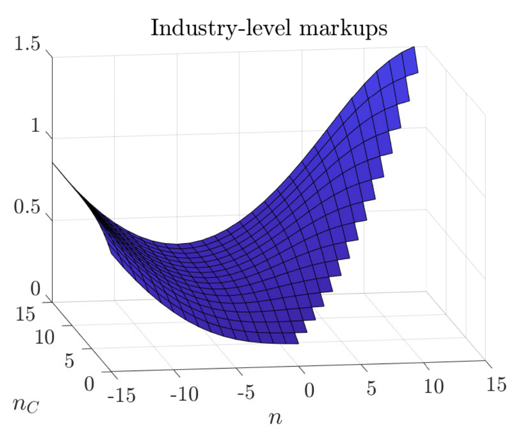

Figure 5: Industry-level markups.

Notes: The figure uses our baseline parameter values, listed in Table 3.

Finally, Figure 5 plots industry-level markups (defined in equation (18)), as a function

[20]Intuitively, the S-shape is due to the fact that markups depend on market shares, and market shares are capped at 1. Thus, the profit function of the Home leader is bound to become concave at some point (and by symmetry, this implies that it must initially be convex).

of the technology gap. Industry markups have a U-shape, being highest in industries in which one leader dominates the market. This is due to the fact that markups are increasing in market shares: when one leader (Home or Foreign) dominates an industry, it has both a high markup and a high weight in industry-level aggregates.

3.3.2 R&D policies and technology gap distribution

Figure 6 shows the value and innovation policy function of Home leaders, as a function of the industry's technology gap. The innovation policy function has an inverted U-shape. This is a direct consequence of the S-shape of the profit function: as innovation is most valuable for leaders with low technology gaps, these leaders invest most in R&D. The figure also shows Home entry rates. The value of leaders is increasing in the technology gap, as greater technology gaps imply higher profits. Thus, entry rates are also increasing, as a higher value of incumbency makes entry more attractive.

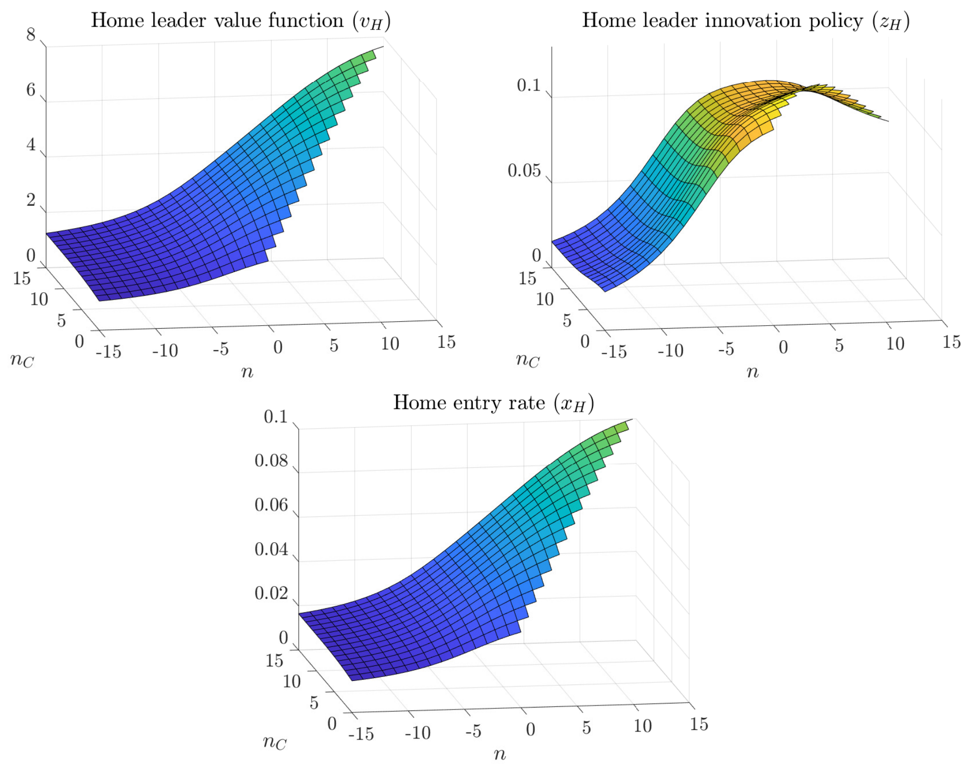

Figure 6: Value function and innovation policy of the Home leader, and Home entry rate.
Notes: The figure uses our baseline parameter values, listed in Table 3.

Finally, the top panel of Figure 7 plots the invariant distribution of technology gaps in the (n, n_C) space, and the lower panels show the corresponding marginal distributions

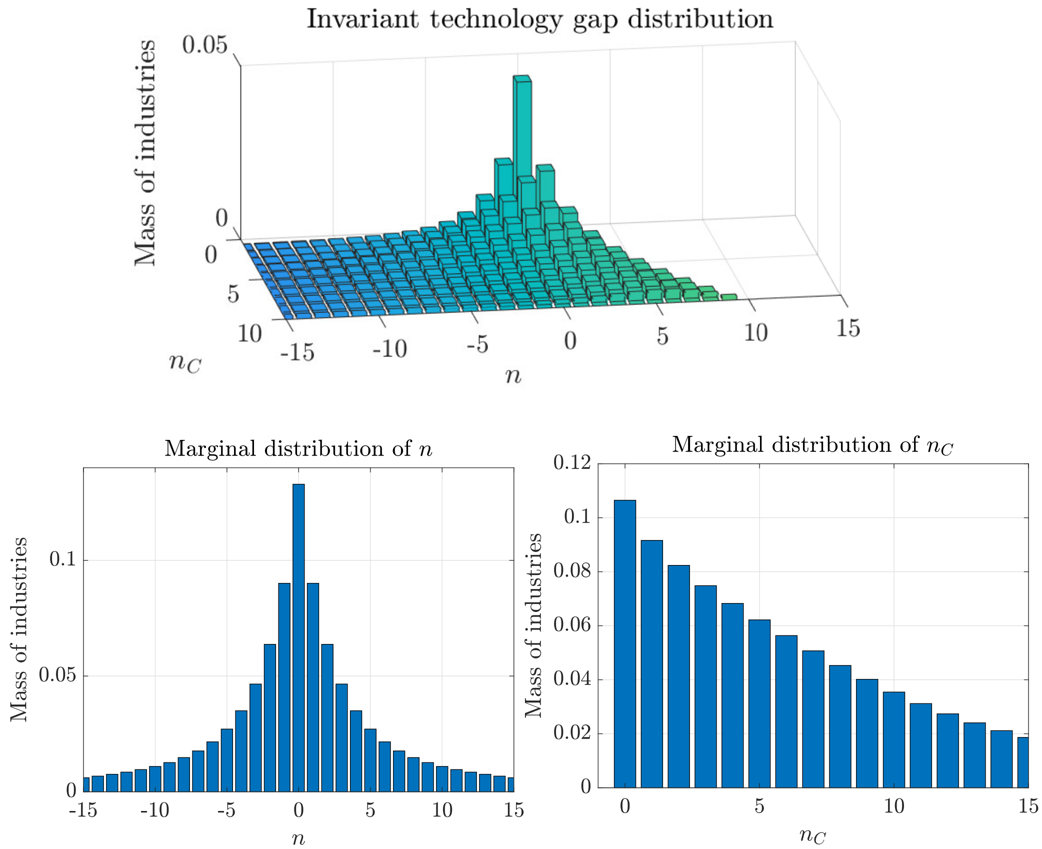

Figure 7: Joint distribution of technology gaps, and marginal distributions of n and n_C.
Notes: The figure uses our baseline parameter values, listed in Table 3.

of n and n_C. The marginal distribution of the technology gap between leaders is bell-shaped, symmetric and centred around $n = 0$ (the neck-to-neck state), while that of relative productivity between Home leader and fringe is left-tailed. These distributions are shaped by innovation. On the one hand, higher innovation rates around small technology gaps generate an inflow of firms toward higher n and n_C states. As an opposing force, leader and fringe catch-up (governed by the parameter ζ) push for higher shares of firms in lower technology gap states. The relative strength of these forces shapes the invariant distribution.

We are now ready to proceed to our quantitative analysis of the effects of trade openness on innovation and markups. The next section lays out our calibration strategy and our main results.

4 Quantitative Analysis

4.1 Calibration strategy and model fit

Our baseline calibration is designed to reflect the current state of the US manufacturing sector. We calibrate our model at the annual frequency, and need to choose ten parameter values: the discount rate ρ, the leader's quality level ω_H, the within-industry elasticity of substitution η, the innovation step size λ, the catch-up rate ζ, the variable trade cost τ, and the scale and curvature parameters in incumbents' and potential entrants R&D cost functions, χ_i, χ_e, ψ_i and ψ_e.

We set the discount rate to $\rho = 0.02$, and the curvature of the R&D cost functions to $\psi_i = \psi_e \equiv \psi = 2$, a standard choice informed by empirical studies on the cost elasticity of R&D spending (Akcigit and Kerr (2018) provide a survey of the evidence). We set the elasticity of substitution between intermediates to $\eta = 7$, as in Burstein *et al.* (2020).[21]

We set the values of the remaining six parameters using indirect inference, choosing parameters in order to minimise the distance between a series of model-generated moments and their data equivalents. We target fifteen moments, summarised in Table 4. Four moments are taken from aggregate data. First, we target the average rate of Total Factor Productivity growth in US manufacturing between 1997 and 2017, which, according to EU KLEMS, was 1.58% per year.[22] Second, we target the ratio of aggregate R&D spending to value added. The average for this ratio between 1997 and 2016, computed with the OECD's ANBERD (for R&D) and STAN (for value added) databases, is 9.8%. Third, we target the aggregate ratio of trade to domestic sales, using the trade data introduced in Section 2. In the data, we compute this number as the ratio between aggregate manufacturing imports and aggregate sales of manufacturing goods in the United States. In the year 2007, roughly the midpoint of the period that we consider, this statistic was equal to 26.1%.

Three more moments are informed by firm-level data. First, to discipline the rate at which leaders are displaced, we target the exit rate of large firms, taken from Garcia-Macia *et al.* (2019), who estimate this number to be 4.9% between 2003 to 2013. Second, we target the total contribution of entrants to TFP growth. Akcigit and Kerr (2018) find that 25.7% of productivity growth in the United States is due to entrants.[23] Finally, we target

[21] In Section 4.5 we conduct robustness checks using both higher and lower values for η and ψ.

[22] See `https://euklems.eu/` (2019 release).

[23] Garcia-Macia *et al.* (2019) reach similar conclusions, finding that entrants accounted on average for 21.1% of US productivity growth between 1993 and 2013.

Table 3: Baseline calibration.

Parameter	Value	Description
Calibrated externally		
ρ	0.02	Discount rate
ψ_i	2	R&D cost elasticity (incumbents)
ψ_e	2	R&D cost elasticity (entrants)
η	7	Within-industry elasticity of substitution
Calibrated internally		
λ	0.085	Innovation step size
χ_i	1.557	R&D cost scale (incumbents)
χ_e	39.328	R&D cost scale (entrants)
τ	1.490	Variable trade cost
ω_H	0.455	Quality of Home leader
ζ	0.016	Catch-up rate

Notes: Internally calibrated parameters are obtained by indirect inference, targeting the moments listed in Table 4.

the employment share of fringe firms. In our model, leaders are characterised by the fact that they invest in R&D, while the fringe does not. Using data from the National Science Foundation to measure the employment of manufacturing firms that do R&D, and the BDS database to measure total employment, we find that between 2008 and 2016, firms spending on R&D represented on average 81.8% of manufacturing employment. Thus, we target a fringe employment share of 18.2%.

Most importantly, we also target the distribution of industry-level markups (computed as specified in Section 2, for the year 2007, and weighted by industry value of shipments). To operationalise this target, we focus on the first nine deciles of the distribution, shown in Table 4. In our model, the distribution of industry markups is a direct function of the productivity gap distribution. Thus, targeting the markup distribution allows us to discipline the underlying distribution of relative productivities.[24]

Appendix C.2 contains further details on the definition of model moments, and on the numerical implementation of the indirect inference algorithm. Table 3 shows the parameter

[24]In our model, all else equal, a more dispersed relative productivity distribution implies a more dispersed distribution of industry-level markups, and vice-versa.

Table 4: Targeted moments: model versus data.

Moment	Model	Data	Data Source
A. From aggregate data			
Productivity growth	1.56%	1.58%	EU KLEMS, 2019 Release
R&D share of value added	13.8%	9.8%	OECD
Import share	26.9%	26.1%	US Census Bureau, NBER-CES
B. From firm-level data			
Exit rate	4.9%	4.9%	US Census Bureau
Contribution of entrants to growth	25.6%	25.7%	Akcigit and Kerr (2018)
Employment share of the fringe	21.1%	18.2%	US Census Bureau, NSF
C. Markup distribution			
1^{st} decile	13.8%	10.1%	Compustat
2^{nd} decile	17.7%	17.8%	Compustat
3^{rd} decile	20.7%	21.9%	Compustat
4^{th} decile	23.6%	25.0%	Compustat
5^{th} decile	27.6%	26.1%	Compustat
6^{th} decile	34.4%	30.2%	Compustat
7^{th} decile	42.3%	40.3%	Compustat
8^{th} decile	57.6%	67.5%	Compustat
9^{th} decile	98.6%	98.6%	Compustat

Notes: All data moments refer to the US manufacturing sector. The Appendix describes how we compute these moments in the model.

values obtained, and Table 4 reports the values of the targeted moments in the model and in the data. Even though the model is over-identified, it fits the data remarkably well. In the next section, we show that it can also reproduce some untargeted moments.

4.2 Untargeted moments

As our paper aims to study the effect of trade on markups, our baseline calibration targets the empirical markup distribution. Nevertheless, we acknowledge the inherent difficulties of markup measurement (which we will also take into account in our robustness checks). Therefore, we examine how well our model matches some untargeted distributions, such as the distribution of concentration ratios and of import shares across industries.

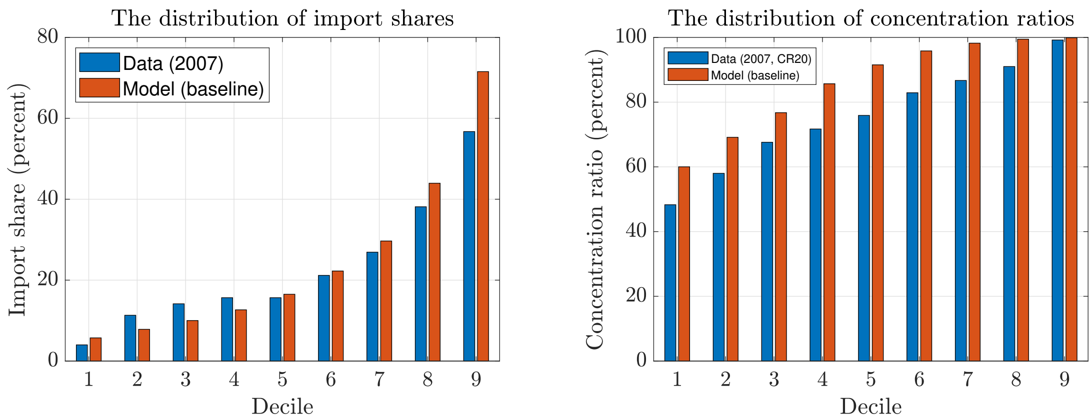

Figure 8: The distribution of import shares and concentration ratios, in the model and in the data. **Notes**: The left panel shows the first nine deciles of the distribution of import shares across industries, in the model and in the data. The import share of an industry is the ratio of imports to total sales on the domestic market. The right panel shows the first nine deciles of the distribution of concentration ratios across industries, in the model and in the data. In the data, concentration ratios are CR20 ratios from the US Census Bureau. Both data distributions are weighted by industry value of shipments.

The left panel of Figure 8 plots the distribution of import shares, showing that our model matches the data very well (except for the ninth decile). Thus, the model does not only reproduce the average level of trade, but also its distribution across industries. The right panel of Figure 8 plots the distribution of domestic concentration ratios. In the data, we consider the US Census Bureau's sales share of the largest 20 firms in an industry (CR20).[25] This sales share focuses on US firms, and sales include both their exports and their domestic sales. We compute an analogous measure in our model by computing the sales share of the Home leader, including its exports, among all Home firms of an industry (i.e., the leader itself and the Home fringe). As Figure 8 shows, our model somewhat overpredicts the level of concentration, but matches the shape of the distribution well.[26]

4.3 Markups and innovation for different levels of trade costs

We are now ready to analyse the effect of trade on innovation and markups. To do so, we first compare the BGP equilibria of our model for different trade costs τ, keeping all other parameters at their baseline values. Our main comparison confronts our baseline BGP (reflecting the current state of US manufacturing) to an alternative BGP in which the

[25]We use the Census data as compiled by Keil (2017). See Appendix A for further details.

[26]The distributions of import shares and domestic concentration ratios are not independent of each other. In industries with high import shares, domestic leaders charge lower markups on domestic sales, and domestic concentration is therefore higher.

trade-to-sales ratio is equal to 15%, its level in 1989.[27] Henceforth, we refer to these two BGPs as the "low trade cost" and the "high trade cost" BGPs.

4.3.1 Market shares, markups and profits

Figure 9 shows the percentage difference in Home market shares and markups when passing from the high to the low trade cost BGP.

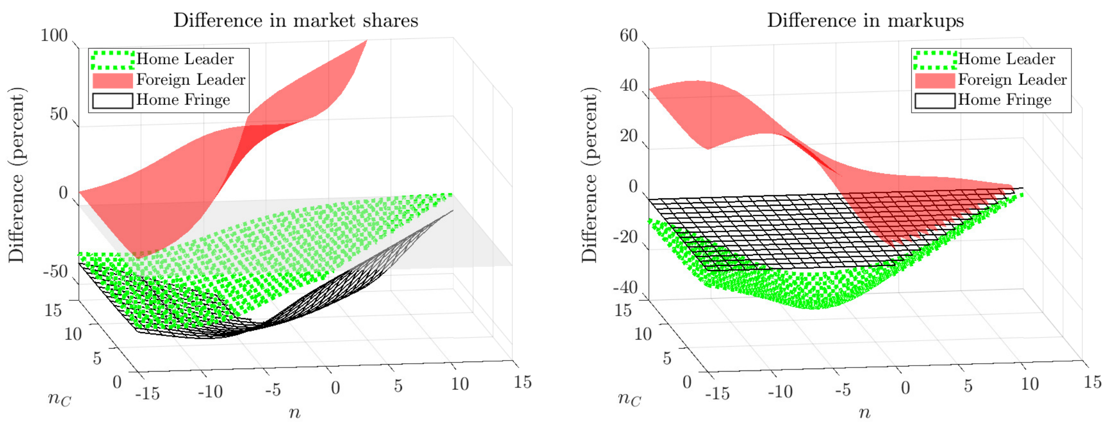

Figure 9: Percentage difference in market shares and markups between BGPs.
Notes: This figure plots the percentage difference in Home market shares and markups between the high trade cost BGP and the low trade cost BGP, for different levels of the technology gap. Percentage differences for a variable x are computed as $100 \cdot \left(x_{\tau_{\text{low}}} / x_{\tau_{\text{high}}} - 1 \right)$.

In the low trade cost BGP, exporters (i.e., Foreign leaders) have lower relative costs. Thus, as shown in the left panel of Figure 9, the market share of Foreign leaders is higher, while the market shares of Home leaders and Home fringes are lower. As a result, markups of Home leaders on Home sales are lower, while markups of Foreign leaders on exports are higher (see the right panel of Figure 9). Thus, the traditional pro-competitive effect of import competition on domestic firms is potentially counteracted by higher markups of foreign firms (as in Arkolakis *et al.*, 2019). Moreover, zero-markup fringe firms lose market share, which tends to increase the aggregate markup through a composition effect. The relative strength of these effects depends on the initial market shares of firms.

Figure 10 provides some perspective on this, by plotting the percentage difference in industry-level markups between the two BGPs. It shows an intuitive pattern: in industries where the Home leader has a high market share (i.e., a large technology gap with respect to the other firms), the pro-competitive effect dominates, and industry-level markups are

[27] In this alternative BGP, we set $\tau = 1.94$, 29.9% higher than in the baseline calibration.

lower in the low trade cost BGP. However, in industries where the Home leader has a small market share, the anti-competitive effects dominate, and industry-level markups are higher in the low trade cost BGP. Figure 10 shows that in our calibration, industry-level markups decrease for most industries (recall that most of the distribution of industries is concentrated around the neck-to-neck state). This is a direct consequence of the (empirically realistic) fact that the Home leader is the firm with the largest Home market share in most industries. As we show more formally later on, this implies that if the technology gap distribution were unchanged between BGPs (i.e., if there were no endogenous innovation), lower trade costs would imply a lower aggregate markup.

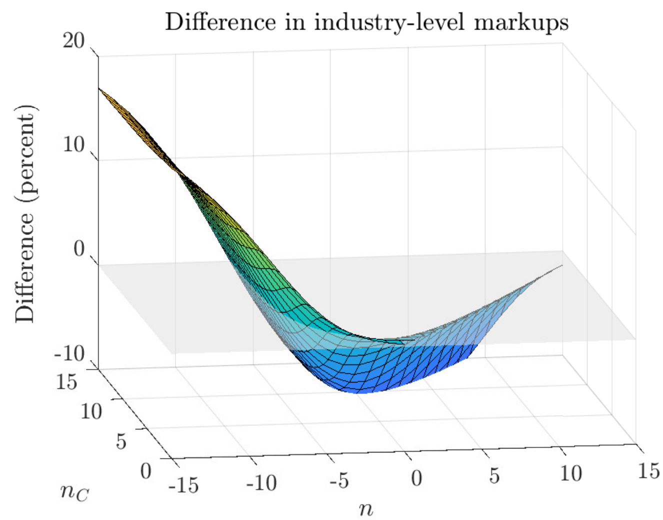

Figure 10: Percentage difference in industry-level markups between BGPs.
Notes: This figure plots the percentage difference in industry-level markups between the high trade cost BGP and the low trade cost BGP, for different levels of the technology gap.

Finally, Figure 11 plots the percentage difference in total profits of Home leaders between both BGPs. Lower trade costs imply that leaders lose market share on their domestic market, but gain market share on their export market. Thus, lower trade costs imply higher profits for firms with high technology gaps (which export a lot), and lower profits for firms with low technology gaps (which mainly sell domestically).

Importantly, these changes accentuate the S-shape of the profit function, increasing the payoffs of high technology gap states and lowering the payoffs of low technology gap states. Intuitively, with lower trade costs, leaders compete on a more equal footing, which increases the importance of relative productivity and the stakes in the innovation race between leaders.[28] These differences in profits are the key driver of differences in

[28]Note that in the extreme case of infinite trade costs, relative productivity between leaders is irrelevant, as

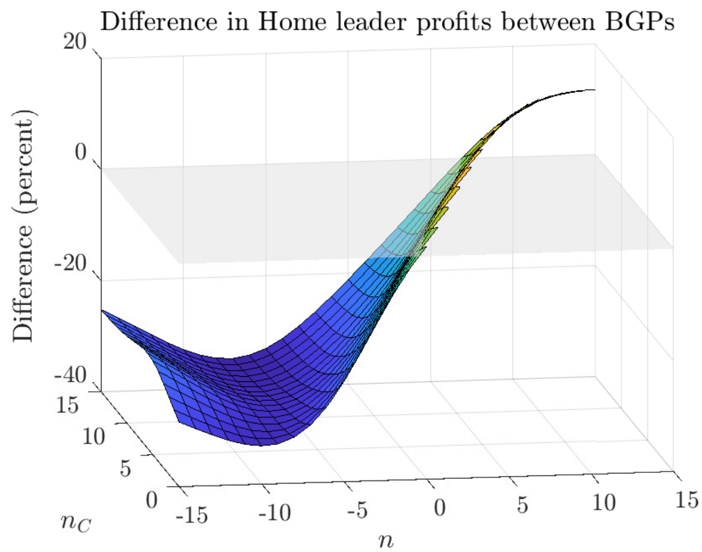

Figure 11: Difference in Home leader profits between BGPs.
Notes: This figure plots the percentage change in total Home leader profits (defined as $\pi_H^H(\underline{n}) + \pi_H^F(\underline{n})$) when passing from the high to the low trade cost BGP, as a function of the industry's technology gap.

innovation behaviour between both BGPs. We turn to this issue next.

4.3.2 R&D choices and the technology gap distribution

Figure 12 plots the percentage differences in the innovation rates of Home leaders and in Home entry rates between the two BGPs. The left panel shows that in the low trade cost BGP, innovation rates are significantly higher for leaders with technology gaps around zero, but decrease for leaders which are either far ahead or far behind. Indeed, as we have shown above, lower trade costs imply a steeper profit function at intermediate ranges of the technology gap. Therefore, leaders in these states have a higher incentive to escape their current state through innovation. The right panel shows that entry rates mimic profits, increasing for industries with high technology gaps (where lower trade costs mainly imply higher export opportunities) and decreasing for industries with low technology gaps (where lower trade costs mainly imply higher import competition).

These differences in innovation behaviour imply important differences in the invariant distribution of technology gaps, shown in Figure 13. The left panel plots the distribution of the technology gap between leaders, n, for both BGPs. As we have just seen, with low trade costs, innovation rates are higher in industries in which the technology gap between the

the Home leader cannot export and is shielded from import competition.

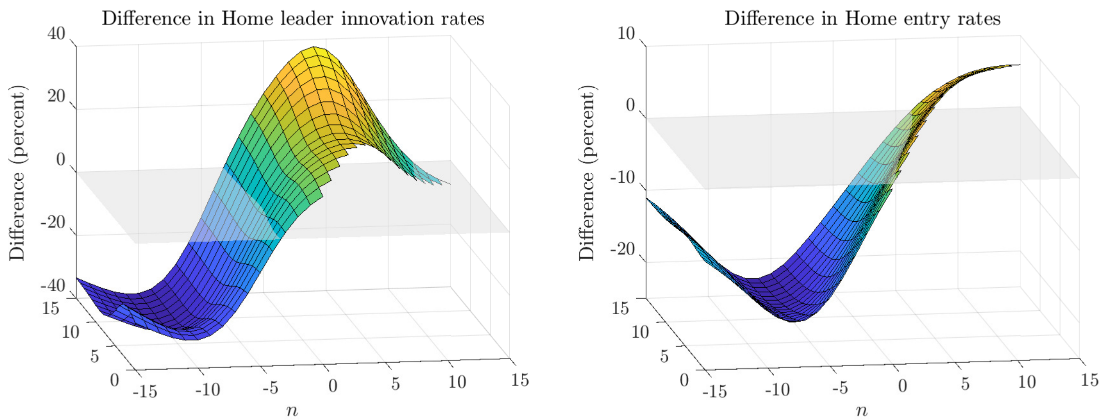

Figure 12: Difference in innovation rates between BGPs.

Notes: This figure plots the percentage difference in the innovation rate of the Home leader (z_H) and the Home entry rate (x_H) between the high and the low trade cost BGP, as a function of the technology gap.

Home and the Foreign leader is low. Therefore, there is a lower mass of such industries in equilibrium, and a higher mass of industries in which one leader has a large technology gap over the other one. In other words, with lower trade costs, the technology gap distribution is more polarised. The right panel plots the distribution of the technology gap between the Home leader and the fringes, n_C. This distribution is shifted to the right with lower trade costs, as leaders innovate more and pull ahead of non-innovating fringe firms.[29]

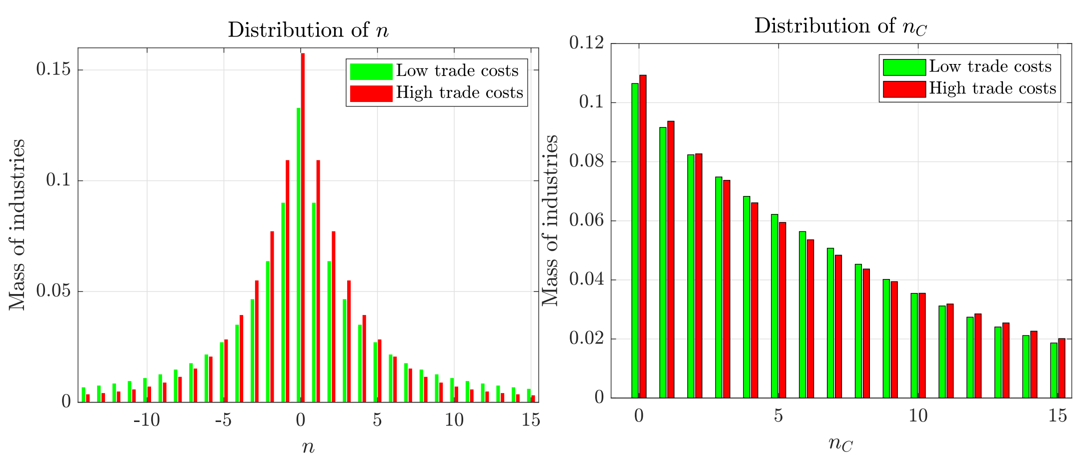

Figure 13: The invariant distribution of technology gaps for different levels of trade costs.

Summing up, lower trade costs induce a more polarised technology gap distribution, with a higher share of industries dominated by one leader. However, as we have seen earlier, these are precisely the industries in which markups are highest. Thus, all else equal, the

[29]Moreover, the catch-up speed of the fringe does not depend on trade costs.

shift in the technology gap distribution (fully driven by innovation) is a force that tends to increase the aggregate markup. We refer to this force as the innovation feedback effect, and examine its quantitative importance in the next sections.

4.3.3 Aggregate outcomes and magnitudes

Figure 14 plots the BGP values of some key aggregate variables for different trade costs. The high and low trade cost BGP values of τ are marked by vertical lines.

The first panel plots the trade share of GDP, which is obviously decreasing in the trade cost τ.[30] The second panel shows the rate of productivity growth, which is higher for lower trade costs. Indeed, as we have shown before, lower trade costs increase innovation incentives for leaders that are technologically close to their rivals. As this is the case for most leaders, aggregate innovation increases. Precisely, the rate of productivity growth is 0.14 percentage points (or 9.9%) higher in our baseline low trade cost BGP than in the high trade cost BGP.

The third panel shows our main result: the aggregate markup (which, as shown in equation (19), equals the inverse of the labour share) increases when passing from the high to the low trade cost BGP, by around 3.5 percentage points (or 7.7%). As we will show more formally later, this increase is entirely due to the innovation feedback effect. Lower trade costs lead to a polarisation of the technology gap distribution (shown in the sixth panel of Figure 14), and increase the incidence of high-markup industries. Accordingly, concentration ratios, as shown in the fifth panel of the figure, also rise.[31] This dominates the direct, pro-competitive effects of trade. However, it is worth noting that the relationship between aggregate markups and trade costs is U-shaped. For high levels of the trade cost (exceeding the value implied by 1989 trade levels), the direct pro-competitive effect can be stronger than the innovation feedback effect.[32] In the next subsection, we make these arguments more precise by explicitly decomposing the aggregate markup change into the part driven by the innovation feedback effect, and the part driven by other factors.

[30] The implied trade elasticity between the high and low trade cost BGPs is 2.2. This is at the lower end of estimates in the literature, but within the range of long-run elasticity estimates in Boehm *et al.* (2021).

[31] Concentration ratios are defined as in Figure 8, and capture concentration among domestic firms.

[32] To recover the baseline level of the aggregate markup with higher trade costs, one would need trade costs in excess of 3, which would correspond to a trade-to-GDP ratio below 4.6%.

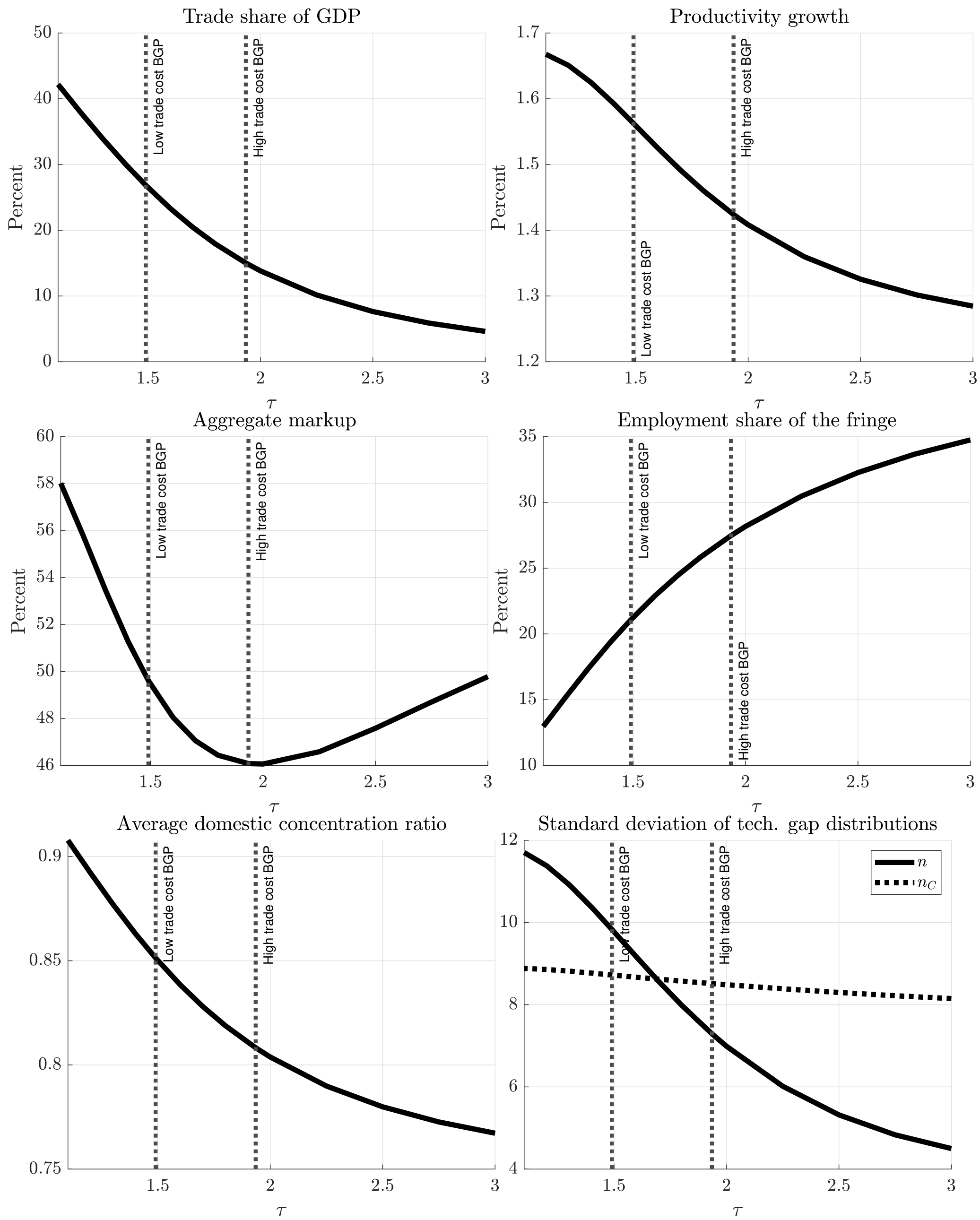

Figure 14: Aggregate BGP outcomes for different trade costs. Notes: The figure shows aggregate outcomes for BGPs obtained with different values of trade costs τ. All other parameter values are at their baseline values throughout.

4.4 Quantifying the innovation feedback effect

In our model, changes in trade costs affect the aggregate markup through two channels: changes conditional on a given technology gap distribution ("direct effects") and changes in the technology gap distribution itself (the innovation feedback effect). To assess the relative contribution of these two channels, it is useful to explicitly consider transition dynamics.

Precisely, we assume that the economy is initially in the high trade cost BGP, and is hit by a permanent and unexpected shock which instantly lowers trade costs to their low trade cost BGP level. The economy then gradually converges to the low trade cost BGP. Appendix C.3 describes how we solve for the transition path. This analysis allows us to distinguish direct and innovation feedback effects. Indeed, on impact, the fall in trade costs changes markups in all industries, but does not affect the technology gap distribution, which is a state variable. Over the transition, markups conditional on the technology gap distribution are fixed, and changes in the aggregate markup are entirely due to shifts in the technology gap distribution. Thus, the impact response captures the direct effect, and the additional change over the transition captures the innovation feedback effect.

Columns (1) and (2) in Table 5 list the values of key aggregate variables in both BGPs, and Column (3) shows the percentage point difference between both BGPs. In Column (4), we report the change in aggregate variables that occurs on impact and is therefore attributable to direct effects. Column (5) instead shows the response that occurs during the transition, due to the innovation feedback effect.

Table 5: The quantitative importance of the innovation feedback effect.

	(1)	(2)	(3)	(4)	(5)
Variable	**BGP$_{\text{initial}}$**	**BGP$_{\text{final}}$**	**Total Change**	**Impact**	**Transition**
Panel 1. *Transition from high trade cost BGP to low trade cost BGP*					
Productivity growth	1.42%	1.56%	+0.14	+0.08	+0.05
Aggregate markup	46.08%	49.61%	+3.54	−4.04	+7.58
Trade share	15.02%	26.85%	+11.82	+9.71	+2.11
Fringe emp. share	27.47%	21.07%	−6.40	−6.44	+0.04
Panel 2. *Transition from high trade cost BGP to free trade BGP*					
Productivity growth	1.42%	1.67%	+0.25	+0.21	+0.04
Aggregate markup	46.08%	59.81%	+13.74	−5.28	+19.01
Trade share	15.02%	46.64%	+31.62	+31.25	+0.37
Fringe emp. share	27.47%	10.73%	−16.74	−16.98	+0.24

Notes: Differences in Columns (3) to (5) are stated in percentage points. The algorithm that computes the transition dynamics between different BGPs is described in the Appendix.

Panel 1 shows that in a transition from the high trade cost BGP to the low trade cost BGP, the aggregate markup falls by 4.04 percentage points on impact. This is due to a classic pro-competitive effect of trade through import competition, lowering the market shares and markups of Home leaders. However, eventually, the aggregate markup ends up being 3.54 percentage points higher in the low trade cost BGP. This increase is entirely due to the shift in the technology gap distribution during the transition, i.e., to the innovation feedback effect. In total, the innovation feedback effect thus accounts for a 7.58 percentage point ($3.54-(-4.04)$) increase in the aggregate markup, about 16.4% of its initial level. Panel 2 in Table 5 considers instead a transition from high trade cost to a free trade BGP (in which $\tau = 1$). Qualitative effects are similar: the aggregate markup falls on impact, but the innovation feedback effect eventually raises it again.

For other aggregate variables, such as the growth rate, the trade share or the employment share of the fringe, the innovation feedback effect is less relevant, as most changes occur on impact. Also, the innovation feedback effect reinforces the impact response, and does not turn it around as it is the case for markups.

As this discussion shows, the innovation feedback effect operates through a polarisation of the productivity distribution, which implies in turn a polarisation of the distribution of industry-level markups. In Section 2, we showed that the distribution of industry-level markups has indeed experienced a polarisation between 1989 and 2007, as can be seen in the shift of the cumulative distribution function of industry-level markups in Figure 1.

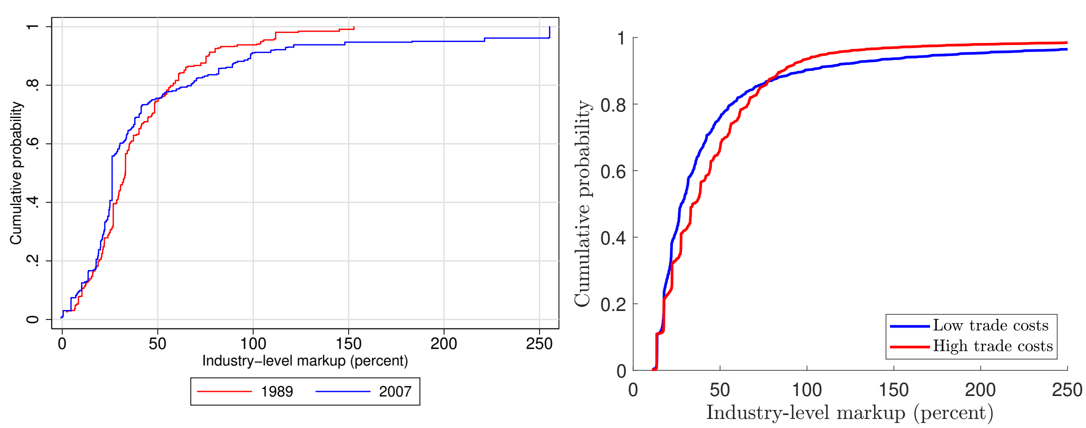

Figure 15: The shift in the industry-level markup distribution in the data and in the model.
Notes: The left panel shows the cumulative distribution function of industry-level markups in the data, as in Figure 1. The right panel shows the model equivalent.

Figure 15 shows that this pattern is quantitatively consistent with the model. As the right panel of this figure shows, the model closely matches the shift of the markup distribution (even though our calibration only targeted the markup distribution in 2007, and not the distribution in 1989). Thus, the magnitude of the innovation feedback effect in our model is consistent with the magnitude of markup polarisation in the data.

More generally, our results are consistent with many of the findings emphasised by the literature on concentration and markups over the last years. For instance, it is important to note that the innovation feedback effect operates entirely through within-industry changes (with some industries experiencing increases and others decreases in markups), and not through a reallocation of activity from low to high markup industries. Indeed, because of Cobb-Douglas aggregation, the GDP shares of all industries are fixed throughout. The fact that changes in the aggregate markup are driven by within-industry rather than across-industry forces is consistent with the results in De Loecker *et al.* (2020), Autor *et al.* (2020b) and Baqaee and Farhi (2020).[33]

4.5 Robustness Checks

In this section, we examine whether our main result (a positive effect of trade on markups through the innovation feedback effect) is robust to some extensions and modifications of our baseline setup. Appendix D provides further details.

Markup targets Markups are difficult to measure, and any point estimate masks considerable uncertainty. The estimates of De Loecker *et al.* (2020), which we use in our baseline calibration, are at the higher end of the literature. For robustness, we therefore consider two alternative calibrations, in which we assume that firm markups are uniformly by 25% or 50% lower than in the baseline. In these alternative calibrations, the initial aggregate markup is equal to 33.4% and 21.2% (as opposed to 46.08% in the baseline).

Using these new calibrations, we again compare the baseline BGP to a high trade cost BGP (where the change in trade between the high and the low trade cost BGPs is the same as in the baseline). Table D.3 in the Appendix shows that the innovation feedback effect raises aggregate markups by 2.4 and 4.7 percentage points in these calibrations. This corresponds to 11.1% and 13.9% of the initial aggregate markup, and is similar to the baseline calibration, where the increase amounted to 16.4% of the initial markup.

[33] Moreover, we find that the markups of the median home leader and the median industry fall when trade costs fall (in line, again, with De Loecker *et al.* (2020)), showing that the increase in the aggregate markup is entirely due to a greater polarisation around the median. Similarly, as in Autor *et al.* (2020b) and Kehrig and Vincent (2021), we find an increase in concentration and a decrease in the labour share which is ultimately due to a reallocation effect, where top leaders increase both markups and market shares.

Within-industry elasticity of substitution Our baseline within-industry elasticity of substitution is $\eta = 7$, as in Burstein *et al.* (2020). Given the uncertainty in elasticity estimates, we consider how our results change when we set η to values of 5 and 10, and recalibrate all internally calibrated parameter values to match the baseline targets. As shown in Table D.3 in the Appendix, markup results for the transition from a high to a low trade cost BGP are comparable to our baseline results. In particular, the innovation feedback is again positive.

R&D cost curvature In the baseline calibration, we have set the parameters related to the curvature of the R&D cost function for both incumbents and entrants to 2, as in Akcigit and Kerr (2018). While this choice is based on a large number of empirical studies on the cost elasticity of R&D, there is obviously some uncertainty about the value of these parameters. Thus, we conduct two robustness checks by setting $\psi_i = \psi_e = 1.5$ or $\psi_i = \psi_e = 2.5$, and recalibrating the internal parameters to match the baseline targets. The results are reported in Table D.3 in the Appendix. As could be expected, a more convex R&D cost function dampens the innovation response, and therefore lowers the magnitude of the innovation feedback effect, while a less convex R&D cost function increases these numbers. However, the quantitative magnitudes remain reasonably close to the baseline results.

Fixed cost of exporting In our model, all leaders are exporters. In the data, this is not the case. To make the model more realistic, we introduce a fixed cost of exporting (as described in Appendix D). We calibrate this extended model targeting the same moments as in the baseline, and adding a new target for the percentage of leaders that export. As Table D.3 in the Appendix shows, our results are again unchanged in this setup.

R&D cost in units of labour In our baseline model, R&D is paid in units of the final good. Our results might in principle be different if R&D were instead paid in units of labour, as changes in trade costs affect the wage-to-GDP ratio. To address this concern, we develop an alternative specification of our model, in which all R&D is paid in units of labour (see Appendix D). As shown in Table D.3 in the Appendix, this does not affect our results.

Overall, our robustness checks therefore confirm the baseline results, indicating that lower trade costs trigger an innovation feedback effect that increases the aggregate markup. Before concluding, the next section examines how lower trade costs affect consumer welfare.

5 Welfare implications

5.1 Consumption-equivalent welfare changes

On the BGP, the welfare of the representative consumer is given by $\boldsymbol{U}_0 = \frac{\ln(\boldsymbol{C}_0)}{\rho} + \frac{g}{\rho^2}$, where $\boldsymbol{C}_0$ is the initial level of consumption and g is the rate of economic growth.[34] In order to compare two BGPs A and B, we derive a consumption equivalent welfare measure γ, defined as the percentage increase in consumption that a household in BGP B would require to be indifferent between living in BGP A or B. It is easy to show that

$$\gamma = \frac{\boldsymbol{C}_0^A e^{\frac{g^A - g^B}{\rho}}}{\boldsymbol{C}_0^B} - 1. \tag{29}$$

Using this formula, the consumption-equivalent welfare gain from moving from the high to the low trade cost BGP is 15.0%, while moving from the high trade cost BGP to free trade generates welfare gains of 44.5%.[35]

Welfare gains are driven by three different mechanisms. First, lower trade costs directly raise the level of consumption. Second, lower trade costs stimulate innovation and therefore raise growth. Third, lower trade costs affect the dispersion of markups and productivity, and change the amount of labour misallocation. To get a sense of the magnitude of these effects, note that the increase in the growth rate accounts for a 7.3% welfare gain when passing from the high to the low trade cost BGP, roughly half of the total gain.[36]

While our model implies large welfare gains from trade, it is worth pointing out that these gains are unequally shared, as the increase in markups implies that profit income increases more strongly than labour income. Considering a fictitious household earning all the labour income of the economy, we find that its consumption-equivalent welfare gain from moving from the high to the low trade cost BGP is 12.3%. By contrast, the corresponding number for a household earning all the profit income is 21.0%, roughly

[34] To compute the initial level of output and consumption, we normalise for each BGP the level of productivity of all Home leaders to one at time $t = 0$, i.e. $q_{jH,0} = 1, \forall j$.

[35] These numbers change only slightly when we take into account transition dynamics: in that case, consumption equivalent welfare increases by 11.9% when passing from the high to the low trade cost BGP. This smaller increase is explained by the fact that the growth rate rises only gradually during the transition. Appendix C.3 describes how we compute welfare during the transition.

[36] These large gains are in line with the literature that studies the interaction between trade and growth. Hsieh *et al.* (2019) find a 18.8% increase in welfare after a 50% reduction in trade costs, and Perla *et al.* (2021) find gains of 10.8% for a 10% reduction in trade costs. Both of these papers are calibrated to US data.

twice as high.

Finally, as our model is calibrated to the manufacturing sector, one might be concerned that the numbers in this section overstate the welfare gains for the overall US economy. Thus, in Appendix Section D.2, we set up a simple extension of our baseline model in which aggregate output is a Cobb-Douglas aggregate of (non-tradable) services and manufacturing. Setting the spending share of manufacturing to 20% (as in Eaton and Kortum, 2012) then roughly divides all our welfare results by a factor of 5. Precisely, the welfare gains from moving to the high to the low trade cost BGP would now be 2.8%, and the welfare gains from moving from high trade costs to free trade would be 7.7%. These numbers could be seen as a lower bound, given that some services are actually tradable and some manufacturing goods are used as inputs in service production.

5.2 Social Planner Solution

To better understand the sources of welfare gains in our model, we finally compare the decentralised equilibrium (DE) with the allocation of a Social Planner (SP) who maximises global welfare. Appendix E derives the SP solution formally and provides additional discussion.

We find that in the SP solution, the growth rate of the economy is roughly invariant to the level of trade costs (increasing by just 0.8% when moving from the high trade cost BGP to the low trade cost BGP). However, as markup dispersion increases, the DE experiences a rise in misallocation when trade costs fall, which limits its welfare gains. Also, the SP takes into account that the shape of the productivity distribution matters for level of output (and can therefore be optimally twisted when trade costs change), while the DE does not. Thus, overall, welfare gains from lower trade costs are roughly equal in the DE and SP solutions (15.0% in the DE solution, 15.8% in the SP solution).

6 Conclusion

The last decades have seen a change in the competitive landscape of the United States, with rising levels of concentration and markups. Over the same period of time, the trade share of GDP has more than doubled. Our paper argues that these trends might be linked. By spurring the innovation efforts of global firms, the increase in trade might have triggered a polarisation of the productivity distribution. This polarisation, in turn, can account for greater markup dispersion and an increase in the aggregate markup.

Our quantitative analysis for the US manufacturing sector shows that this innovation feedback effect can be substantial. This suggests that globalisation may have contributed to the large increase in concentration and markups observed in many developed economies over the last decades.

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International Trade and Innovation Dynamics with Endogenous Markups

by Laurent Cavenaile, Pau Roldan-Blanco and Tom Schmitz

Appendix Materials

A Data sources

Most data sources are cited directly in the paper, and are publicly accessible. This appendix provides a short overview of the main sources.

Trade Our trade data comes from the US Census Bureau. It was first compiled by Schott (2008) and can be downloaded at `https://faculty.som.yale.edu/peterschott/international-trade-data/`.

Industry sales Industry sales (shipments) come from the NBER-CES Manufacturing Industry Database (`https://www.nber.org/research/data/nber-ces-manufacturing-industry-database`).

Markups Our markup estimates combine the production function estimates of De Loecker *et al.* (2020), available at `https://dataverse.harvard.edu/dataset.xhtml?persistentId=doi:10.7910/DVN/5GH8XO`, and data from Compustat. Compustat is a commercial database maintained by Standard & Poor's, accessible at `http://www.compustat.com`.

Using this data, we compute firm-level markups as the ratio of the De Loecker *et al.* (2020) elasticity of sales to variable inputs (cost of goods sold) to the variable input share (cost of goods sold divided by sales). To obtain industry-level markups, we follow Edmond *et al.* (2021) and compute a weighted average of firm level markups, where weights are given by the cost of goods sold.

Concentration ratios Concentration ratios are computed every five years by the US Census Bureau. We use a compiled version of these statistics by Keil (2017). The data can be downloaded at `https://sites.google.com/site/drjankeil/data`.

B Derivations and Proofs

B.1 Solution of the static Bertrand game

In this section, we derive the solution of the Bertrand game between the Home and the Foreign leader. The static problem of the Home leader is

$$\max_{\{p^H_{jH,t},p^F_{jH,t}\}} \left\{ \underbrace{\left(p^H_{jH,t} - \frac{w_t}{q_{jH,t}}\right) y^H_{jH,t}}_{\text{Domestic profits}} + \underbrace{\left(p^F_{jH,t} - \tau\frac{w_t}{q_{jH,t}}\right) y^F_{jH,t}}_{\text{Foreign profits}} \right\}, \tag{B.1}$$

where the quantities sold at Home and in Foreign, $y^H_{jH,t}$ and $y^F_{jH,t}$, are given by the demand function (10). As specified in the main text, the Home leader takes the price of the Foreign leader and the fringes as given. The first-order optimality conditions of Problem (B.1) are

$$\text{Domestic price } (p^H_{jH,t}): \qquad y^H_{jH,t} + p^H_{jH,t}\frac{\partial y^H_{jH,t}}{\partial p^H_{jH,t}} = \frac{w_t}{q_{jH,t}}\frac{\partial y^H_{jH,t}}{\partial p^H_{jH,t}}, \tag{B.2}$$

$$\text{Export price } (p^F_{jH,t}): \qquad y^F_{jH,t} + p^F_{jH,t}\frac{\partial y^F_{jH,t}}{\partial p^F_{jH,t}} = \tau\frac{w_t}{q_{jH,t}}\frac{\partial y^F_{jH,t}}{\partial p^F_{jH,t}}. \tag{B.3}$$

These optimality conditions show that decisions on the Home and on the Foreign market are independent from each other. In each country, the leader equates its marginal cost to the marginal benefit of increasing prices by one unit. Combining the first-order conditions with the demand function (equation (10)) and the definition of market shares (equation (12)), we obtain equation (11) and an expression for the export price of the Home leader, which by symmetry yields equation (13).

Finally, replacing equations (10), (11) and (13) into the leader's profit function, defined as the value of Problem (B.1), we get the expression for profits stated in equation (15).

B.2 Proof of Lemma 3.1

In this proof, we focus on the Home country throughout. Using the definitions in equations (3) and (4), we can express total output as

$$\begin{aligned}\ln Y^H_t &= \int_0^1 \ln\left(Y^H_{jt}\right)\mathrm{d}j = \frac{\eta}{\eta-1}\int_0^1 \ln\left(\sum_{c=H,C_H,F}(\omega_c)^{\frac{1}{\eta}}\left(y^H_{jc,t}\right)^{\frac{\eta-1}{\eta}}\right)\mathrm{d}j \\ &= \frac{\eta}{\eta-1}\int_0^1\left[\ln\left(\left(y^H_{jH,t}\right)^{\frac{\eta-1}{\eta}}\right) + \ln\left(\sum_{c=H,C_H,F}(\omega_c)^{\frac{1}{\eta}}\left(\frac{y^H_{jc,t}}{y^H_{jH,t}}\right)^{\frac{\eta-1}{\eta}}\right)\right]\mathrm{d}j\end{aligned} \tag{B.4}$$

$$= \int_0^1 \ln(y^H_{jH,t})\mathrm{d}j + \underbrace{\frac{\eta}{\eta-1}\int_0^1 \ln\left(\sum_{c=H,C_H,F}(\omega_c)^{\frac{1}{\eta}}\left(\frac{y^H_{jc,t}}{y^H_{jH,t}}\right)^{\frac{\eta-1}{\eta}}\right)\mathrm{d}j}_{\equiv\Psi^H_t}.$$

Using the fact that relative outputs only depend on technology gaps, we can express Ψ^H_t as

$$\Psi^H_t = \frac{\eta}{\eta-1}\sum_{n=-\infty}^{+\infty}\sum_{n_C=0}^{+\infty}\left(\varphi(n,n_C)\ln\left(\sum_{c=H,C_H,F}(\omega_c)^{\frac{1}{\eta}}\left(\frac{y^H_c(n,n_C)}{y^H_H(n,n_C)}\right)^{\frac{\eta-1}{\eta}}\right)\right). \tag{B.5}$$

Conditional on the technology gap, output ratios are constant over time. The technology gap distribution is constant over time as well, and therefore, Ψ^H_t is a constant.

Furthermore, we can rewrite

$$\begin{aligned}\int_0^1 \ln(y^H_{jH,t})\mathrm{d}j &= \int_0^1 \ln(q_{jH,t})\mathrm{d}j + \int_0^1 \ln(\ell^H_{jH,t})\mathrm{d}j \\ &= \int_0^1 \ln(q_{jH,t})\mathrm{d}j + \sum_{n=-\infty}^{+\infty}\sum_{n_C=0}^{+\infty}\left(\varphi(n,n_C)\ln\left(\ell^H_H(n,n_C)\right)\right).\end{aligned}$$

Again, the second term of the above equation is a constant. These intermediate results imply that output growth in Home is given by

$$\frac{\dot{Y}^H_t}{Y^H_t} = \dot{\Theta}^H_t, \tag{B.6}$$

where

$$\Theta^H_t = \int_0^1 \ln(q_{jH,t})\mathrm{d}j. \tag{B.7}$$

The productivity of Home leaders increases because of Home innovations (done by incumbents and potential entrants) and lagging leaders who catch up. The law of large numbers implies that in a short time interval of length $\Delta > 0$, the mass of Home innovations realised in an industry with technology gap $\underline{n}$ is $i_H(\underline{n})\Delta$. The total mass of Home innovations and catch-up realised in all industries is $\Delta \cdot \left(\sum_{n=-\infty}^{+\infty}\sum_{n_C=0}^{+\infty}\varphi(n,n_C)i_H(n,n_C) + \zeta\sum_{n=-\infty}^{-1}\sum_{n_C=0}^{+\infty}\varphi(n,n_C)|n|\right)$, where $|.|$ denotes the absolute value. The second term in the parenthesis is because when a lagging leader with gap $n<0$ catches up at rate ζ, it does so by $|n|$ steps (i.e. the technology gap is reset to zero). Each one of these innovations increases productivity by a factor $1+\lambda$. Therefore, we have

$$\Theta^H_{t+\Delta} = \Delta\cdot\left(\sum_{n=-\infty}^{+\infty}\sum_{n_C=0}^{+\infty}\varphi(n,n_C)i_H(n,n_C) + \zeta\sum_{n=-\infty}^{-1}\sum_{n_C=0}^{+\infty}\varphi(n,n_C)|n|\right)\cdot\ln(1+\lambda) + \Theta^H_t. \tag{B.8}$$

To arrive at this expression, we have used the log-linearity of Θ^H_t,which implies that the effect of an innovation does not depend on the productivity level of the industry to which it applies. Subtracting Θ^H_t from both sides of the previous expression, dividing by Δ and taking the limit for $\Delta \to 0$, we get

$$\dot{\Theta}_t^H = \left(\sum_{n=-\infty}^{+\infty} \sum_{n_C=0}^{+\infty} \varphi(n, n_C) i_H(n, n_C) + \zeta \sum_{n=-\infty}^{-1} \sum_{n_C=0}^{+\infty} \varphi(n, n_C)|n| \right) \cdot \ln(1+\lambda),$$

which gives the expression for aggregate growth stated in Lemma 3.1. It is easy to verify that aggregate symmetry implies that Foreign must grow at the same rate. ■

C Numerical Appendix

C.1 The numerical solution of the model

This section provides further details on the numerical solution of our model. This solution is greatly simplified by the fact that static decisions are independent of innovation choices, and innovation choices are themselves independent of the technology gap distribution.

As technology gaps between firms can a priori become infinitely large, we impose an upper bound $n_{\max} = 40$ on both n (the technology gap between Home leader and Foreign leader) and on $\max(n_C, n_C - n)$ (the technology gap between the fringes and the most productive leader). Imposing these bounds a priori changes firm behaviour, as leaders recognise that they can never acquire an advantage exceeding $n_{\max}$. However, we make sure that bounds are irrelevant in practice, by verifying that the mass of firms in states in which the technology gap is maximal is always sufficiently small. For robustness, we have repeated all of our main quantitative experiments with higher choices for $n_{\max}$ (75 and 100), and the results remain unchanged. These results are available upon request.

The static solution To solve for relative prices, markups and market shares in an industry with technology gap $\underline{n} = (n, n_C)$, we reduce equations (11) to (14) to a system of two equations in two unknowns. Simple algebra shows

$$\frac{p_F^H(\underline{n})}{p_H^H(\underline{n})} = (1+\lambda)^n \cdot \tau \cdot \frac{\left(\frac{\eta}{\eta-1} - \sigma_F^H(\underline{n})\right)\left(1 - \sigma_H^H(\underline{n})\right)}{\left(\frac{\eta}{\eta-1} - \sigma_H^H(\underline{n})\right)\left(1 - \sigma_F^H(\underline{n})\right)}, \tag{C.1}$$

where we have used that $q_H / q_F = (1+\lambda)^n$. Likewise, the relative price of the Home competitive fringe with respect to the Home leader holds

$$\frac{p_{C_H}^H(\underline{n})}{p_H^H(\underline{n})} = (1+\lambda)^{n_C} \cdot \frac{1 - \sigma_H^H(\underline{n})}{\frac{\eta}{\eta-1} - \sigma_H^H(\underline{n})}. \tag{C.2}$$

As shown in equation (12), market shares are themselves a function of relative prices. For instance, the market share of the Home leader on the Home market is equal to

$$\sigma_H^H(\underline{n}) = \frac{1}{1 + \frac{\omega_F}{\omega_H}\left(\frac{p_F^H(\underline{n})}{p_H^H(\underline{n})}\right)^{1-\eta} + \frac{\omega_C}{\omega_H}\left(\frac{p_{C_H}^H(\underline{n})}{p_H^H(\underline{n})}\right)^{1-\eta}}, \tag{C.3}$$

and the market share of the Foreign leader on the Home market is equal to

$$\sigma_F^H(\underline{n}) = \frac{\frac{\omega_F}{\omega_H}\left(\frac{p_F^H(\underline{n})}{p_H^H(\underline{n})}\right)^{1-\eta}}{1 + \frac{\omega_F}{\omega_H}\left(\frac{p_F^H(\underline{n})}{p_H^H(\underline{n})}\right)^{1-\eta} + \frac{\omega_C}{\omega_H}\left(\frac{p_{C_H}^H(\underline{n})}{p_H^H(\underline{n})}\right)^{1-\eta}}. \tag{C.4}$$

Thus, we can use equations (C.3) and (C.4) to substitute out market shares from equations (C.1) and (C.2). This yields a system of two equations with two unknowns, the relative prices $p_F^H(\underline{n})/p_H^H(\underline{n})$ and $p_{C_H}^H(\underline{n})/p_H^H(\underline{n})$. This system can be solved numerically. Furthermore, once we know relative prices, we can immediately deduce market shares and markups for all firms.

Dynamic innovation choices To solve for the value functions of Home leaders in both sectors, we use a simple Value Function Iteration algorithm, described below.

1. Guess an initial value function $\left(v_H^{(k=0)}(\underline{n})\right)$, where the superscript k stands for the current iteration.
2. For any given iteration $k \in \mathbb{N}$, deduce the optimal R&D choices of entrants and incumbents in both countries, using equations (24) and (25), and the symmetry implied by equation (22).
3. Deduce the new implied values of the value function $v_H^{new}(\underline{n})$, given by equation (23).
4. Update the guess for the value function according to

$$v_H^{(k+1)}(\underline{n}) = \iota v_H^{new}(\underline{n}) + (1-\iota)\, v_H^{(k)}(\underline{n}),$$

   where $\iota \in (0,1)$ is a dampening parameter.
5. Iterate on 2-4 until the difference between $v_H^{new}(\underline{n})$ and $v_H^{(k)}(\underline{n})$ is sufficiently small.

The invariant distribution of technology gaps Once all innovation rates are known, equations (26), (27) and the condition that the distribution sums up to 1 form a linear system of equations, which can be easily solved numerically.

C.2 Internal calibration

In order to estimate the vector of internal parameters $\boldsymbol{\theta}$, we define a model-data distance function

$$\mathcal{D}(\boldsymbol{\theta}) \equiv \sum_{m=1}^{M} \left| \frac{\text{Moment}_m\,(\text{Data}) - \text{Moment}_m\,(\text{Model}, \boldsymbol{\theta})}{\frac{1}{2}\Big(\text{Moment}_m\,(\text{Data}) + \text{Moment}_m\,(\text{Model}, \boldsymbol{\theta})\Big)} \right|.$$

where M is the number of moments. We find the vector $\boldsymbol{\theta}$ that minimises this function using a Differential Evolution algorithm, developed by Markus Buehren and available for download at `https://it.mathworks.com/matlabcentral/fileexchange/18593-differential-evolution`. This method searches for the global optimum that minimises our criterion function $\mathcal{D}(\boldsymbol{\theta})$.

Next, we describe how we compute the different targeted moments in the model.

1. **Import share:** The aggregate import share is computed as the sales of foreign firms in the Home economy, on average across industries (using the equilibrium technology gap distribution):

$$\frac{M_t}{Y_t} = \sum_{n=-\infty}^{+\infty} \sum_{n_C=0}^{+\infty} \varphi(n, n_C) \sigma_F^H(n, n_C).$$

2. **Exit rate:** The exit rate is computed using entry rates in the Home economy, as in the BGP the exit and entry rates must coincide:

$$ExitRate = \sum_{n=-\infty}^{+\infty} \sum_{n_C=0}^{+\infty} \varphi(n, n_C) x_H(n, n_C).$$

3. **Contribution of entrants to growth:** We define the contribution of entrants to growth as the percentage of innovations realised by entrants:

$$ContEntGrowth = \frac{\sum_{n=-\infty}^{+\infty} \sum_{n_C=0}^{+\infty} \varphi(n, n_C) x_H(n, n_C)}{\sum_{n=-\infty}^{+\infty} \sum_{n_C=0}^{+\infty} \varphi(n, n_C) i_H(n, n_C)}.$$

4. **Employment share of the fringe:** We use the labour demand of the fringe (equation (16)) and the aggregate markup $\boldsymbol{\mu}$ to compute the share of employment accounted for by the fringe (our measure of relative fringe size) as:

$$EmpShareFringe = \boldsymbol{\mu} \sum_{n=-\infty}^{+\infty} \sum_{n_C=0}^{+\infty} \varphi(n, n_C) \sigma_{C_H}^H(n, n_C).$$

5. **Distribution of markups:** To compute the deciles of the distribution of industry employment-weighted average markups in the model, first we define the employment-weighted markup of domestically-produced goods (either sold at home or abroad) in industry state $\underline{n} \equiv (n, n_C)$ as:

$$\overline{\mu}^{EW}(n, n_C) \equiv \epsilon^H(n, n_C)\Big(\mu_H^H(n, n_C) - 1\Big) + \epsilon^F(n, n_C)\Big(\mu_H^F(n, n_C) - 1\Big),$$

where

$$\epsilon^H(n, n_C) \equiv \frac{l_H^H(n, n_C)}{l_H^H(n, n_C) + l_{C_H}^H(n, n_C) + l_H^F(n, n_C)}, \quad \text{and} \quad \epsilon^F(n, n_C) \equiv \frac{l_H^F(n, n_C)}{l_H^H(n, n_C) + l_{C_H}^H(n, n_C) + l_H^F(n, n_C)}.$$

are the employment weights, and $l_c^k(n, n_C) = \sigma_c^k(n, n_C)\left(\frac{\mu_c^k(n, n_C)}{\boldsymbol{\mu}}\right)^{-1}$ is the labour demand from each domestic firm $c \in \{H, C_H\}$ selling in market $k = H, F$. Then, we extract quantiles from the distribution of these markups by sorting industries from lowest to highest markup, and extracting the deciles using the equilibrium technology gap distribution.

C.3 Transition Dynamics Algorithm

To compute transition dynamics, we assume that the economy is initially on a given BGP. At some time t_0, it is hit by a permanent and unexpected shock which lowers trade costs τ. Eventually, the economy will then converge to a new low trade cost BGP. This section describes how we compute outcomes during the transition period.

We assume that the transition is completed after some long period of time T (in practice, we set $T = 500$). Furthermore, we discretise the model, considering short time intervals of length $\Delta = 0.05$. Thus, we solve the model on a time grid $\mathcal{T} = \{\Delta, 2\Delta, 3\Delta, \ldots, T - \Delta, T\}$.

1. Set the iteration counter to $k = 0$. Guess paths for the growth rate of output and the interest rate, $\underline{g}_Y^{(k=0)} = \left\{g_{Y,t}^{(k=0)} : t \in \mathcal{T}\right\}$ and $\underline{r}^{(k=0)} = \left\{r_t^{(k=0)} : t \in \mathcal{T}\right\}$.
2. For any given iteration $k \in \mathbb{N}$, and going backwards from $t = T - \Delta$ to $t = \Delta$:

   (a) Solve for the innovation rates $\left\{z_{H,t}^{(k)}(\underline{n}), x_{H,t}^{(k)}(\underline{n})\right\}$ using:

$$z_{H,t}^{(k)}(\underline{n}) = \left(e^{-\Delta r_{t+\Delta}^{(k)}} \frac{v_{H,t+\Delta}^{(k)}(n+1, n_C+1) - v_{H,t+\Delta}^{(k)}(\underline{n})}{\psi_i \chi_i}\right)^{\frac{1}{\psi_i - 1}},$$

$$x_{H,t}^{(k)}(\underline{n}) = \left(e^{-\Delta r_{t+\Delta}^{(k)}} \frac{v_{H,t+\Delta}^{(k)}(n+1, n_C+1)}{\psi_e \chi_e}\right)^{\frac{1}{\psi_e - 1}},$$

   with initial condition (i.e. at $t = T - \Delta$) given by $v_{H,T}^{(k)}(\underline{n}) = v_H^{final}(\underline{n})$, the value function from the final BGP.

   (b) Use the results from (a) to get $v_{H,t}^{(k)}(\underline{n})$ from the discretized Bellman equation:

$$\begin{aligned}
v_{H,t}^{(k)}(\underline{n}) = {} & \pi_H^{H,final}(\underline{n})\Delta + \pi_H^{F,final}(\underline{n})\Delta - \chi_i \left(z_{H,t}^{(k)}(\underline{n})\right)^{\psi_i} \Delta \\
& + \Delta e^{\Delta(g_{Y,t+\Delta}^{(k)} - r_{t+\Delta}^{(k)})} \Bigg[- x_{H,t}^{(k)}(\underline{n}) v_{H,t+\Delta}^{(k)}(\underline{n}) \\
& \qquad + z_{H,t}^{(k)}(\underline{n}) \left(v_{H,t+\Delta}^{(k)}(n+1, n_C+1) - v_{H,t+\Delta}^{(k)}(\underline{n})\right) \\
& \qquad + \left(x_{F,t}^{(k)}(\underline{n}) + z_{F,t}^{(k)}(\underline{n})\right)\left(v_{H,t+\Delta}^{(k)}(n-1, n_C) - v_{H,t+\Delta}^{(k)}(\underline{n})\right) \\
& \qquad + \zeta\left(v_{H,t+\Delta}^{(k)}(0,0) - v_{H,t+\Delta}^{(k)}(\underline{n})\right) + \frac{v_{H,t+\Delta}^{(k)}(\underline{n})}{\Delta}\Bigg],
\end{aligned}$$

   where $\pi_H^{H,final}(\underline{n})$ and $\pi_H^{F,final}(\underline{n})$ are the domestic and foreign profits from the final BGP.
3. Using $\left\{z_{H,t}^{(k)}(\underline{n}), x_{H,t}^{(k)}(\underline{n}) : t \in \mathcal{T}\right\}$ and $\left\{\varphi^{initial}(\underline{n})\right\}$, the stationary technology gap distribution from the initial BGP, compute $\left\{\varphi_t^{(k)}(\underline{n}) : t \in \mathcal{T}\right\}$ using the flow equations.

4. Compute the implied aggregate output and aggregate consumption levels, for each $t \in \mathcal{T}$.

   (a) For aggregate output, use Equation (B.4):

$$\boldsymbol{Y}_t^{(k)} = \exp\left(\Theta_t^{(k)} + \sum_{n=-\infty}^{+\infty}\sum_{n_C=0}^{+\infty} \varphi_t^{(k)}(\underline{n}) \ln\left(\ell_H^{H,final}(\underline{n})\right) + \Psi_t^{H,(k)}\right),$$

   where $\Psi_t^{H,(k)}$ is defined as in Equation (B.5), $\Theta_t^{(k)}$ is defined as in Equation (B.7), and $\ell_H^{H,final}(\underline{n})$ is the labour allocation from the final BGP. To compute $\{\Theta_t^{(k)} : t \in \mathcal{T}\}$, we normalise $\Theta_0^{(k)} = 0$ and use Equation (B.8) to update.

   (b) For aggregate consumption, use $\boldsymbol{C}_t^{(k)} = \boldsymbol{Y}_t^{(k)}\left(1 - \frac{\boldsymbol{R}_t^{(k)}}{\boldsymbol{Y}_t^{(k)}}\right)$ by the resource constraint, where

$$\frac{\boldsymbol{R}_t^{(k)}}{\boldsymbol{Y}_t^{(k)}} = \sum_{n=-\infty}^{+\infty}\sum_{n_C=0}^{+\infty} \varphi_t^{(k)}(\underline{n})\left(\chi_i\left(z_{H,t}^{(k)}(\underline{n})\right)^{\psi_i} + \chi_e\left(x_{H,t}^{(k)}(\underline{n})\right)^{\psi_e}\right).$$

5. For each $t \in \mathcal{T}$, compute the implied growth rates of output and consumption, and get a new interest rate from the Euler equation:

$$g_{Y,t}^{new} = \frac{1}{\Delta}\left(\frac{\boldsymbol{Y}_t^{(k)}}{\boldsymbol{Y}_{t-\Delta}^{(k)}} - 1\right), \qquad \text{and} \qquad r_t^{new} = \frac{1}{\Delta}\left(\frac{\boldsymbol{C}_t^{(k)}}{\boldsymbol{C}_{t-\Delta}^{(k)}} - 1\right) + \rho.$$

6. Collect these results in vectors $\underline{g}_Y^{new} = \{g_{Y,t}^{new} : t \in \mathcal{T}\}$ and $\underline{r}^{new} = \{r_t^{new} : t \in \mathcal{T}\}$. Stop if $\|\underline{g}_Y^{new} - \underline{g}_Y^{(k)}\| < \varepsilon$ and $\|\underline{r}^{new} - \underline{r}^{(k)}\| < \varepsilon$ for some small tolerance $\varepsilon > 0$, where $\|\cdot\|$ denotes the sup-norm. Otherwise, define:

$$\underline{g}_Y^{(k+1)} = \iota \underline{g}_Y^{new} + (1-\iota)\underline{g}_Y^{(k)}, \qquad \text{and} \qquad \underline{r}^{(k+1)} = \iota \underline{r}^{new} + (1-\iota)\underline{r}^{(k)},$$

   where $\iota \in (0,1)$ is a dampening parameter, and go back to 2. with $[k] \leftarrow [k+1]$.

Welfare during the transition To compute welfare gains during the transition, we compare the transitioning economy (labelled A) to a counterfactual economy (labelled B) which remains on the high trade cost BGP throughout. Our consumption-equivalent welfare gain γ, making a representative household in B indifferent between the two regimes, is then given by:

$$\frac{\ln(\boldsymbol{C}_0^B)}{\rho} + \frac{g^B}{\rho^2} = \int_0^\infty e^{-\rho t}\ln(\boldsymbol{C}_t^A(1+\gamma))\mathrm{d}t. \tag{C.5}$$

D Robustness Checks

D.1 Alternative models and calibrations

In this section, we first describe the robustness checks among those described in Section 4.5 that require modifications of our baseline model, and then present the calibration results and our main quantitative results for each case.

Fixed costs of exporting In the model with fixed costs of exporting, we assume that leaders need to pay a flow cost $\kappa \boldsymbol{Y}_t$, with $\kappa > 0$, in order to export their product. In equilibrium, the Home leader exports if and only if its export profits are sufficiently high, that is, if and only if $\pi_H^F(\underline{n}) \geq \kappa$. Using Equation (15), this yields a threshold market share: the Home leader exports if its market share $\sigma_H^F(\underline{n})$ exceeds

$$\widehat{\sigma} = \frac{\kappa\eta}{1+\kappa(\eta-1)}.$$

All other equilibrium conditions are unchanged in this extended model. To discipline the new parameter κ, we target the share of leaders that export. Bernard *et al.* (2018) show that in 2007, 35% of US manufacturing firms exported. Harris and Moffat (2011) show that over the period 2004-2008, the prevalence of exporting was about 41% higher among R&D performing firms (the equivalent of leaders in our model) than in the general population of manufacturing firms in the United Kingdom. Assuming that the same relationship also holds for the United States, we obtain that 49% of R&D-performing firms export.

R&D costs paid in labour As another robustness check, we derive and calibrate a version of the model in which R&D uses labour as input instead of units of the final good. The main structure of the model is kept unchanged except for the cost of R&D for both incumbents and entrants, which is now paid in wages. We assume that it requires $\chi_i z^{\psi_i}$ units of labour for an incumbent large firm to generate a Poisson rate z of being successful at innovation. Similarly, a potential entrant needs to hire $\chi_e x^{\psi_e}$ units of labour to generate a Poisson rate x of being successful at innovation. As a result, labour is either used in the production sector or in innovation. Both types of labour are paid the same wage, w_t.

The static part of the model remains unchanged in terms of market shares, profits and markups as a function of the industry state. The dynamic part changes and the value functions of the leaders (normalised by Y_t) can be written as follows:

$$\Big(\rho + x_H(\underline{n})\Big) v_H(\underline{n}) = \max_{z_H(\underline{n})} \Bigg\{ \pi_H^H(\underline{n}) + \pi_H^F(\underline{n}) - \chi_i \left(z_H(\underline{n})\right)^{\psi_i} \widehat{w} + z_H(\underline{n}) \Big(v_H(n+1, n_C+1) - v_H(\underline{n}) \Big) \\ + \left(x_F(\underline{n}) + z_F(\underline{n})\right) \Big(v_H(n-1, n_C) - v_H(\underline{n}) \Big) + \zeta \Big(v_H(0,0) - v_H(\underline{n}) \Big) \Bigg\},$$

and the problem of the potential entrants is:

$$\max_{x_H(\underline{n})} \left\{ x_H(\underline{n}) v_H(n+1, n_C+1) - \chi_e \left(x_H(\underline{n})\right)^{\psi_e} \widehat{w} \right\}.$$

Both problems now explicitly depend on the relative wage, $\widehat{w} \equiv \frac{w_t}{\boldsymbol{Y}_t}$. The equilibrium policy functions are:

$$z_H(\underline{n}) = \left(\frac{v_H(n+1, n_C+1) - v_H(\underline{n})}{\chi_i \psi_i \widehat{w}} \right)^{\frac{1}{\psi_i - 1}},$$

$$x_H(\underline{n}) = \left(\frac{v_H(n+1, n_C+1)}{\chi_e \psi_e \widehat{w}} \right)^{\frac{1}{\psi_e - 1}}.$$

We normalise the labour supply $\boldsymbol{L} = 1$, so that the labour market clearing condition is given by:

$$1 = \sum_{n=-\infty}^{+\infty} \sum_{n_C=0}^{+\infty} \varphi_t(\underline{n}) \left(\ell_H^H(\underline{n}) + \ell_H^F(\underline{n}) + \ell_C^H(\underline{n}) + \chi_i z(\underline{n})^{\psi_i} + \chi_e x(\underline{n})^{\psi_e} \right),$$

where the demands for labour used in production are, as before:

$$\ell_{H,t}^H(\underline{n}) = \frac{\sigma_H^H(\underline{n})}{\mu_H^H(\underline{n})\widehat{w}}, \qquad \ell_{H,t}^F(\underline{n}) = \frac{\sigma_H^F(\underline{n})}{\mu_H^F(\underline{n})\widehat{w}}, \qquad \ell_{C_H,t}^H(\underline{n}) = \frac{\sigma_{C_H}^H(\underline{n})}{\widehat{w}}.$$

To remain consistent with our baseline model, the aggregate markup is now defined as the inverse of the labour share of production workers (i.e. excluding research labour). To calibrate this economy, we use the same set of parameters and moments as in the baseline calibration.

Robustness results Table D.1 lists the internally calibrated parameter values for each of the different robustness checks, and Table D.2 shows how the different calibrations fit the targeted moments. External parameter values are set to their baseline values described in the main text, and targets are the same as the baseline ones in Table 4 (with the exception of the low markup calibration). Table D.3 summarises the results of the different robustness checks. It presents, for each robustness check, the analogue of Panel 1 of Table 5. That is, we consider for each case a transition from a high trade cost BGP (in which the trade-to-GDP ratio is half as high as in the baseline) to the baseline low trade cost BGP.[37]

[37] We do not report the decomposition results for productivity growth with R&D cost in labour as solving for the transition dynamics is more challenging and computationally intensive.

Table D.1: Internally calibrated parameters for the different calibrations.

Parameter	**Baseline**	**(1)**	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**	**(7)**	**(8)**
λ	0.085	0.086	0.081	0.085	0.088	0.087	0.133	0.086	0.087
χ_i	1.557	1.520	1.375	1.318	1.803	2.264	1.555	4.410	1.997
χ_e	39.33	23.70	31.40	38.08	37.88	39.72	20.56	137.07	49.19
τ	1.490	1.192	1.357	1.477	1.52	1.239	1.396	1.425	1.484
ω_H	0.455	0.374	0.423	0.392	0.468	0.476	0.472	0.455	0.451
ζ	0.016	0.031	0.021	0.015	0.017	0.025	0.019	0.016	0.017
κ	.	.	.	.	.	0.054	.	.	.
τ_{high}	1.94	1.48	1.73	2.06	1.87	1.46	1.79	1.90	1.92

Notes: Internally calibrated parameters are obtained by indirect inference, targeting the data moments listed in Table D.2. Additionally, for the model with fixed export costs, we target the share of exporting firms. τ_{high} corresponds to the level of trade costs in the "high trade cost" equilibrium. Each numbered column corresponds to one robustness check. (1) All markups 50% lower than in the baseline; (2) All markups 25% lower than in the baseline; (3) Within-industry elasticity of substitution set to $\eta = 5$; (4) Within-industry elasticity of substitution set to $\eta = 10$; (5) Model with a fixed cost of exporting κ; (6) R&D cost curvatures set to $\psi_i = \psi_e = 1.5$; (7) R&D cost curvatures set to $\psi_i = \psi_e = 2.5$; (8) All R&D costs paid in units of labour.

Table D.2: Targeted moments: model versus data for all robustness checks.

Moment	Baseline	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	Data	Data Source
A. From aggregate data											
Productivity growth	1.56%	1.58%	1.56%	1.57%	1.58%	1.58%	1.58%	1.58%	1.58%	1.58%	EU KLEMS
R&D share	13.8%	8.6%	11.4%	13.0%	13.8%	14.3%	17.1%	11.1%	13.6%	9.8%	OECD
Import share	26.9%	26.1%	26.2%	26.1%	23.9%	26.6%	30.4%	28.8%	26.7%	26.1%	US Census, NBER-CES
Share of exporting firms	.	.	.	.	.	49.0%	.	.	.	49.0%	Bernard *et al.* (2018)
B. From firm-level data											
Exit rate	4.9%	4.9%	4.9%	4.9%	4.9%	4.9%	3.3%	4.9%	4.8%	4.9%	US Census
Contr. entrants to growth	25.6%	25.7%	24.6%	25.6%	25.9%	25.9%	25.7%	25.7%	25.4%	25.7%	Akcigit and Kerr (2018)
Emp. share of fringe	21.1%	40.0%	29.3%	34.6%	18.2%	22.7%	18.2%	19.7%	18.5%	18.2%	US Census, NSF
C. Markup distribution											
1^{st} decile	13.8%	7.2%	9.9%	11.4%	11.1%	18.2%	17.1%	13.8%	13.0%	10.1%	Compustat
2^{nd} decile	17.7%	7.3%	13.0%	16.6%	12.8%	19.5%	17.8%	17.7%	17.0%	17.8%	Compustat
3^{rd} decile	20.7%	10.0%	16.1%	20.5%	15.8%	21.3%	19.9%	20.6%	20.2%	21.9%	Compustat
4^{th} decile	23.6%	12.5%	18.2%	24.8%	20.9%	24.5%	24.0%	23.0%	22.6%	25.0%	Compustat
5^{th} decile	27.6%	13.5%	20.7%	29.2%	26.1%	26.8%	26.3%	27.0%	27.0%	26.1%	Compustat
6^{th} decile	34.4%	17.3%	24.9%	34.9%	31.5%	30.2%	31.8%	32.8%	33.5%	30.2%	Compustat
7^{th} decile	42.3%	21.4%	32.0%	43.6%	40.9%	40.3%	40.3%	41.2%	41.6%	40.3%	Compustat
8^{th} decile	57.6%	29.5%	43.9%	59.2%	54.2%	60.7%	56.8%	56.5%	56.4%	67.5%	Compustat
9^{th} decile	98.6%	49.3%	74.0%	98.5%	98.3%	98.5%	98.1%	98.6%	99.7%	98.6%	Compustat

Notes: All data moments refer to the US manufacturing sector. Each column refers to a different robustness check, as listed in the notes to Table D.1.

Table D.3: The innovation feedback effect across different robustness checks.

	(1)	(2)	(3)	(4)	(5)
Variable	**$\text{BGP}_{\text{initial}}$**	**$\text{BGP}_{\text{final}}$**	**Total change**	**Impact**	**Transition**
Panel 1. *All markups 50% lower*					
Productivity growth	1.44%	1.58%	+0.14	+0.08	+0.06
Aggregate markup	21.65%	25.24%	+3.59	+1.18	+2.41
Trade share	14.61%	26.10%	+11.49	+10.46	+1.03
Panel 2. *All markups 25% lower*					
Productivity growth	1.43%	1.56%	+0.13	+0.08	+0.06
Aggregate markup	33.38%	36.99%	+3.60	−1.05	+4.65
Trade share	14.67%	26.15%	+11.48	+9.98	+1.50
Panel 3. *Fixing $\eta = 5$*					
Productivity growth	1.43%	1.57%	+0.14	+0.08	+0.06
Aggregate markup	41.45%	46.20%	+4.75	−0.87	+5.62
Trade share	14.59%	26.11%	+11.52	+10.05	+1.47
Panel 4. *Fixing $\eta = 10$*					
Productivity growth	1.45%	1.58%	+0.13	+0.07	+0.06
Aggregate markup	44.91%	48.68%	+3.77	−4.88	+8.65
Trade share	13.39%	23.91%	+10.52	+7.47	+3.05
Panel 5. *Fixed export cost κ*					
Productivity growth	1.38%	1.58%	+0.20	+0.27	−0.07
Aggregate markup	45.23%	51.27%	+6.04	−0.18	+6.22
Trade share	14.91%	26.60%	+11.69	+8.98	+2.71
Panel 6. *Setting $\psi = 1.5$*					
Productivity growth	1.31%	1.58%	+0.27	+0.18	+0.10
Aggregate markup	44.45%	51.47%	+7.02	−2.69	+9.72
Trade share	17.01%	30.36%	+13.36	+10.84	+2.51
Panel 7. *Setting $\psi = 2.5$*					
Productivity growth	1.48%	1.58%	+0.10	+0.05	+0.04
Aggregate markup	46.83%	49.06%	+2.23	−4.18	+6.41
Trade share	16.15%	28.81%	+12.66	+11.10	+1.55
Panel 8. *R&D in labor*					
Productivity growth	1.44%	1.58%	+0.14	.	.
Aggregate markup	44.84%	48.75%	+3.91	−3.59	+7.50
Trade share	14.96%	26.65%	+11.69	+9.56	+2.13

Notes: This table shows the evolution of markups and other variables in the transition between a high and a low trade cost BGP, for different parameter values (corresponding to different robustness checks). Parameter values are given in Table D.1. All differences in Columns (3) to (5) are stated in percentage points. The baseline calibration results are shown in Table 5. The algorithm that computes the transition dynamics between BGPs is described in Appendix C.3.

D.2 Welfare gains in manufacturing and in the overall economy

To adjust our welfare computations for the fact that manufacturing does not represent the entire economy, we consider the following extension of our baseline model. We assume that aggregate output is produced as Cobb-Douglas aggregation of manufacturing and service sector outputs:

$$\boldsymbol{Y} = \boldsymbol{Y}_M^{\alpha} \boldsymbol{Y}_S^{1-\alpha},$$

where $\alpha \in (0,1)$ is a parameter, M stands for the manufacturing sector, and S stands for the service sector. We assume that labour is sector-specific, and that manufacturing-sector R&D and entry costs are proportional to total manufacturing sales.[38] Finally, assume that service output grows exogenously at rate g_S. In this case, we can solve for manufacturing output and its growth rate exactly in the baseline model (because there are no interactions between the manufacturing and the service sector). However, the formula for welfare gains becomes

$$\gamma = \left(\frac{\boldsymbol{C}_{M,0}^{A}}{\boldsymbol{C}_{M,0}^{B}} \right)^{\alpha} e^{\frac{\alpha\left(g_M^A - g_M^B\right)}{\rho}} - 1.$$

Following Eaton and Kortum (2012), we set the spending share of manufacturing to $\alpha = 0.2$.

E Social Planner Problem

This section analyses the problem of a global social planner who sets equal weights on the welfare at Home and Foreign. As in the decentralised solution, we first solve for the optimal allocation of labour across firms, and then for the optimal innovation policies.

E.1 Optimal labour allocation

First, we determine the optimal allocation of labour across firms at any given point in time, conditional on the technology gap distribution. Lemma E.1 characterises the optimal allocation for the Home market. Allocations for the Foreign market are exactly analogous.

Lemma E.1 *The planner allocates labour $\ell_c^{H*}(\underline{n}) = \sigma_c^{H*}(\underline{n})$ to firm $c = H, C_H, F$ of industry $\underline{n} \equiv (n, n_C)$, where*

$$\sigma_H^{H*}(\underline{n}) = \left(1 + \frac{\omega_C}{\omega_H} (1+\lambda)^{-n_C(\eta-1)} + \frac{\omega_F}{\omega_H} \left(\frac{(1+\lambda)^{-n}}{\tau} \right)^{\eta-1} \right)^{-1}, \tag{E.1}$$

$$\sigma_{C_H}^{H*}(\underline{n}) = \left(1 + \frac{\omega_H}{\omega_C} (1+\lambda)^{n_C(\eta-1)} + \frac{\omega_F}{\omega_C} \left(\frac{(1+\lambda)^{n_C-n}}{\tau} \right)^{\eta-1} \right)^{-1}, \tag{E.2}$$

[38] Under this assumption, both profits and costs are proportional to αY_t. Then, the solution to the HJB equation and the free entry problem does not change.

$$\sigma_F^{H*}(\underline{n}) = \left(1 + \frac{\omega_H}{\omega_F}\left(\frac{\tau}{(1+\lambda)^{-n}}\right)^{\eta-1} + \frac{\omega_C}{\omega_F}\left(\frac{\tau}{(1+\lambda)^{n_C-n}}\right)^{\eta-1}\right)^{-1}. \tag{E.3}$$

Proof. The planner seeks to maximize world output, defined by $\boldsymbol{Y}^W = (\boldsymbol{Y}^H)^{\frac{1}{2}}(\boldsymbol{Y}^F)^{\frac{1}{2}}$, and chooses $\mathbb{L}_j = \left\{\ell_{jc}^k : c = H, C_k, F; k = H, F\right\}$ for each industry $j \in [0,1]$ in order to solve:

$$\max_{(\mathbb{L}_j)_{j\in[0,1]}} \left\{ \exp\left[\int_0^1 \ln\left((\omega_H)^{\frac{1}{\eta}}\left(q_{jH}\ell_{jH}^H\right)^{\frac{\eta-1}{\eta}} + (\omega_C)^{\frac{1}{\eta}}\left(q_{jC_H}\ell_{jC_H}^H\right)^{\frac{\eta-1}{\eta}} + (\omega_F)^{\frac{1}{\eta}}\left(\frac{q_{jF}\ell_{jF}^H}{\tau}\right)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \mathrm{d}j\right]^{\frac{1}{2}} \right.$$
$$\left. \times \exp\left[\int_0^1 \ln\left((\omega_F)^{\frac{1}{\eta}}\left(q_{jF}\ell_{jF}^F\right)^{\frac{\eta-1}{\eta}} + (\omega_C)^{\frac{1}{\eta}}\left(q_{jC_F}\ell_{jC_F}^F\right)^{\frac{\eta-1}{\eta}} + (\omega_H)^{\frac{1}{\eta}}\left(\frac{q_{jH}\ell_{jH}^F}{\tau}\right)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \mathrm{d}j\right]^{\frac{1}{2}} \right\},$$

subject to:

$$\int_0^1 (\ell_{jH}^H + \ell_{jC_H}^H + \ell_{jH}^F)\mathrm{d}j \le 1,$$
$$\int_0^1 (\ell_{jF}^F + \ell_{jC_F}^F + \ell_{jF}^H)\mathrm{d}j \le 1,$$
$$\ell_{jH}^H, \ell_{jC_H}^H, \ell_{jF}^H, \ell_{jF}^F, \ell_{jC_F}^F, \ell_{jH}^F \ge 0, \quad \forall j$$

Let $\beta^k \ge 0$ be the Lagrange multiplier on the feasibility constraint for country $k = H, F$. The optimality conditions yield:

$$\ell_{jH}^H = \left(\frac{\frac{1}{2}\boldsymbol{Y}^W}{\beta^H}\right)^{\eta}\omega_H\left(\frac{q_{jH}}{Y_j^H}\right)^{\eta-1}, \quad \ell_{jC_H}^H = \left(\frac{\frac{1}{2}\boldsymbol{Y}^W}{\beta^H}\right)^{\eta}\omega_C\left(\frac{q_{jC_H}}{Y_j^H}\right)^{\eta-1}, \quad \ell_{jH}^F = \left(\frac{\frac{1}{2}\boldsymbol{Y}^W}{\beta^H}\right)^{\eta}\omega_H\left(\frac{q_{jH}/\tau}{Y_j^F}\right)^{\eta-1},$$
$$\ell_{jF}^F = \left(\frac{\frac{1}{2}\boldsymbol{Y}^W}{\beta^F}\right)^{\eta}\omega_F\left(\frac{q_{jF}}{Y_j^F}\right)^{\eta-1}, \quad \ell_{jC_F}^F = \left(\frac{\frac{1}{2}\boldsymbol{Y}^W}{\beta^F}\right)^{\eta}\omega_C\left(\frac{q_{jC_F}}{Y_j^F}\right)^{\eta-1}, \quad \ell_{jF}^H = \left(\frac{\frac{1}{2}\boldsymbol{Y}^W}{\beta^F}\right)^{\eta}\omega_F\left(\frac{q_{jF}/\tau}{Y_j^H}\right)^{\eta-1}.$$

Using the formula for H's industry output we obtain:

$$\left(\frac{Y_j^H}{\frac{1}{2}\boldsymbol{Y}^W}\right)^{\eta-1} = \left(\frac{1}{\beta^H}\right)^{\eta-1}\omega_H(q_{jH})^{\eta-1} + \left(\frac{1}{\beta^H}\right)^{\eta-1}\omega_C(q_{jC_H})^{\eta-1} + \left(\frac{1}{\beta^F}\right)^{\eta-1}\omega_F\left(\frac{q_{jF}}{\tau}\right)^{\eta-1}. \tag{E.4}$$

Imposing feasibility in H, i.e. $\int_0^1\left(\ell_{jH}^H + \ell_{jC_H}^H + \ell_{jH}^F\right)\mathrm{d}j = 1$, gives:

$$1 = \left(\frac{\frac{1}{2}\boldsymbol{Y}^W}{\beta^H}\right)^{\eta}\int_0^1\left(\omega_H\left(\frac{q_{jH}}{Y_j^H}\right)^{\eta-1} + \omega_C\left(\frac{q_{jC_H}}{Y_j^H}\right)^{\eta-1} + \omega_F\left(\frac{q_{jH}/\tau}{Y_j^F}\right)^{\eta-1}\right)\mathrm{d}j. \tag{E.5}$$

The counterparts of (E.4)-(E.5) for F give us four equations and four unknowns, $(\beta^H, \beta^F, Y_j^H, Y_j^F)$. Plugging (E.4) into (E.5) yields:

$$(\beta^H)^\eta = \frac{1}{2}\boldsymbol{Y}^W \int_0^1 \left(\frac{\omega_H (q_{jH})^{\eta-1}}{\left(\frac{1}{\beta^H}\right)^{\eta-1} \omega_H (q_{jH})^{\eta-1} + \left(\frac{1}{\beta^H}\right)^{\eta-1} \omega_C (q_{jC_H})^{\eta-1} + \left(\frac{1}{\beta^F}\right)^{\eta-1} \omega_F \left(\frac{q_{jF}}{\tau}\right)^{\eta-1}} \right.$$
$$+ \frac{\omega_C (q_{jC_H})^{\eta-1}}{\left(\frac{1}{\beta^H}\right)^{\eta-1} \omega_H (q_{jH})^{\eta-1} + \left(\frac{1}{\beta^H}\right)^{\eta-1} \omega_C (q_{jC_H})^{\eta-1} + \left(\frac{1}{\beta^F}\right)^{\eta-1} \omega_F \left(\frac{q_{jF}}{\tau}\right)^{\eta-1}}$$
$$\left. + \frac{\omega_F \left(\frac{q_{jH}}{\tau}\right)^{\eta-1}}{\left(\frac{1}{\beta^F}\right)^{\eta-1} \omega_F (q_{jF})^{\eta-1} + \left(\frac{1}{\beta^F}\right)^{\eta-1} \omega_C (q_{jC_F})^{\eta-1} + \left(\frac{1}{\beta^H}\right)^{\eta-1} \omega_H \left(\frac{q_{jH}}{\tau}\right)^{\eta-1}} \right) \mathrm{d}j. \tag{E.6}$$

As in the decentralised allocation, we focus on a symmetric solution where $\beta^H = \beta^F = \beta$. Then Equation (E.6) gives $\beta = \frac{1}{2}\boldsymbol{Y}^W$. Given this, Equation (E.4) gives:

$$Y_j^H = \left[\omega_H (q_{jH})^{\eta-1} + \omega_C (q_{jC_H})^{\eta-1} + \omega_F \left(\frac{q_{jF}}{\tau} \right)^{\eta-1} \right]^{\frac{1}{\eta-1}}, \tag{E.7a}$$

and similarly for F:

$$Y_j^F = \left[\omega_F (q_{jF})^{\eta-1} + \omega_C (q_{jC_F})^{\eta-1} + \omega_H \left(\frac{q_{jH}}{\tau} \right)^{\eta-1} \right]^{\frac{1}{\eta-1}}. \tag{E.7b}$$

Total world output $\ln \boldsymbol{Y}^W = \frac{1}{2} \ln \boldsymbol{Y}^H + \frac{1}{2} \ln \boldsymbol{Y}^F$ is then given by:

$$\ln \boldsymbol{Y}^W = \frac{1}{2} \left(\int_0^1 \ln Y_j^H \mathrm{d}j + \int_0^1 \ln Y_j^F \mathrm{d}j \right). \tag{E.8}$$

The labour allocation within industries and across countries is, then:

$$\ell_{jH}^H = \omega_H \left(\frac{q_{jH}}{Y_j^H} \right)^{\eta-1}, \quad \ell_{jC_H}^H = \omega_C \left(\frac{q_{jC_H}}{Y_j^H} \right)^{\eta-1}, \quad \ell_{jH}^F = \omega_H \left(\frac{q_{jH}/\tau}{Y_j^F} \right)^{\eta-1},$$
$$\ell_{jF}^F = \omega_F \left(\frac{q_{jF}}{Y_j^F} \right)^{\eta-1}, \quad \ell_{jC_F}^F = \omega_C \left(\frac{q_{jC_F}}{Y_j^F} \right)^{\eta-1}, \quad \ell_{jF}^H = \omega_F \left(\frac{q_{jF}/\tau}{Y_j^H} \right)^{\eta-1}. \tag{E.9}$$

In particular, labour for use in the domestic market (by the leader and the fringe) is:

$$\ell_{jH}^H = \frac{\omega_H}{\omega_H + \omega_C \left(\frac{q_{jC_H}}{q_{jH}} \right)^{\eta-1} + \omega_F \left(\frac{q_{jF}/\tau}{q_{jH}} \right)^{\eta-1}}, \tag{E.10a}$$

$$\ell^H_{jC_H} = \frac{\omega_C}{\omega_H \left(\frac{q_{jH}}{q_{jC_H}}\right)^{\eta-1} + \omega_C + \omega_F \left(\frac{q_{jF}/\tau}{q_{jC_H}}\right)^{\eta-1}}, \tag{E.10b}$$

$$\ell^F_{jF} = \frac{\omega_F}{\omega_F + \omega_C \left(\frac{q_{jC_F}}{q_{jF}}\right)^{\eta-1} + \omega_H \left(\frac{q_{jH}/\tau}{q_{jF}}\right)^{\eta-1}}, \tag{E.10c}$$

$$\ell^F_{jC_F} = \frac{\omega_C}{\omega_F \left(\frac{q_{jF}}{q_{jC_F}}\right)^{\eta-1} + \omega_C + \omega_H \left(\frac{q_{jH}/\tau}{q_{jC_F}}\right)^{\eta-1}}. \tag{E.10d}$$

To find labour use for the export market, first use (E.7b) to note that:

$$\left(\frac{Y^F_j}{q_{jH}/\tau}\right)^{\eta-1} = \omega_F \left(\frac{q_{jF}}{q_{jH}/\tau}\right)^{\eta-1} + \omega_C \left(\frac{q_{jC_F}}{q_{jH}/\tau}\right)^{\eta-1} + \omega_H,$$
$$\left(\frac{Y^H_j}{q_{jF}/\tau}\right)^{\eta-1} = \omega_F \left(\frac{q_{jH}}{q_{jF}/\tau}\right)^{\eta-1} + \omega_C \left(\frac{q_{jC_H}}{q_{jF}/\tau}\right)^{\eta-1} + \omega_H.$$

Therefore, labour use for exports by H and F firms, respectively, is:

$$\ell^F_{jH} = \frac{\omega_H}{\omega_F \left(\frac{q_{jF}}{q_{jH}/\tau}\right)^{\eta-1} + \omega_C \left(\frac{q_{jC_F}}{q_{jH}/\tau}\right)^{\eta-1} + \omega_H}, \tag{E.10e}$$

$$\ell^H_{jF} = \frac{\omega_F}{\omega_H \left(\frac{q_{jH}}{q_{jF}/\tau}\right)^{\eta-1} + \omega_C \left(\frac{q_{jC_H}}{q_{jF}/\tau}\right)^{\eta-1} + \omega_F}. \tag{E.10f}$$

In the BGP, we may identify each industry j by a pair of technology gaps $\underline{n} = (n, n_C)$, holding $\frac{q_{jH}}{q_{jF}} = (1+\lambda)^n$ and $\frac{q_{jH}}{q_{jC_H}} = \frac{q_{jH}}{q_{jC_F}} = (1+\lambda)^{n_C}$. Using these into Equations (E.10a)-(E.10f) completes the proof of Lemma E.1. ■

The planner allocates equal labour to all industries, as $\sigma^{H*}_H(\underline{n}) + \sigma^{H*}_{C_H}(\underline{n}) + \sigma^{H*}_F(\underline{n}) = 1$. Labour within each industry is allocated according to a weighted average of relative qualities (the term involving a ratio of ω's), relative productivities (the term involving λ) and trade costs (the term involving τ) across firms. Statically, this allocation differs from the DE solution because of dispersion in markups between firms. Indeed, recall that the DE labour allocation (equation (16)) holds

$$\ell^H_c(\underline{n}) = \sigma^H_c(\underline{n}) \cdot \left(\frac{\mu^H_c(\underline{n})}{\boldsymbol{\mu}}\right)^{-1}, \tag{E.11}$$

where $\boldsymbol{\mu}$ is the aggregate markup, defined in equation (19). Moreover, by definition of $\sigma^H_c(\underline{n})$ and the formula for equilibrium prices we have:

$$\sigma^H_H(\underline{n}) = \left(1 + \frac{\omega_C}{\omega_H}(1+\lambda)^{-n_C(\eta-1)} \left(\frac{\mu^H_H(\underline{n})}{\mu^H_{C_H}(\underline{n})}\right)^{\eta-1} + \frac{\omega_F}{\omega_H}\left(\frac{(1+\lambda)^{-n}}{\tau}\right)^{\eta-1}\left(\frac{\mu^H_H(\underline{n})}{\mu^H_F(\underline{n})}\right)^{\eta-1}\right)^{-1},$$

and similarly for $\sigma^H_{C_H}(\underline{n})$ and $\sigma^H_F(\underline{n})$. Notice that the only difference between $\ell^H_c(\underline{n})$ from Equation (E.11) and $\ell^{H*}_c(\underline{n})$ from Lemma E.1 are the terms involving ratios of markups, i.e. the presence of markup dispersion. Indeed, the two allocations coincide when $\mu^H_H(\underline{n}) = \mu^H_{C_H}(\underline{n}) = \mu^H_F(\underline{n}) = \boldsymbol{\mu}$. Moreover, as $\mu^H_{C_H}(\underline{n}) = 1$ by assumption, then the static labour allocation in the DE coincides with the SP solution when all firms charge zero markups in all industries.

Dispersion in markups generates both within- and across-industry misallocation. To study within-industry misallocation, we can show that industry-level TFP (the ratio of industry output to industry labour use) for the DE can be written in terms of market shares and markups as follows:[39]

$$TFP(\underline{n}) = \left(\frac{\omega_H}{\sigma^H_H(\underline{n})}\right)^{\frac{1}{\eta-1}} \left(\sigma^H_H(\underline{n}) + \sigma^H_{C_H}(\underline{n})\frac{\mu^H_H(\underline{n})}{\mu^H_{C_H}(\underline{n})} + \sigma^H_F(\underline{n})\frac{\mu^H_H(\underline{n})}{\mu^H_F(\underline{n})}\right)^{-1}. \tag{E.12}$$

Setting markups to zero and using $\sigma^{H*}_H(\underline{n}) + \sigma^{H*}_{C_H}(\underline{n}) + \sigma^{H*}_F(\underline{n}) = 1$ gives us the TFP level in the SP allocation:

$$TFP^*(\underline{n}) = \left(\frac{\omega_H}{\sigma^{H*}_H(\underline{n})}\right)^{\frac{1}{\eta-1}}. \tag{E.13}$$

The green surface on the left panel of Figure E.1 plots TFP losses across industries, computed as $\frac{TFP(\underline{n})}{TFP^*(\underline{n})}$, for our set of calibrated parameters. As seen in the figure, TFP in the DE is lowest in industries where markups are more dispersed, i.e. in industries where the technology gap between firms is the largest. TFP losses are entirely driven by within-industry markup dispersion: in industries with large technology gaps, static TFP losses are as high as 14%, where the planner would choose to give all production to one firm but this firms finds it more profitable to charge high markups instead. In contrast, in industries where the markup dispersion is lowest, TFP losses are close to zero. This occurs for a slightly negative value for n, as in this industry the slight technological disadvantage of the Home leader exactly offsets the trade cost disadvantage of the Foreign leader.

Labour is also misallocated across industries, as seen on the right panel of Figure E.1. As noted before, the planner wants to allocate equal labour to all industries. In the DE, however, industries with higher within-industry misallocation receive too little labour, as technological leaders in these industries are too small relative to the social optimum. This labour is allocated into neck-to-neck industries instead, where firms are inefficiently large.

Trade has interesting effects on the amount of static misallocation and labour reallocation across industries. In Figure 10 we saw that moving from high to low trade costs in BGP depresses markups for the industries with larger technological leads, thereby reducing their levels of misallocation. As seen on the right panel of Figure E.1, labour is reallocated away from industries where the Home leader is lagging the most. This occurs because, as discussed in the previous section, a decrease in trade costs increases the innovation incentives of neck-to-neck firms, leading to a shift in the composition of firms toward states with larger technology gaps.

[39] To obtain this formula, we use Equation (4) and normalise $q_H = 1$ to arrive at industry output $Y(\underline{n}) = \left(\frac{\omega_H}{\sigma^H_H(\underline{n})}\right)^{\frac{1}{\eta-1}} \frac{\boldsymbol{\mu}}{\mu^H_H(\underline{n})}$. Moreover, from Equation (E.11) we have that total industry labour use is $L(\underline{n}) = \sum_{c=H,C_H,F} \sigma^H_c(\underline{n}) \left(\frac{\mu^H_c(\underline{n})}{\boldsymbol{\mu}}\right)^{-1}$. Taking the ratio gives Equation (E.12).

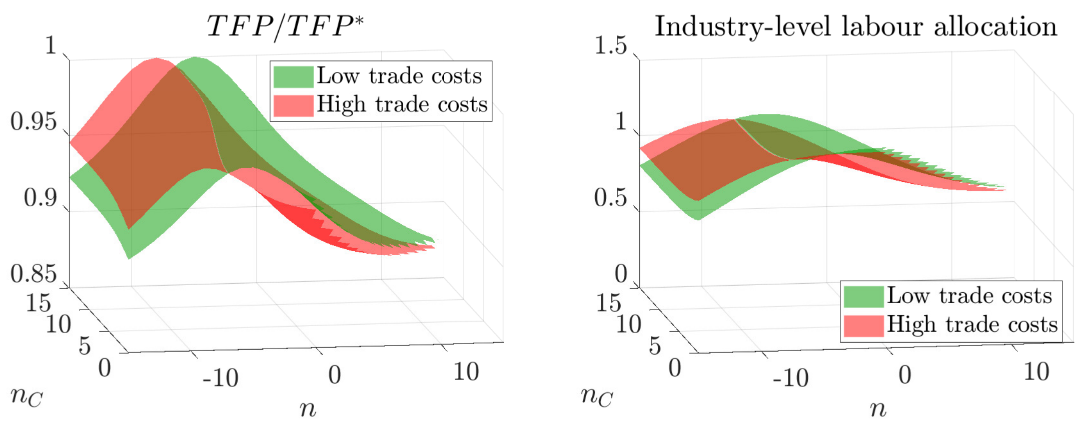

Figure E.1: Comparison between high and low trade costs for the industry TFP relative to efficient level (left) and labour allocation across industries (right).

Notes: The parameter values are given in Table 3.

Computing misallocation as the gap between the DE and SP initial levels of output, or $1 - \boldsymbol{Y}_0/\boldsymbol{Y}_0^*$, we find that moving from high to low trade costs increases this gap by 8.5% (from a loss due to static misallocation of 11.68% under the high trade costs to a loss of 12.67% in the BGP with low trade costs). This long-run increase in misallocation is once again due to the innovation feedback effect: as a result of the increase in the polarisation of industries, markup dispersion increases, which implies a larger static loss in allocative efficiency. This is in contrast to trade models with endogenous markups that do not feature the feedback effect, where trade is usually found to reduce misallocation. For instance, Edmond *et al.* (2015) find that opening up to trade (from autarky) reduces misallocation by one-fifth.

E.2 Optimal innovation policies

We now turn to characterising the optimal innovation choices of the planner. Given the optimal labour allocation (Lemma E.1), the planner chooses R&D investment for each firm, $\{z_H^*(n,n_C), x_H^*(n,n_C)\}$, and the distribution of firms across states, $\{\varphi^*(n,n_C)\}$, to maximise social welfare in BGP. The following lemma describes the optimal dynamic choices of the planner (once again, we focus on country H as solutions for F are symmetric).

Lemma E.2 *The dynamic choices of the constrained SP in country H satisfy*

$$\frac{\chi_i\psi_i\,[z_H^*(n,n_C)]^{\psi_i-1}}{1-\boldsymbol{r}^{H*}} = \frac{\chi_e\psi_e\,[x_H^*(n,n_C)]^{\psi_e-1}}{1-\boldsymbol{r}^{H*}} = \frac{\ln(1+\lambda)}{\rho} + \nu(n+1,n_C+1) - \nu(n,n_C), \tag{E.14}$$

where $\boldsymbol{r}^{H*}$ is the aggregate R&D share of GDP (from Equation (28)), and $\nu(n,n_C)$ is the shadow value of an innovation.

Proof. We solve for the planner's solution under balanced growth, so aggregate world output must grow at a constant rate, g^W. The objective function is:

$$\int_0^{+\infty} e^{-\rho t} \ln(\boldsymbol{C}_t^W) \mathrm{d}t = \frac{1}{\rho}\left(\ln(\boldsymbol{C}_0^W) + \frac{g^W}{\rho}\right),$$

as $\boldsymbol{C}_t^W = \boldsymbol{C}_0^W e^{g^W t}$. The planner chooses:

1. An initial consumption level, $\boldsymbol{C}_0^W$, and a growth rate, g^W.
2. Innovation intensities in each country: $\Big\{(z_k(n,n_C), x_k(n,n_C)) : (n,n_C) \in \mathbb{Z} \times \mathbb{Z}_+, k = H, F\Big\}$.
3. A stationary distribution of firms across states in each country: $\Big\{\varphi_k(n,n_C) : (n,n_C) \in \mathbb{Z} \times \mathbb{Z}_+, k = H, F\Big\}$.

The planner is subject to the following constraints:

- First, the planner recognises the symmetric nature of the technology gap distribution: for every leader that is ahead in one country and given industry, there must be a leader that is behind in the other country in the same industry. Formally:

$$\varphi_H(n,n_C) = \varphi_F(-n, n_C - n).$$

  Given this symmetry, henceforth we will write $\varphi(n,n_C)$ generically to speak about the distribution from the point of view of a domestic firm in H.

- Second, the planner takes into account each country's resource constraint: $\frac{\boldsymbol{C}_t^k}{\boldsymbol{Y}_t^k} + \frac{\boldsymbol{R}_t^k}{\boldsymbol{Y}_t^k} \leq 1$, where:

$$\boldsymbol{r}^k \equiv \frac{\boldsymbol{R}_t^k}{\boldsymbol{Y}_t^k} = \sum_n \sum_{n_C} \varphi(n,n_C) \left[\chi_i \left(z_k(n,n_C)\right)^{\psi_i} + \chi_e \left(x_k(n,n_C)\right)^{\psi_e}\right]. \tag{E.15}$$

  Therefore, $\ln \boldsymbol{C}_t^W = \ln \boldsymbol{Y}_t^W + \frac{1}{2}\sum_{k=H,F} \ln(1 - \boldsymbol{r}^k)$, where $\ln \boldsymbol{Y}_t^W = \ln \boldsymbol{Y}_0^W + g^W t$, and:

$$\ln \boldsymbol{Y}_0^W = \frac{1}{2}\sum_k \sum_n \sum_{n_C} \varphi(n,n_C) \ln Y^k(n,n_C),$$

  from the static planner solution (Equation (E.8)). Using (E.7a)-(E.7b) and normalizing $q_H = 1$, we have:

$$Y^H(n,n_C) = \left[\omega_H + \omega_C (1+\lambda)^{-n_C(\eta-1)} + \omega_F \left(\frac{(1+\lambda)^{-n}}{\tau}\right)^{\eta-1}\right]^{\frac{1}{\eta-1}},$$

$$Y^F(n,n_C) = \left[\omega_F (1+\lambda)^{-n(\eta-1)} + \omega_C (1+\lambda)^{(n-n_C)(\eta-1)} + \omega_H \left(\frac{1}{\tau}\right)^{\eta-1}\right]^{\frac{1}{\eta-1}}.$$

- Third, the planner takes into account two laws of motion: (i) the law of motion for the φ distribution (written below in the planner's program); and (ii) the law of motion for aggregate output, $\dot{\boldsymbol{Y}}_t^W = \boldsymbol{Y}_t^W g^W$. For the latter, we have:

$$g^W \equiv \frac{\partial}{\partial t} \ln \mathbf{Y}_t^W = \frac{1}{2}\Bigg(\underbrace{\frac{\partial}{\partial t} \int_0^1 \ln Y_j^H \mathrm{d}j}_{\equiv g^H} + \underbrace{\frac{\partial}{\partial t} \int_0^1 \ln Y_j^F \mathrm{d}j}_{\equiv g^F} \Bigg) = \frac{g^H + g^F}{2},$$

where

$$\begin{aligned}
\int_0^1 \ln Y_j^H \mathrm{d}j &= \int_0^1 \left(\frac{1}{\eta-1}\right) \ln \left(\omega_H (q_{jH})^{\eta-1} + \omega_C (q_{jC_H})^{\eta-1} + \omega_F \left(\frac{q_{jF}}{\tau}\right)^{\eta-1} \right) \mathrm{d}j \\
&= \int_0^1 \left(\frac{1}{\eta-1}\right) \left[\ln\left((q_{jH})^{\eta-1}\right) + \ln \left(\omega_H + \omega_C \left(\frac{q_{jC_H}}{q_{jH}}\right)^{\eta-1} + \omega_F \left(\frac{q_{jF}/\tau}{q_{jH}}\right)^{\eta-1} \right) \right] \mathrm{d}j \\
&= \int_0^1 \ln(q_{jH}) \mathrm{d}j + \int_0^1 \left(\frac{1}{\eta-1}\right) \left[\ln \left(\omega_H + \omega_C \left(\frac{q_{jC_H}}{q_{jH}}\right)^{\eta-1} + \omega_F \left(\frac{q_{jF}/\tau}{q_{jH}}\right)^{\eta-1} \right) \right] \mathrm{d}j,
\end{aligned}$$

and similarly for F. In a balanced-growth solution, the second additive term on the last line is constant. Thus, following the same logic as in the proof of Lemma 3.1:

$$g^W = \frac{\ln(1+\lambda)}{2} \sum_{k=H,F} \left(\sum_{n=-\infty}^{+\infty} \sum_{n_C=0}^{+\infty} \varphi(n,n_C) i_k(n,n_C) + \zeta \sum_{n=-\infty}^{-1} \sum_{n_C=0}^{+\infty} \varphi(n,n_C)|n| \right).$$

We are now ready to write down the dynamic problem:

$$\begin{aligned}
\max_{\substack{\{z_H(n,n_C),x_H(n,n_C)\} \\ \{z_F(n,n_C),x_F(n,n_C)\} \\ \{\varphi(n,n_C)\}}} \Bigg\{ & \sum_{k=H,F} \sum_{n=-\infty}^{+\infty} \sum_{n_C=0}^{+\infty} \varphi(n,n_C) \ln Y^k(n,n_C) \\
& + \frac{\ln(1+\lambda)}{\rho} \sum_{k=H,F} \left(\sum_{n=-\infty}^{+\infty} \sum_{n_C=0}^{+\infty} \varphi(n,n_C) i_k(n,n_C) + \zeta \sum_{n=-\infty}^{-1} \sum_{n_C=0}^{+\infty} \varphi(n,n_C)|n| \right) \\
& + \sum_{k=H,F} \ln \left(1 - \sum_{n=-\infty}^{+\infty} \sum_{n_C=0}^{+\infty} \varphi(n,n_C) \left[\chi_i \left(z_k(n,n_C)\right)^{\psi_i} + \chi_e \left(x_k(n,n_C)\right)^{\psi_e} \right] \right) \Bigg\}
\end{aligned}$$

subject to:

$$0 = -\varphi(0,0) \sum_{k=H,F} i_k(0,0) + \zeta\Big(1 - \varphi(0,0)\Big) + i_F(1,0)\varphi(1,0) \tag{E.16a}$$

$$\begin{aligned}
\forall (n,n_C) \neq (0,0): \; 0 = & -\varphi(n,n_C) \left(\sum_{k=H,F} i_k(n,n_C) + \zeta \right) + \mathbf{1}_{(n_C>0)} i_H(n-1,n_C-1)\varphi(n-1,n_C-1) \\
& + i_F(n+1,n_C)\varphi(n+1,n_C)
\end{aligned} \tag{E.16b}$$

$$0 \leq \varphi(n,n_C), z_H(n,n_C), x_H(n,n_C), z_F(n,n_C), x_F(n,n_C) \tag{E.16c}$$

$$1 = \sum_{n=-\infty}^{+\infty} \sum_{n_C=0}^{+\infty} \varphi(n,n_C); \tag{E.16d}$$

where we have defined the total innovation rates of each country as $i_H(n,n_C) \equiv z_H(n,n_C) + x_H(n,n_C)$ and $i_F(n,n_C) \equiv z_F(n,n_C) + x_F(n,n_C)$. The Lagrangian is:

$$\begin{aligned}
\mathcal{L} = & \sum_k \sum_{n=-\infty}^{+\infty} \sum_{n_C=0}^{+\infty} \varphi(n,n_C) \ln Y^k(n,n_C) + \frac{\ln(1+\lambda)}{\rho} \sum_k \left(\sum_{n=-\infty}^{+\infty} \sum_{n_C=0}^{+\infty} \varphi(n,n_C) i_k(n,n_C) + \zeta \sum_{n=-\infty}^{-1} \sum_{n_C=0}^{+\infty} \varphi(n,n_C)|n| \right) \\
& + \sum_k \ln \left(1 - \sum_{n=-\infty}^{+\infty} \sum_{n_C=0}^{+\infty} \varphi(n,n_C) \left[\chi_i \left(z_k(n,n_C)\right)^{\psi_i} + \chi_e \left(x_k(n,n_C)\right)^{\psi_e} \right] \right) \\
& + \nu(0,0) \left[-\varphi(0,0) \sum_k i_k(0,0) + \zeta \Big(1 - \varphi(0,0)\Big) + i_F(1,0)\varphi(1,0) \right] \\
& + \sum_{(n,n_C) \neq (0,0)} \nu(n,n_C) \Bigg[-\varphi(n,n_C) \left(\sum_k i_k(n,n_C) + \zeta \right) \\
& \qquad + \mathbf{1}_{(n_C>0)} i_H(n-1,n_C-1)\varphi(n-1,n_C-1) + i_F(n+1,n_C)\varphi(n+1,n_C) \Bigg] \\
& - \sum_{n=-\infty}^{+\infty} \sum_{n_C=0}^{+\infty} \left[\vartheta^{\varphi}(n,n_C)\varphi(n,n_C) + \sum_k \Big(\vartheta^{z_k}(n,n_C) z_k(n,n_C) + \vartheta^{x_k}(n,n_C) x_k(n,n_C) \Big) \right] \\
& + \phi \left[1 - \sum_{n=-\infty}^{+\infty} \sum_{n_C=0}^{+\infty} \varphi(n,n_C) \right],
\end{aligned}$$

where $\nu(n,n_C) \geq 0$ is the multiplier on (E.16a)-(E.16b); $\vartheta^{\varphi}(n,n_C), \vartheta^{z_k}(n,n_C), \vartheta^{x_k}(n,n_C) \geq 0$ are the multipliers on (E.16c); and $\phi \geq 0$ is the multiplier on (E.16d). The first-order conditions for z_k and x_k yield:

$$\frac{\chi_i \psi_i \left[z_k(n,n_C)\right]^{\psi_i - 1}}{1 - \boldsymbol{r}^k} + \frac{\vartheta^{z_k}(n,n_C)}{\varphi(n,n_C)} = \frac{\ln(1+\lambda)}{\rho} + \Delta_k(n,n_C), \tag{E.17a}$$

$$\frac{\chi_e \psi_e \left[x_k(n,n_C)\right]^{\psi_e - 1}}{1 - \boldsymbol{r}^k} + \frac{\vartheta^{x_k}(n,n_C)}{\varphi(n,n_C)} = \frac{\ln(1+\lambda)}{\rho} + \Delta_k(n,n_C), \tag{E.17b}$$

respectively, where we have defined:

$$\Delta_k(n,n_C) \equiv \begin{cases} \nu(n+1,n_C+1) - \nu(n,n_C) & \text{if } k = H \\ \nu(n-1,n_C) - \nu(n,n_C) & \text{if } k = F \end{cases}$$

and $\boldsymbol{r}^k$ is given by Equation (E.15). The result in Lemma E.2 corresponds to the interior equilibrium of the above optimality conditions, where $\vartheta^{z_k} = \vartheta^{x_k} = 0$. ■

Discussion All in all, the DE solution differs from the SP allocation because of static and dynamic inefficiencies. Statically, labour is misallocated across firms and industries in the DE because different firms charge different markups. Dynamically, the DE solution has a suboptimal allocation of resources between consumption and R&D, and of R&D between firms. The social and private returns to innovation differ for two reasons: (i) firms do not internalise that future innovators will benefit from their own innovations (a positive externality); and (ii) firms do not internalise that part of their (private) gains from innovation are associated with a decrease in the value of other firms through business stealing (a negative externality).

In the SP solution, consumption-equivalent welfare gains for moving from the high to the low trade cost BGP are equal to 15.75%. This number is similar to the the one we found for the DE solution (15.04%). Even though the welfare gains are similar, they are the result of three main forces in the DE solution that differ from those in the SP solution. First, there are direct gains from trade coming from lower (iceberg) trade costs which are present both in the DE and SP solutions. Second, there are welfare losses in the DE coming from an increase in misallocation as we move from high to low trade costs, as we discussed at the end of Section E.1. Third, there is underinvestment in terms of R&D in the DE, and productivity growth is inefficiently low. In particular, we find that the SP growth rate is almost invariant to the level of trade costs (increasing only by 0.8% when moving from high to low trade cost BGP). In the DE, lower trade costs alleviate this problem, making R&D investment closer to the first-best as we move from high to low trade costs.