Advertising, Innovation and Economic Growth*

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Abstract

This paper analyzes the implications of advertising for firm dynamics and economic growth through its interaction with R&D. We develop a model of endogenous growth with firm heterogeneity that incorporates advertising decisions and calibrate it to match several empirical regularities across firm size. Our model provides microfoundations for the empirically observed negative relationship between both firm R&D intensity and growth, and firm size. In the calibrated model, about half of the deviation from proportional firm growth is attributed to our novel advertising channel. In addition, R&D and advertising are substitutes, a prediction for which we find evidence in the data.

JEL codes: E20; L10; M30; O31, O32, O33, and O41.

Keywords: Endogenous Growth; Advertising; Innovation; Research and Development; Firm Dynamics; Policy.

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1 Introduction

Ever since the seminal contributions of Romer (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992), the economic literature has emphasized the role of innovation in the process of economic growth. Research and Development (R&D) introduces goods of higher-quality and enhanced production technologies which raise living standards. From a firm's perspective, innovation is used strategically to increase profits: by performing R&D, firms can increase their market share by selling goods of higher quality and diverting demand away from lower-quality ones. Yet, while this process and its implications for economic growth are well understood in the literature, there exists a variety of other tools that firms may use to direct demand towards their products.[1]

One non-negligible example is advertising. Like R&D, advertising expenditures represent a sizable share of aggregate economic activity. While the share of R&D expenditures over GDP fluctuated between 2.27% and 2.82% in the period 1980-2013, over the same time window firms in the United States spent on average around 2.2% of GDP on advertising each year.[2] By advertising their products, firms can alter consumer preferences and ultimately increase product-specific profits. From this point of view, innovation and advertising may be substitutable tools in firms' quest for higher profits, as both shift demand toward specific products. From another perspective, however, advertising and innovation expenditures may also be complementary, as the former raises the return to innovation by increasing market shares of new products. In either interpretation, advertising decisions are not neutral in terms of innovation decisions and may, therefore, impact global economic growth. Yet, the literature has remained relatively silent on the interaction between R&D and advertising expenditures and its potential impact on firm growth, firm dynamics, and economic development.

In this paper, we fill this gap by asking how advertising affects R&D investment decisions at the firm level, and study the implications for firm and aggregate dynamics. Our main

[1] A recent trend in macroeconomic has investigated several types of intangible investment and their potential implications for the overall economy. See for instance, McGrattan and Prescott (2014), McGrattan (2017), Gourio and Rudanko (2014a), Hall (2014), Molinari and Turino (2015), Gourio and Rudanko (2014b), Arkolakis (2010, 2016), and Atkeson and Kehoe (2005).

[2] *R&D data source:* OECD data, available at http://data.oecd.org/rd/gross-domestic-spending-on-r-d.htm. *Advertising data source:* Coen Structured Advertising Expenditure Dataset, extracted from the McCann Erikson advertising agency (available at http://www.purplemotes.net/2008/09/14/us-advertising-expenditure-data/).

contribution is twofold. First, we show that the interaction between R&D and advertising has important implications for firm dynamics and can explain a significant share of the deviation from the so-called Gibrat's law of proportional firm growth. Second, we find that advertising and R&D are substitutes both in our model and in the data. This has consequences for innovation and economic policy as advertising is shown to be detrimental to growth.

We build on the Akcigit and Kerr (2018) model of endogenous growth through R&D, which we extend to incorporate explicit advertising decisions. Firms are heterogenous in their portfolios of goods, which they monopolistically supply. Each product's quality grows on a ladder through innovation arising from investment in R&D, which can take two forms. Through internal R&D, firms can increase the quality of their own goods. External R&D, on the other hand, enables incumbent firms and potential entrants to improve on the quality of a good that they do not own and displace the former producer through creative destruction. New to our model, firms can also use advertising to expand their market shares and profits. Advertising is used to influence the perception that consumers have of product quality, thereby altering preferences and ultimately shifting demand toward those goods.

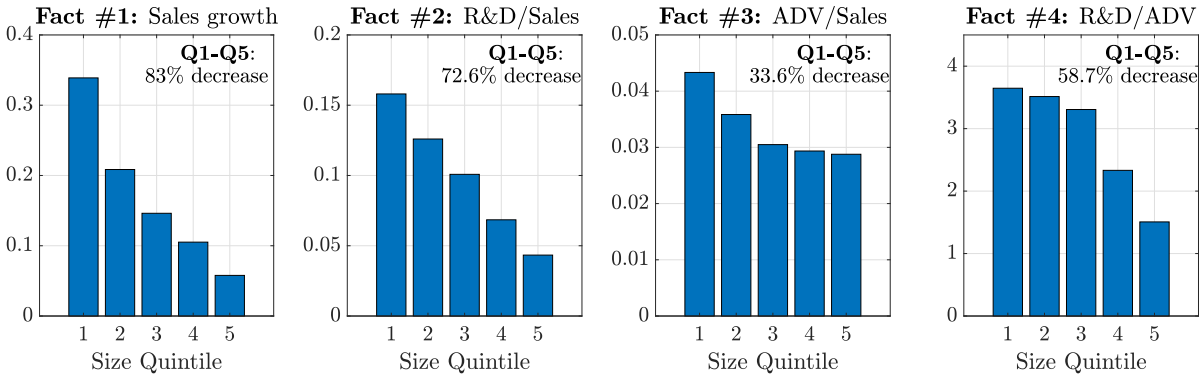

Figure 1: Average firm growth, R&D intensity, advertising intensity, and R&D-advertising ratio, across firm sales quintiles, and percentage decrease from first to fifth quintile.
Notes: Firms are ranked in sales quintiles according to their normalized level of sales (sales as a ratio of average sales in the same year). R&D and advertising intensities are measured as the ratio of total R&D and advertising expenditures to total sales within each group.

We calibrate the model via indirect inference to match four empirical facts related to firm dynamics and expenditure across firm size. In Section 2 we present these facts in detail and show that they are robust findings of the U.S. economy. Figure 1 offers a graphical representation.[3] The first two facts have been documented before. First, as seen in the left-

[3]The data include U.S. listed companies performing R&D and advertising between 1980 and 2015 from the

most panel, there is a deviation from Gibrat's law: growth rates are higher for smaller firms. The second panel shows that small firms also tend to be relatively more R&D intensive. Taken together, these facts suggest that smaller firms experience higher growth through relatively higher R&D investments, an observation that has sometimes been interpreted in the existing literature as evidence for differences in R&D technology across firm size. This interpretation can have important implications in terms of policy recommendations (e.g. Akcigit (2009), Acemoglu *et al.* (2018) or Akcigit and Kerr (2018)). If small firms are more efficient at innovating, R&D subsidies targeted at small firms may be optimal. An important issue is then to understand the source of the deviation from constant R&D intensity across firm size.

Our paper contributes to this debate by providing microfoundations for this deviation. We show that the observation that R&D intensity diminishes with firm size can arise as the result of the optimal allocation of resources between R&D and advertising at the firm level. Specifically, we discipline our calibration by two new facts about advertising. First, we find a negative relationship between advertising intensity (i.e., advertising expenditures normalized by size) and firm size, which can be observed in the third panel of Figure 1. Second, we find that larger firms rely relatively more on advertising compared to R&D (fourth panel in Figure 1).

The main mechanism in our model is based on well-established empirical observations from the marketing literature. There is ample evidence that larger firms have a cost advantage in terms of advertising compared to firms with fewer products. One reason for this comes from a spillover effect through different goods under the same brand name. By advertising one product, a firm can influence not only the perception of the quality of the good being advertised, but also of other goods sharing the same brand name. Our model captures this fact in a reduced-form way by allowing the per-product return to advertising to depend on firm size. The existence of this size-dependence (henceforth called the "advertising spillover effect") alters firms' dynamic incentives to engage in R&D. As smaller firms gain relatively more in terms of advertising spillover from acquiring an additional product, they optimally choose to perform relatively more external innovation in order to expand into new product markets. As a result, these firms grow relatively faster than large ones.

Compustat database. More details on the sample selection can be found in Section 2.

Using the calibrated model, we conduct a number of quantitative exercises to explore the implications of this R&D-advertising interaction. First, we show that the advertising channel can account for about half of the observed deviation in Gibrat's law, with the remainder accounted for by differences in the R&D technology between firms. Second, using simulated data from the calibrated model, we show that advertising is responsible for almost a third of the cross-sectional dispersion in firm- and product-level sales. Third, in the calibrated model, we find the existence of a substitution effect between R&D and advertising at the firm level: a decrease in the cost of advertising leads to an increase in the entry rate and in creative destruction. This decreases the incentive for incumbent firms to invest in R&D and shifts the firm size distribution toward the lower end. Overall, investment in R&D decreases, and this leads to a decrease in economic growth. We predict that the growth rate of the economy would be 0.22 percentage points higher in an economy without advertising.

To validate our calibration, we show that the model matches several untargeted moments, among which the observed decrease in the cross-sectional dispersion of firm growth, R&D, and advertising intensity, with firm size, and the degree of Gibrat's law deviation across firms with different advertising intensities. To provide empirical evidence for the finding that R&D and advertising are substitutes at the firm level, we exploit exogenous variation in the cost of R&D arising from changes in the tax treatment of R&D expenditures across U.S. states and over time. We find that an increase in tax credits leads to significant decreases in advertising intensity at the firm level, lending support to our quantitative findings.

In the last part of the paper, we study the policy implications of our model. First, we show that an extremely high tax rate on advertising would be required to achieve non-negligible gains in terms of economic growth, because the elasticity of advertising to sales is low. Second, we show that identifying the source of the observed decrease in R&D intensity with firm size is relevant for R&D policy. R&D subsidies are more effective at promoting economic growth if the deviation from constant R&D intensity comes from the interaction between R&D and advertising rather than from technological differences in terms of R&D efficiency across firm size. For example, for a 50% subsidy, the increase in growth is 0.3 percentage point higher in the first case. We further show that the model with advertising performs better in terms of matching moments related to firm growth and R&D intensity

across firm size.

Related literature Our work is related to several strands of the literature. First and foremost, we build upon models of endogenous growth through product innovation with heterogeneous firms. This field was pioneered by Klette and Kortum (2004), who embed a Schumpeterian growth model into a multi-product firm dynamics setting. Subsequent research (e.g. Lentz and Mortensen (2008), Acemoglu and Cao (2015), Acemoglu *et al.* (2018), and Akcigit and Kerr (2018)) has used quantitative versions of this model to rationalize empirical regularities regarding the lifecycle of innovative firms. Empirically, various studies have found that there exist deviations from Gibrat's law among these type of firms, with an important contributor being size differences in innovation intensity across firms.[4] To explain these facts, the quantitative literature has typically considered heterogeneity among innovation technologies, which generates imperfect scaling in growth and R&D via size-dependent R&D technological efficiency.[5] In contrast to these studies, the size-dependence in our model can emerge through the interaction between innovation and advertising decisions.

To build the advertising technology into the framework, we rely on a well-documented fact from the marketing literature: larger firms experience higher returns to advertising expenditures. The literature has found that there exist spillovers between goods within the firm, in that increasing advertising expenditures on a product not only increases its sales, but also indirectly those of all the other goods under the same brand. This phenomenon has been named *umbrella branding* in the literature (e.g. Erdem and Sun (2002)).[6] An implication of umbrella branding is that firms with more products will reach higher per-product returns to advertising for the same advertising expenses. In our model, we capture this dependence

[4]See Hall (1987), Sutton (1997), Caves (1998), Geroski (1998), and Santarelli *et al.* (2006) (survey), for empirical studies on firm growth. The literature has connected growth and differences in R&D intensity across firm size. Cohen and Klepper (1996b) show that small innovating firms generate more innovations per dollar spent in R&D, and Akcigit and Kerr (2018) show that small firms spend more in R&D per dollar of sales. Argente *et al.* (2019) show that the rate at which firms introduce new products is decreasing in firm size as measured by sales.

[5]For instance, Akcigit and Kerr (2018) assume weak scalability in R&D costs, Acemoglu and Cao (2015) assume that small firms experience larger technological jumps, and Acemoglu *et al.* (2018) assume the existence of a highly absorbing state with low productive capacity.

[6]The marketing literature on the spillover effects of advertising is vast, including Morrin (1999), Balachander and Ghose (2003), Dacin and Smith (1994), Erdem (1998), Sullivan (1990), Büschken (2007), and Suppliet (2016). Moreover, brand developments have been found to decrease marketing costs (Lane and Jacobson (1995) and Tauber (1981, 1988)), enhance marketing productivity (Rangaswamy *et al.* (1993)), and capture greater market share (Smith and Park (1992)).

in a reduced-form way by assuming that per-product advertising costs are decreasing in the number of products of the firm.

In the model, advertising acts as a demand shifter in equilibrium, in line with a long tradition in economics of modeling advertising as explicitly affecting consumer tastes (Dorfman and Steiner (1954), Dixit and Norman (1978), Becker and Murphy (1993) and Benhabib and Bisin (2002)). More broadly, we relate to a large literature exploring the role of intangibles for firm and industry dynamics, such as customer capital (e.g. Fishman and Rob (2003), Gourio and Rudanko (2014b), Hall (2008)), costs to market penetration (e.g. Arkolakis (2010, 2016) and Eaton *et al.* (2014)), or informational frictions (Dinlersoz and Yorukoglu (2012) and Perla (2017)). We contribute to this literature by exploring the interaction of advertising with R&D. To our knowledge, only Grossmann (2008) has considered this interaction theoretically in general equilibrium before. Unlike Grossmann (2008), we quantitatively study the implications of advertising for firm dynamics, innovation and economic growth.

Finally, our paper is related to a growing literature studying the sources of firm growth. In recent years, empirical evidence based on micro-level data has shown that there exists a large and persistent degree of dispersion in revenue among firms, over and above differences in physical productivity. The literature customarily attributes this large residual dispersion to idiosyncratic demand fundamentals (e.g. Foster *et al.* (2008), Hottman *et al.* (2016), Argente *et al.* (2018), Eslava and Haltiwanger (2019)). Some of the leading explanations for this behavior are slow learning (e.g. Arkolakis *et al.* (2018), Argente and Yeh (2018)), pricing to accumulate a customer base (e.g. Foster *et al.* (2016), Roldan-Blanco and Gilbukh (Forthcoming)), and marketing and advertising activities (e.g. Drozd and Nosal (2012), Fitzgerald *et al.* (2017)).[7] We contribute to this literature by providing a general-equilibrium model with demand-side sources of growth in quantities, which allows us to quantify the role of each channel for firm dynamics and sales dispersion (Section 4.2).

[7]Most relatedly, Fitzgerald *et al.* (2017) find empirically that quantities, but not prices, increase with the firm's tenure in the market after controlling for supply-side variation, and argue that these dynamics are consistent with a model in which firms grow through marketing and advertising expenditures. Our model, in which markups are constant but marketing allows firms to shift demand as they grow, is consistent with this finding.

2 Empirical Findings

In this section, we present empirical results on the relation between firm size and firm growth, R&D, and advertising expenditures.

Data We use annual data from Compustat on listed companies from all non-financial sectors of the U.S. economy over the period 1980-2015.[8] Our focus, as is standard in the endogenous growth literature, is on innovative firms. Even though Compustat is not representative of the whole economy, it offers a better representation of R&D firms, as R&D is typically performed by larger firms.[9] Among firms reporting positive R&D, we look at companies who report positive advertising expenditures. Compustat covers a sizable share of aggregate advertising activity in the U.S.[10] Innovative firms reporting advertising expenditures comprise a significant share of innovative firms in Compustat (63.3%), and represent a large share of their sales (65.1% each year on average). More importantly, these firms collectively account for 76.4% of total R&D expenditures in Compustat. These numbers and the fact that we see Compustat as a good representation of R&D firms suggest that the mechanisms that we highlight in this paper might be relevant for a large share of innovative activity in the U.S. economy as a whole. Descriptive statistics can be found in Table A.1 of Appendix A.

Key facts Our main focus is on regression counterparts for the four facts presented in Figure 1. We use firm sales as our baseline measure of firm size. Our baseline regression specification is:

$$y_{ij,t} = \alpha_0 + \beta \log(Sales_{ij,t}) + \boldsymbol{X}'_{ij,t}\boldsymbol{\gamma} + \alpha_j + \alpha_t + u_{ij,t}$$

for firm i in 4-digit SIC industry j and year t, where α_t and α_j are time and industry

[8] Firms reporting negative sales or negative employment are excluded from the sample. To exclude outliers, we ignore firms experiencing year-on-year sales growth rates of more than 1,000%. We exclude mergers and acquisitions. We also exclude firms with R&D-to-sales and/or advertising-to-sales ratios of more than 100%.

[9] Compustat also provides a good R&D coverage relative to aggregate data, and closely matches trends in aggregate R&D expenditures over time (see the discussion in Barlevy (2007)) in spite of potential reporting issues.

[10] Compustat item *xad* represents the cost of advertising media (including radio, television, newspapers, and periodicals). Despite potential reporting issues, aggregate advertising expenses in Compustat account for 61.25% of expenses on average at the economy-wide level over the 1980-2007 period (from McCann Erikson's Coen Structured Advertising Expenditure Dataset, available at https://www.purplemotes.net/2008/09/14/us-advertising-expenditure-data/).

fixed effects. The dependent variable is:

$$y_{ij,t} \in \left\{ \frac{\Delta Sales_{ij,t}}{Sales_{ij,t}}, \log\left(\frac{R\&D_{ij,t}}{Sales_{ij,t}}\right), \log\left(\frac{Adv_{ij,t}}{Sales_{ij,t}}\right), \log\left(\frac{R\&D_{ij,t}}{Adv_{ij,t}}\right) \right\}$$

for each of the four facts, respectively. To control for firm characteristics, the vector $\boldsymbol{X}_{ij,t}$ includes firm age, which we measure as the number of years elapsed since the firm first appeared in the sample;[11] and firm leverage, defined as the ratio of total debt (long term and current liabilities) to equity. We choose these controls as they are alternative candidates in the literature to explain why firms of different sizes may exhibit different growth paths.[12]

	(1) $\frac{\Delta Sales}{Sales}$	(2) $\log\left(\frac{R\&D}{Sales}\right)$	(3) $\log\left(\frac{Adv}{Sales}\right)$	(4) $\log\left(\frac{R\&D}{Adv}\right)$
$\log(Sales)$	–0.0331	–0.1053	–0.0360	–0.0693
	(0.00282)	(0.00868)	(0.00990)	(0.01167)
Firm Age	–0.0048	0.0031	0.0005	0.0026
	(0.00037)	(0.00157)	(0.00187)	(0.00215)
Leverage	–0.0001	0.0001	–0.0002	0.0003
	(0.00004)	(0.00019)	(0.00012)	(0.00025)
Time FE	✓	✓	✓	✓
Industry FE	✓	✓	✓	✓
Observations	26666	26666	26666	26666
R^2	0.0922	0.5080	0.2781	0.4467

Table 1: Firm-level regressions.
Notes: Compustat data (1980-2015). The sample is restricted to non-financial firms reporting strictly positive sales, strictly positive employment, strictly positive R&D and advertising expenditures, with year-on-year sales growth less than 1,000%, and R&D-to-sales and advertising-to-sales ratios of less than 100%. Mergers and acquisitions have been excluded from the sample. Age is measured as the elapsed time since the first observation in the data. Our measure of financial constraints is leverage, computed as total debt (the sum of long-term and current liabilities) divided by equity. Sales and advertising expenditures are in thousands of U.S. dollars. Standard errors are clustered by firm (in parentheses).

Column (1) in Table 1 shows that there exists a significant deviation from Gibrat's law among firms in our sample. Larger firms experience lower growth rates on average, with a 1% increase in sales translating into a 0.0331% average decrease in sales growth. This evidence is similar to what is reported in Akcigit and Kerr (2018). An emerging trend in the economic

[11] This measure of firm age in the Compustat database is standard and has been used among others by Shumway (2001), Lubos and Veronesi (2003) and Fama and French (2004).

[12] For instance, Haltiwanger *et al.* (2013) show that age has a predominant role for explaining firm growth, supporting theories of age-driven firm dynamics such as Jovanovic (1982) or Clementi and Palazzo (2016). On the other hand, Cooley and Quadrini (2001) and Clementi and Hopenhayn (2006) link the dependence of firm growth on size and age to financial market frictions.

growth literature has investigated this phenomenon by linking it to another cross-sectional empirical fact related to firm size: the higher R&D intensity of smaller firms. As larger firms invest relatively less in innovation, they experience relatively lower growth rates.[13] Column (2) in Table 1 shows that this pattern holds in our sample as well. Larger firms are less R&D intensive, with a 1% increase in sales translating into a 0.1053% average decrease in the R&D-to-sales ratio.

Thus, smaller innovative firms grow relatively faster, and these firms spend relatively more on R&D. While we are not the first ones to make these observations using Compustat data (e.g. Akcigit (2009) and Itenberg (2015)), we show in addition that size remains significant after controlling for age and financial constraints in both the growth and R&D intensity regressions, suggesting that explanations based on the firm's product life-cycle and financial constraints cannot fully account for the deviation from Gibrat's law observed in the data.

We propose an alternative explanation for this deviation based on the interaction between R&D and advertising in firms' optimal decisions. To build this intuition, we establish two new facts. First, column (3) in Table 1 shows that advertising intensity is also decreasing in firm size. Smaller firms spend more in advertising per dollar in sales: a 1% increase in sales translates into a 0.0360% decrease in the advertising-to-sales ratio. The coefficients on size in columns (2) and (3) also tell us that larger firms have higher R&D and advertising expenditures in levels.[14] Second, column (4) shows that larger firms tend to use relatively more advertising than R&D: a 1% increase in sales leads to a 0.0693% decrease in the R&D-to-advertising ratio. This suggests that as innovative firms grow larger, they tend to substitute advertising for R&D.

To confirm the validity of these results, we run a series of robustness checks. The full description of those robustness checks as well as the corresponding tables can be found in Appendix D. We show that our results are robust to the inclusion of firm fixed effects, state- and industry-specific time trends, to alternative measures of financial constraints (investment rates and the Kaplan-Zingales index) and firm size (employment and assets), and that they

[13] See, for example, Akcigit (2009), Acemoglu *et al.* (2014), Acemoglu and Cao (2015), Acemoglu *et al.* (2018), and Akcigit and Kerr (2018).

[14] Specifically, a 1% increase in firm sales is associated with a $1 - 0.1053 = 0.8947\%$ and a $1 - 0.0360 = 0.9640\%$ increases in total R&D and advertising expenditures, respectively, on average.

are not driven by outliers.

Advertising and Gibrat's law deviation In our model, the existence of advertising will create an interaction with R&D that leads to a deviation from Gibrat's law. In this section, we provide evidence that advertising intensity affects this deviation in a significant way, in line with the predictions of our model.

	(1) $\frac{\Delta Sales}{Sales}$	(2) $\frac{\Delta Sales}{Sales}$	(3) $\frac{\Delta Sales}{Sales}$
$\log(Sales)$	–0.0861	–0.0631	–0.0585
	(0.00886)	(0.00295)	(0.00199)
$\log(Sales) \times \log\left(\frac{Adv}{Sales}\right)$	–0.0137		
	(0.00192)		
$\log(Sales)\times$ Share of Adv. Firms		–0.0182	
		(0.00802)	
$\log(Sales)\times$ Adv. Intensity			–0.246
			(0.0338)
Controls	✓	✓	✓
Industry FE	✓	✓	✓
Time FE	✓	✓	✓
Observations	26666	164894	164894
R^2	0.10	0.08	0.08

Table 2: Gibrat's law deviation as a function of firm-level advertising intensity (column (1)) and industry-level measures of advertising use (columns (2) and (3)).
Notes: Controls include firm-level age and leverage. Additionally, in column (1) we control for log advertising intensity at the firm level. Standard errors are clustered by firm (in parentheses).

Table 2 shows results for Gibrat's law regressions where we interact firm size and different measures of advertising intensity. In column (1) of Table 2, we find that while all innovative firms exhibit a deviation from Gibrat's law regardless of their advertising intensity, those firms with a higher usage of advertising experience, on average, a stronger deviation. Columns (2) and (3) show that similar patterns hold at the industry level as well: firms in industries that rely more on the use of advertising tend to deviate more from Gibrat's law. Column (2) measures advertising reliance by the share of firms within the industry with positive advertising expenditures in at least one year of the sample. Column (3) uses the average advertising-to-sales ratio of the industry instead. In both cases, we observe stronger Gibrat's law deviations for firms in more advertising-intensive industries. Overall, these results reflect

sizable differences in Gibrat's law deviations across firms with different advertising intensities. For instance, using Column (1) and the distribution of advertising intensity, we find that firms at the 10th percentile experience a modest deviation of –0.005, while those at the 90th percentile experience a much larger deviation of –0.052. Table D.5 in Appendix D shows that these results are robust to firm fixed effects and state and industry trends. Finally, Table D.6 shows results for the Gibrat's law regression on four groups of Compustat firms: those reporting no R&D nor advertising expenses, those with positive expenses in either R&D or advertising, and those with positive expenditures in both (i.e. our sample). We find that Gibrat's law deviation is weakest among firms reporting neither expense and, importantly, it is strongest amongst firms reporting positive expenditures in both R&D and advertising.

3 An Endogenous Growth Model with Advertising

Inspired by these empirical findings, we build a theory of advertising into a model of firm dynamics and endogenous growth. Overall, we aim to explain the micro-level facts discussed above, and assess the macroeconomic implications.

3.1 Environment

Preferences Time is continuous, infinite, and indexed by $t \in \mathbb{R}_+$. The economy is populated by a measure-one of identical, infinitely-lived individuals with discount rate $\rho > 0$. A representative household has preferences:

$$U = \int_0^{+\infty} e^{-\rho t} \ln\left(C_t\right) \mathrm{d}t$$

where C_t denotes consumption of the final good, whose price is normalized to one. The household is endowed with one unit of time every instant, supplied inelastically to firms as labor, and earning a wage w_t which clears the labor market. The household owns all the firms in the economy and carries a stock of wealth A_t each period, equal to the total value of corporate assets. Wealth earns a time-varying rate of return r_t. The flow budget constraint is $\dot{A}_t = r_t A_t + w_t - C_t$, with $A_0 \geq 0$ given.

Production The final good is produced by a representative final good firm using a measure-one continuum of input varieties, indexed by $j \in [0,1]$, with technology:

$$Y_t = \frac{1}{1-\beta} \int_0^1 \widetilde{q}_{jt}^{\beta} y_{jt}^{1-\beta} \mathrm{d}j$$

where $\beta \in (0,1)$, and y_{jt} is the quantity of intermediate good j. The term $\widetilde{q}_{jt}$ stands for the *total quality* of the good perceived by agents in the economy, defined by:

$$\widetilde{q}_{jt} \equiv q_{jt}(1 + d_{jt}) \tag{1}$$

Total perceived quality includes two components. The first component ($q_{jt} > 0$), called the *intrinsic quality*, is built over time through a process of innovation, which is driven by expenditures on R&D. The second component is the so-called *extrinsic quality*, given by $\phi_{jt} \equiv q_{jt} d_{jt}$, where $d_{jt} \geq 0$.[15] This component is the part of total quality that is affected by the producer's advertising efforts on that specific product.

At any instant, there is an endogenous measure $F > 0$ of active intermediate-good producers operating in a monopolistically competitive environment. Each good $j \in [0,1]$ is produced using the technology:

$$y_{jt} = \bar{Q}_t l_{jt}$$

where l_{jt} is labor input and $\bar{Q}_t \equiv \int_0^1 q_{jt} \mathrm{d}j$ is the average intrinsic quality in the economy. Firms may own multiple goods, and a typical firm has size $n \equiv |\mathcal{J}| \in \mathbb{Z}_+$, where $\mathcal{J} \subseteq [0,1]$ is the set of products owned by the firm. We denote the intrinsic quality portfolio of the firm by $\boldsymbol{q} \equiv \{q_j : j \in \mathcal{J}\}$. A firm may not own any products ($n = 0$), in which case it belongs to the pool of potential entrants. The measure of potential entrants is normalized to one, and the actual measure of entrants is determined by a free-entry condition.

Product ownership is determined on the basis of intrinsic quality advantage. To keep the problem tractable, we make the following two assumptions. First, we assume that advertising does not shift market shares within a product line, so that agents choose the product that

[15] We borrow the terms "intrinsic", "extrinsic", and "perceived" from the marketing literature (Zeithaml (1988)).

has the highest (fully observable) intrinsic quality. Therefore, advertising can only be used by firms to shift preferences between imperfect substitute product lines of different intrinsic qualities. Second, we assume a two-stage game between the leader and the follower, as in Akcigit and Kerr (2018). In the first stage, if the former incumbent wants to compete with the new leader, it has to pay a (arbitrarily small) fee. In the second stage, the two firms compete in prices à la Bertrand. Because the new leader has a higher intrinsic quality, it sets the limit price such that the former leader makes no profit. As a result, the former leader chooses not to pay the fee in the first stage, and in equilibrium the new leader will charge the monopoly price.[16]

R&D technology Firms can invest in R&D to improve the intrinsic quality q of their own product lines (*internal* R&D), and to acquire and build upon goods that are being produced by other incumbent firms (*external* R&D).[17] Firms move endogenously through the size space on the basis of the latter type of innovations, and *creative destruction* occurs when a good is taken over by a successful innovator. A firm endogenously exits when it loses its last remaining product.[18] For internal innovation, a firm must spend:

$$R_z(z_j) = \widehat{\chi} q_j z_j^{\widehat{\psi}}$$

units of the final good to generate a Poisson rate z_j of innovating upon product $j \in \mathcal{J}$, where $\widehat{\chi} > 0$ and $\widehat{\psi} > 1$. If successful, internal R&D improves intrinsic quality by a factor of $(1 + \lambda^I) > 1$. External R&D is undirected, so that each innovator draws a good upon which to innovate at random from the entire product space $[0, 1]$. Incumbents and entrants face different innovation technologies. On the one hand, potential entrants must spend:

[16] The combination of these two assumptions simplifies the profit function as being linear in the intrinsic quality of the product, and allows us not to keep track of the size of the follower in each industry. As a result, the model cannot speak to any potential strategic use of advertising within product lines. There is evidence, however, that a significant effect of advertising is to shift market share between product categories (see for instance Lancaster (1984) and Hanssens *et al.* (2012) for a discussion and empirical evidence).

[17] Garcia-Macia *et al.* (2019) show that both types of innovation contribute significantly to aggregate growth.

[18] In Online Appendix G.1, we extend our baseline model with a theory of patent citations. This extended model can deliver two other empirical regularities (see Akcigit and Kerr (2018)): smaller firms tend to produce relatively more patents, and those patents tend to be of higher quality.

$$R_e(x_e) = \nu \bar{Q} x_e$$

units of the final good to generate a Poisson rate x_e of acquiring their first product, where $\nu > 0$. On the other hand, to create a rate X_n of innovation, an incumbent firm with $n \geq 1$ products must spend $R_x(X_n, n)$ units of the final good, where:

$$R_x(X_n, n) = \widetilde{\chi} \bar{Q} X_n^{\widetilde{\psi}} n^{\sigma}$$

with $\widetilde{\chi} > 0$, $\widetilde{\psi} > 1$, and $\sigma \leq 0$. If successful, external R&D improves intrinsic quality by a factor of $(1 + \lambda^E) > 1$. Henceforth, we will say that the R&D technology exhibits *decreasing*, *constant*, or *increasing returns to scale* if an n-product firms finds it respectively more expensive, as expensive, or less expensive to grow in size by a given rate than n firms of one product each. Formally:

Definition 1 (Returns to Scale in R&D) *Define total R&D expenditure at the firm level as $R_n \equiv \sum_{j=1}^{n} R_z(z_j) + R_x(X_n, n)$, for a given innovation policy $(\{z_j\}_{j=1}^{n}, X_n)$. Then, there are* decreasing *(or* increasing*) returns to scale if $R_n > nR_1$ (or $R_n < nR_1$), for all n. There are* constant *returns if $R_n = nR_1$.*

Advertising technology Besides R&D, firms can undertake advertising at the product level to enhance demand along the extrinsic margin of total quality. For a firm of size $n \geq 1$ with an intrinsic quality portfolio $\boldsymbol{q}$, we assume the following technology:

$$\phi_j = \widetilde{\theta}_j m_j^{\zeta} n^{\eta} \tag{2}$$

where m_j is the expenditure (in terms of final good units) into advertising good j, with $(\zeta, \eta) \in \mathbb{R}_+^2$, and $\widetilde{\theta}_j$ is a product-firm-specific efficiency component, defined by $\widetilde{\theta}_j \equiv \theta \frac{q_j}{Q_{\boldsymbol{q}}} \bar{Q}^{1-\zeta}$.[19] The object $Q_{\boldsymbol{q}} \equiv \left(\sum_{q \in \boldsymbol{q}} q^{\frac{1}{\alpha}} \right)^{\alpha}$ is a within-firm measure of aggregate intrinsic quality, and $\theta \geq 0$ is a component of advertising efficiency that is constant across time, goods, and firms.

[19] The term $\bar{Q}^{1-\zeta}$ inside $\widetilde{\theta}_j$ is needed for the existence of a Balanced Growth Path.

Discussion Let us briefly discuss some of the novel aspects of our theory.

The functional form for quality (Equation (1)) implies that intrinsic (q_j) and extrinsic ($\phi_j \equiv q_j d_j$) quality are substitutes at the product level, as they enter additively into total quality. However, note that we also allow for some degree of complementarity coming from the fact that ϕ_j is itself an increasing function of intrinsic quality (Equation (2)). Namely, advertising is more effective in raising perceived quality of goods with higher intrinsic quality.[20] At the firm level, R&D and advertising may be complements or substitutes in equilibrium, depending on parameter values (see Online Appendix E for a discussion).

Importantly, advertising acts as a preference shifter. This idea follows Dixit and Norman (1978), Becker and Murphy (1993), Benhabib and Bisin (2002) and Molinari and Turino (2015), among others, who model advertising through product-specific taste parameters or as an explicit argument in the utility function. We further adopt the view that advertising is *persuasive*, and not purely *informative*. By this we mean that consumers cannot choose what information to be exposed to, but rather behave according to the shifts in tastes induced by advertising, which they take as given. Finally, although we model advertising as a static component of firm profits, advertising critically changes the dynamic incentives of acquiring new product lines through R&D. This implies that advertising has an indirect effect on firm dynamics and economic growth. In Online Appendix G.2, we present various alternative ways of modeling advertising and show that, in all cases, advertising appears as a demand shifter and, therefore, would give rise to similar qualitative predictions.[21]

Finally, our choice for the advertising technology (Equation (2)) embodies the main effects of advertising identified by the marketing literature. First, per-product advertising returns are increasing in expenditure (m_j), where $\zeta > 0$ denotes the elasticity. Ample evidence in marketing establishes diminishing returns to advertising expenditures, so we impose $\zeta < 1$.[22] Second, the marketing return increases in the object $q_j/Q_{\boldsymbol{q}}$, which is a measure of the

[20]Note that, in equilibrium, all incumbent firms do both R&D and advertising. The presence of firms that do not do R&D or advertising in the data might suggest the existence of fixed costs. In this respect, our model may be interpreted as a model of firms that have already paid these fixed costs.

[21]These extensions include: goodwill accumulation of advertising (Online Appendix G.2.1); advertising in the utility function (Online Appendix G.2.2); advertising as a wasteful activity (Online Appendix G.2.3); informative advertising (Online Appendix G.2.4); and advertising as altering the price-elasticity of demand (Online Appendix G.2.5).

[22]See Simon and Arndt (1980), Sutton (1991), Jones (1995) and Bagwell (2007). Similar concavity assumptions

relative quality of the good within the firm. Advertising is more effective for goods that are intrinsically of relatively higher quality in the firm's portfolio, so that firms will spend relatively more on those.[23] Third, and critically for our mechanism, larger firms have an absolute marketing advantage over smaller firms: the return to advertising is greater for higher n, as $\eta > 0$. This dependence of advertising return on size is meant to capture the value of firm branding, reminiscent of the "umbrella branding" spillover effect discussed in the empirical marketing literature.[24]

3.2 Equilibrium

Utility optimization leads to the Euler equation $\frac{\dot{C}_t}{C_t} = r_t - \rho$, with the usual transversality condition $\lim_{t\to+\infty} e^{-\int_0^t r_s \mathrm{d}s} A_t = 0$. The final good producer's cost-minimization problem leads to the iso-elastic demand function $p_{jt} = \widetilde{q}_{jt}^{\beta} y_{jt}^{-\beta}$. Since $\widetilde{q}_{jt} = q_{jt}(1 + d_{jt})$, advertising effectively works as a demand shifter that alters consumption decisions.

Production decision A monopolist chooses labor, quantities, prices, R&D and advertising expenditures over each good in its portfolio in order to maximize the present discounted value of the total future stream of profits. Our set-up allows us to break this problem into a static part, in which the firm sets price, quantity and advertising expenditures over the goods that it currently owns, and a dynamic part, in which R&D decisions are made.

Before R&D and advertising choices, the static maximization problem is:

$$\pi(\widetilde{q}_{jt}) = \max_{y_{jt}, p_{jt}} \left\{ p_{jt} y_{jt} - w_t l_{jt} \right\} \qquad \text{s.t. } y_{jt} = \bar{Q}_t l_{jt} \text{ and } p_{jt} = \widetilde{q}_{jt}^{\beta} y_{jt}^{-\beta}$$

The optimality condition implies $p_{jt} = \left(\frac{1}{1-\beta}\right) \frac{w_t}{\bar{Q}_t}$, so prices are constant across products, $p_{jt} = p_t, \forall j$. Using labor market clearing, $\int_0^1 l_{jt} \mathrm{d}j \leq 1$, we can find the market-clearing wage $w_t = (1-\beta)\left[\bar{Q}_t + \bar{\Phi}_t\right]^{\beta} \bar{Q}_t^{1-\beta}$, where $\bar{\Phi}_t \equiv \int_0^1 \phi_{jt} \mathrm{d}j$ denotes aggregate extrinsic quality. The

have been made in economic models with marketing, e.g. Stigler (1961), Arkolakis (2010) and Dinlersoz and Yorukoglu (2012).

[23] This effect has also been identified in the data (e.g. Archibald *et al.* (1983), Caves and Greene (1996), Marquardt and McGann (1975), Rotfeld and Rotzoll (1976), Bagwell (2007) and Kirmani and Rao (2000)).

[24] We model this spillover effect from umbrella branding in a reduced-form way which is similar to the way R&D spillovers between products are captured in the model.

price can then be written as $p_t = \left(\frac{\bar{Q}_t+\bar{\Phi}_t}{\bar{Q}_t}\right)^{\beta}$, and flow operating profits before advertising costs become $\pi_{jt} = \widetilde{\pi}_t \tilde{q}_{jt}$, where $\widetilde{\pi}_t = \beta\left(\frac{\bar{Q}_t}{\bar{Q}_t+\bar{\Phi}_t}\right)^{1-\beta}$.

Advertising decision Henceforth, we drop time subscripts unless otherwise needed. When choosing advertising expenditures on each product (m_j), a firm solves:

$$\pi^{adv} \equiv \max_{\{m_j : j \in \mathcal{J}\}} \sum_{q_j \in \boldsymbol{q}} \left[\widetilde{\pi}(q_j + \phi(m_j)) - m_j\right] \text{ s.t. } \phi(m_j) = \theta \frac{q_j}{\left(\sum_{q \in \boldsymbol{q}} q^{1/\alpha}\right)^{\alpha}} \bar{Q}^{1-\zeta} m_j^{\zeta} n^{\eta}$$

where π^{adv} denotes post-advertising flow profits. We assume that $\alpha = 1-\zeta$ in order to reduce the dimensionality of the parameter space and be able to find closed-form expressions.[25] The optimality condition gives firm-level advertising expenditures:

$$M_n \equiv \sum_{q_j \in \boldsymbol{q}} m_j = \left(\zeta\theta\widetilde{\pi}\right)^{\frac{1}{1-\zeta}} \bar{Q} n^{\frac{\eta}{1-\zeta}} \tag{3}$$

Recall from our empirical analysis that (i) advertising expenditures are increasing in size, and (ii) advertising intensity is decreasing in size. The first observation is delivered directly by our assumption that $\eta > 0$. Defining advertising intensity as M_n/n, in order to satisfy the second requirement it must be that M_n is concave in n, which means $\frac{\eta}{1-\zeta} < 1$. Because we imposed diminishing returns to advertising ($\zeta < 1$), this implies that $\eta < 1$. In other words, in order to replicate the decreasing advertising intensity, the spillover effect from size must be marginally stronger for smaller firms. Namely, the gain in advertising return for an n-to-$(n+1)$ transition is higher when n is smaller. As this makes smaller firms relatively more concerned with expanding to new product markets, these firms choose a relatively higher external innovation intensity in equilibrium.[26] As a consequence, this mechanism can generate a decreasing R&D intensity with firm size and hence a deviation from Gibrat's law,

[25] Our results would not be qualitatively affected by the choice of a different value of α because the main mechanism regarding advertising and R&D intensity works through firm size n. In addition, this assumption implies a positive relationship between advertising expenditure and quality at the product level but not at the firm level. This is in line with empirical results in Archibald *et al.* (1983), who study this relationship at both the product and brand level. A similar result is obtained in Caves and Greene (1996) at the brand level.

[26] This idea is also consistent with empirical findings from Argente *et al.* (2018), who find that the share of revenue coming from newly introduced products is decreasing in the firm's tenure in the market (which is, in turn, highly positively correlated with the number of products in the firm's portfolio).

even in the presence of non-decreasing returns to scale in R&D.

The overall static profits of the firm are:

$$\pi^{adv} = \widetilde{\pi} \sum_{q_j \in \boldsymbol{q}} q_j + \gamma \bar{Q} n^{\frac{\eta}{1-\zeta}} \tag{4}$$

where $\gamma \equiv \frac{1-\zeta}{\zeta}(\theta\widetilde{\pi}\zeta)^{\frac{1}{1-\zeta}}$ is constant across goods and firms, and time-varying only through $\widetilde{\pi}$. Static profits have two components. The first one grows linearly with the aggregate intrinsic quality within the firm, with a factor of proportionality that does not vary across firms. The second one is invariant to the firm's intrinsic quality portfolio, but is increasing and concave in firm size because $\eta + \zeta < 1$. This component comes from advertising and the extrinsic margin of product quality.

R&D decision To analyze dynamic decisions, we now focus on a Balanced Growth Path (BGP), defined as an equilibrium in which aggregate output grows at a constant rate $g \equiv \dot{Y}/Y$. To derive the BGP, we guess-and-verify that aggregate extrinsic and intrinsic qualities grow at the same pace, so $\bar{\Phi} = \Phi^* \bar{Q}$ for some Φ^* which is constant across products, firms, and time. From labor market clearing, one can obtain $Y = \left(\frac{1}{1-\beta}\right)(1+\Phi^*)^{\beta}\bar{Q}$. Therefore, $g = \dot{\bar{Q}}/\bar{Q}$. Flow operating profits are then $\pi_j = \widetilde{\pi}\widetilde{q}_j$, where $\widetilde{\pi} = \beta(1+\Phi^*)^{\beta-1}$ is a constant. By construction, we can express total aggregate extrinsic quality on the BGP as $\bar{\Phi} = \sum_{n=1}^{+\infty} F\mu_n\Phi_n$, where μ_n is the invariant share of size-n firms (derived below). Combining $\bar{\Phi} = \Phi^*\bar{Q}$ with the equilibrium firm-level extrinsic quality:

$$\Phi^* = \theta^{\frac{1}{1-\zeta}} \left(\frac{\zeta\beta}{(1+\Phi^*)^{1-\beta}}\right)^{\frac{\zeta}{1-\zeta}} \sum_{n=1}^{+\infty} F\mu_n n^{\frac{\eta}{1-\zeta}} \tag{5}$$

which defines Φ^* implicitly. Aggregate marketing expenditures, M, grow at the rate g, since $M = \sum_{n=1}^{+\infty} F\mu_n M_n$, and M_n is linear in $\bar{Q}$ by Equation (3). One can show that total R&D expenditures Z grow with $\bar{Q}$ as well. As the economy is closed, GDP equals $Y = C + Z + M$. Thus, aggregate consumption C grows at rate g. From the Euler equation, we then have that $r = g + \rho$.

We may now describe the R&D choices of firms. Denote by τ the (endogenous) rate of creative destruction along the BGP, and by $x_n \equiv \frac{X_n}{n}$ the external R&D *intensity* of a firm

of size n. Taking (r, τ, g) as given, the firm chooses external R&D intensity x_n and internal R&D intensities $\{z_j : j \in \mathcal{J}_f\}$ to solve the HJB equation (see Appendix B.1 for a derivation):

$$
\begin{aligned}
rV_n(\boldsymbol{q}) = \max_{x_n, \{z_j\}} \Bigg\{ \sum_{q_j \in \boldsymbol{q}} \Bigg[& \widetilde{\pi} q_j - \widehat{\chi} z_j^{\widehat{\psi}} q_j + z_j \Big(V_n\big(\boldsymbol{q} \backslash_- \{q_j\} \cup_+ \{q_j(1+\lambda^I)\}\big) - V_n(\boldsymbol{q}) \Big) \\
& + \tau \Big(V_{n-1}\big(\boldsymbol{q} \backslash_- \{q_j\}\big) - V_n(\boldsymbol{q}) \Big) \Bigg] \\
& + n x_n \Big(\int_0^1 V_{n+1}\big(\boldsymbol{q} \cup_+ \{q_j(1+\lambda^E)\}\big) \mathrm{d}j - V_n(\boldsymbol{q}) \Big) \\
& - \widetilde{\chi} n^{\sigma + \widetilde{\psi}} x^{\widetilde{\psi}} \bar{Q} + \gamma \bar{Q} n^{\frac{\eta}{1-\zeta}} \Bigg\} + \dot{V}_n(\boldsymbol{q})
\end{aligned} \tag{6}
$$

subject to Equation (2), where $\cup_+$ and $\backslash_-$ are multiset union and difference operators.[27] The first two terms on the first line of Equation (6) are good-specific intrinsic flow operating profits net of internal R&D costs. The third term is the change in value due to the internal improvement over a currently owned good. The second line captures the change in value when losing good j through creative destruction, either to another incumbent or to an entrant. The third line captures successful external innovation by the incumbent. The last line includes the flow resources spent in external R&D, extrinsic flow profits, and the instantaneous change in firm value due to economic growth. Similarly, for entrants:

$$
rV_0 = \max_{x_e > 0} \left\{ x_e \left[\int_0^1 V_1(\{q_j(1+\lambda^E)\}) \mathrm{d}j - V_0 \right] - \nu x_e \bar{Q} \right\} + \dot{V}_0 \tag{7}
$$

where V_0 denotes firm value at $(n, \boldsymbol{q}) = (0, \emptyset)$. In an equilibrium with positive entry $x_e > 0$, the free-entry condition imposes $V_0 = 0$. Our first proposition shows that the growth rate of the economy is measured as the combined contributions of internal and external innovators, advancing aggregate productivity by λ^I and λ^E, respectively.

Proposition 1 (Growth rate) *The growth rate of the economy along the BGP is:*

$$
g = \tau \lambda^E + z \lambda^I \tag{8}
$$

[27] These operators are defined by $\{a, b\} \cup_+ \{b\} = \{a, b, b\}$, and $\{a, b, b\} \backslash_- \{b\} = \{a, b\}$, and they are needed because the set $\boldsymbol{q}$ may include more than one instance of the same element.

For the proof, see Appendix B.2. The rate of creative destruction τ is given by the aggregate Poisson rate of external innovation coming from entrants and incumbents:

$$\tau = x_e + \sum_{n=1}^{+\infty} F \mu_n n x_n \tag{9}$$

In turn, we can find an expression for the invariant firm size distribution along the BGP:

Proposition 2 (Invariant firm-size distribution) *The invariant firm-size distribution is:*

$$\mu_n = \frac{x_e}{F} \frac{\prod_{i=1}^{n-1} x_i}{n \tau^n}; \;\; \forall n \geq 1 \tag{10}$$

with $\mu_n \in [0,1], \forall n \geq 1$, *and* $\sum_{n=1}^{+\infty} \mu_n = 1$.

For the proof, see Appendix B.3. We are now ready to solve for the value function and find optimal R&D intensities. For this, we guess $V_n(\boldsymbol{q}) = \Gamma \sum_{q_j \in \boldsymbol{q}} q_j + \Upsilon_n \bar{Q}$, so that a firm derives value from the intrinsic quality of the products it is selling (first term), but also from the option value of advertising its goods in order to increase demand and revenue (second term).[28] We find $\Gamma \in \mathbb{R}_+$ and the sequence $\{\Upsilon_n\}_{n=1}^{+\infty}$ by the method of undetermined coefficients. The following proposition summarizes the solution:

Proposition 3 (Value functions) *In an equilibrium with positive entry ($x_e > 0$), the value of a firm is* $V_n(\boldsymbol{q}) = \Gamma \sum_{q_j \in \boldsymbol{q}} q_j + \Upsilon_n \bar{Q}$, *where* Γ *and* $\{\Upsilon_n\}_{n \geq 1}$ *satisfy:*

$$\Gamma = \frac{\nu - \Upsilon_1}{1 + \lambda^E} \tag{11}$$

$$\Upsilon_{n+1} = \Upsilon_n - \Gamma(1+\lambda^E) + \widetilde{\vartheta} \left(\rho \Upsilon_n n^{\frac{\sigma}{\widetilde{\psi}-1}} - (\Upsilon_{n-1} - \Upsilon_n) \tau n^{\frac{\sigma + \widetilde{\psi} - 1}{\widetilde{\psi}-1}} - \gamma n^{\frac{\eta}{1-\zeta} + \frac{\sigma}{\widetilde{\psi}-1}} \right)^{\frac{\widetilde{\psi}-1}{\widetilde{\psi}}} \tag{12}$$

with $\widetilde{\vartheta} \equiv \widetilde{\psi} \left(\frac{\widetilde{\chi}}{(\widetilde{\psi}-1)^{\widetilde{\psi}-1}} \right)^{\frac{1}{\widetilde{\psi}}}$, *and* $\{\Upsilon_n\}$ *has boundary condition* $\Upsilon_0 = 0$.

[28] When $\widetilde{\psi} + \sigma \neq 1$, Υ_n also captures the dependence of external R&D on firm size.

For the proof, see Appendix B.4. In that appendix, we derive the optimal R&D intensities:

$$z_j = \left(\frac{\lambda^I(\nu - \Upsilon_1)}{\widehat{\psi}\widehat{\chi}(1+\lambda^E)}\right)^{\frac{1}{\widehat{\psi}-1}} \tag{13a}$$

$$x_n = n^{\frac{1-(\sigma+\widetilde{\psi})}{\widetilde{\psi}-1}} \left(\frac{\nu - \Upsilon_1 + \Upsilon_{n+1} - \Upsilon_n}{\widetilde{\psi}\widetilde{\chi}}\right)^{\frac{1}{\widetilde{\psi}-1}} \tag{13b}$$

for internal and external innovations, respectively. Note that internal R&D intensity is constant across goods ($z_j = z$), making internal R&D scale proportionally with size.[29] Thus, in equilibrium, the overall degree of returns to scale in innovation (recall Definition 1) is determined through the $(\widetilde{\psi}, \sigma)$ parameters in the external R&D technology, with $\widetilde{\psi} + \sigma > 1$ (or $\widetilde{\psi} + \sigma < 1$) corresponding to decreasing (or increasing) returns. Importantly, we see that n affects external innovation intensities through two distinct channels: (i) the degree of scalability in external R&D technology (i.e. σ and $\widetilde{\psi}$); and (ii) the degree of decreasing returns in advertising through the branding effect (i.e. ζ and η). The former channel features both explicitly in the exponent of the first multiplicative term of (13b), and implicitly within the $\{\Upsilon_n\}$ sequence in (12). The advertising channel, however, only features through Υ_n. Thus, we can obtain the desired inverse dependence of innovation intensity to size through the advertising channel.[30]

To build intuition, let us focus on the case in which there are constant returns in external R&D ($\sigma + \widetilde{\psi} = 1$). In the absence of the advertising channel ($\theta = 0$), the model delivers $\Upsilon_n = n\Upsilon$ for some constant $\Upsilon > 0$ (see Appendix B.4 for the derivation). This implies that the gain in value from acquiring one additional good (i.e.,$\Upsilon_{n+1} - \Upsilon_n$) is constant across firm size, and thus (i) external R&D investment scales one-for-one with size, and consequently (ii) firm growth is independent of size (Gibrat's law), as in Klette and Kortum (2004). When $\theta > 0$, however, deviations from Gibrat's law can occur even when $\sigma + \widetilde{\psi} = 1$: small firms might still be more innovation-intensive than large firms because they benefit marginally more in terms of advertising efficiency gains from the new product lines.

[29] This follows the structure in Akcigit and Kerr (2018), who provide empirical evidence that internal innovations scale strongly with firm size, in line with previous work by Cohen and Klepper (1996a) and Klepper (1996).

[30] This allows for the existence of a BGP in some regions of the parameter space with increasing returns to scale in external R&D. See Online Appendix E for a discussion.

We are now ready to define an equilibrium:

Definition 2 *A* Balanced Growth Path Equilibrium *is, for all $q_j \geq 0$, $j \in [0,1]$, $n \in \mathbb{Z}_+$, and $\bar{Q} > 0$: allocations y_j, l_j and m_j; extrinsic quality ϕ_j; aggregates Y, C, Z, M, A; a mass of incumbents F; a constant Γ and a sequence $\{\Upsilon_n\}$; a firm size distribution $\{\mu_n\}$; prices w, p, and r; and rates g, z, x_n, x_e, and τ; such that: (i) given prices, final good producers maximize profits; (ii) y_j and p solve the intermediate sector problem; (iii) m_j and ϕ_j solve the advertising problem; (iv) z and x_n solve the innovation problem; (v) μ_n satisfies (10); (vi) x_e and F solve the entry problem and satisfy $V_0 = 0$ and the restriction $\sum_{n=1}^{+\infty} \mu_n = 1$; (vii) Γ and $\{\Upsilon_n\}$ satisfy (11) and (12), respectively; (viii) g and τ satisfy (8) and (9), respectively; (ix) aggregates Y, M, Z, and C satisfy the resource constraint; (x) A satisfies $A = \sum_n F\mu_n V_n$ and the transversality condition; (xi) r satisfies $r = g + \rho$, and w clears the labor market.*

4 Quantitative Analysis

In this section, we calibrate the model using the micro data introduced in Section 2, as well as aggregate moments for the U.S. for the period 1980-2015. We conduct three main exercises. First, we quantify the share of the deviation from proportional growth and constant R&D intensity that stems from returns to scale in innovation vis-a-vis the spillover effect in advertising. To this end, in our calibration strategy we do not impose any ex-ante restriction on the parameters controlling for the degree of scalability in the R&D ($\widetilde{\psi}$ and σ) and advertising (η and ζ) technologies. Instead, we calibrate these four parameters freely to match both R&D- and advertising-related moments. Second, we study the contribution of advertising to generating sales dispersion, and relate it to recent findings in the literature. Finally, we quantify the aggregate effects of advertising on innovation and economic growth.

4.1 Calibration

External identification We have 13 parameters to identify: the preference parameters, (ρ, β); the R&D parameters, $(\lambda^E, \lambda^I, \widetilde{\chi}, \widehat{\chi}, \widehat{\psi}, \nu)$; the advertising efficiency parameter, θ; and the scalability parameters, $(\widetilde{\psi}, \sigma)$ and (η, ζ). The parameters $(\rho, \beta, \widehat{\psi}, \widetilde{\psi}, \zeta)$ are calibrated

externally. We set $\rho = 0.02$, which approximately corresponds to a discount factor of 97% annually. From the static equilibrium conditions, we obtain $\beta = \frac{\int_0^1 \pi_{jt} \ \mathrm{d}j}{\int_0^1 p_{jt} y_{jt} \ \mathrm{d}j}$, so we set β to the ratio of average operating income before depreciation to average sales, equal to $\beta = 0.165$. We impose $\widehat{\psi} = \widetilde{\psi} \equiv \psi = 2$, following Akcigit and Kerr (2018) and prior empirical literature estimating the cost curvature of different types of R&D. We impose that neither type of innovation is more radical than the other: $\lambda^E = \lambda^I \equiv \lambda$. Finally, the elasticity of sales to advertising expenditures is set to $\zeta = 0.1$, in line with estimations from the empirical marketing literature (see Tellis (2009) for a review of the literature).

Internal identification We are left with the parameters $(\lambda, \widetilde{\chi}, \widehat{\chi}, \nu, \theta, \eta, \sigma)$, which are calibrated internally. For this, we find parameter values that match moments from the stationary solution of the model, as well as OLS regression coefficients from model simulation, to those observed in the data.[31] Our selection is based on an unweighted minimum absolute-distance criterion.[32]

Due to the high non-linearities of the model, all moments are affected (to various degrees) by all parameters, making identification challenging. However, we can provide intuition for how each moment is most informed by each parameter. From aggregate data, we target the long-run average growth rate of per-capita GDP in the U.S. and the aggregate firm entry rate from the Business Dynamics Statistics (BDS) data of the U.S. Census Bureau.[33] These moments are informed, respectively, by the innovation step λ (as this parameter is a shifter in the growth rate by Equation (8)), and the entry cost ν (as this parameter affects the amount

[31] The numerical implementation is described in Online Appendix F. The simulation of the model uses 1,000 firms and discretizes time to $T = 100$ periods of length $\Delta t = 0.01$ each. All firms are assumed to be identical in period zero, with qualities $q_0 = 1$. In the event of ties, namely multiple successful firms drawing the same good over which to perform an external innovation, we assign equal probabilities to each firm and draw the new monopolist from the corresponding discrete uniform distribution of tied firms. Cross-sectional moments are time-series averages where, to allow for convergence in the distribution, we ignore the first 20% of the time series. In both the simulation and in the data, we use sales as the baseline measure of firm size. In the model, size is measured by the number of products. Since we do not have data about the number of products, we rely on firm sales. Numerous studies find a strong positive correlation between firm sales and number of products (e.g. Scherer (1983), Katila and Ahuja (2002) and Plehn-Dujowich (2013)).

[32] Specifically, the vector of parameters Ψ is chosen to minimize $\sum_{m=1}^{7} \left| \frac{m_{model}(\boldsymbol{\Psi}) - m_{data}}{m_{data}} \right|$, where m_{model} and m_{data} denote a moment in the model or the data.

[33] As is common in the literature, we assume that all of productivity growth and entry come from innovation, and abstract from other sources of growth such as diffusion or learning-by-doing. The BDS data are available at http://www.census.gov/ces/dataproducts/bds/data.html.

of entrants by the free-entry condition). For firm-level moments, we use Compustat as being a good representation of R&D firms in the whole U.S. economy (see our discussion in Section 2). We target the average ratios of R&D-to-sales and R&D-to-advertising expenditures, as well as the degree of scalability in firm growth from the Gibrat's law regression in Section 2 (column (1) in Table 1). These three moments are informed by the R&D and advertising technological parameters. In particular, the advertising efficiency shifter θ is informative about the average R&D-to-advertising ratio, whereas the R&D cost scale parameters $(\widehat{\chi}, \widetilde{\chi})$ jointly pin down the average R&D-to-sales ratio and the Gibrat's law OLS coefficient. The external R&D cost $\widetilde{\chi}$ directly affects the deviation from Gibrat's law as all of this deviation in the model comes from external innovation. Finally, we target the OLS coefficients from the R&D and advertising intensity regressions (columns (2) and (3) in Table 1), which are informed by the degree of scalability in the R&D and advertising technologies (σ and η, respectively).

Parameter		**Value**	**Target** (Source)	**Data**	**Model**
Externally identified					
Time discount rate	ρ	0.02	Standard		
R&D cost curvature	ψ	2	R&D elasticity (Akcigit and Kerr (2018))		
Quality share in Y	β	0.165	Profitability ratio (Compustat)		
Returns to ADV	ζ	0.1	Sales-ADV elasticity (Tellis (2009))		
Internally identified					
Innovation step	λ	0.0444	Average growth rate (Standard)	0.020	0.020
Entry cost	ν	0.9825	Firm entry rate (BDS)	0.098	0.100
Internal R&D cost scale	$\widehat{\chi}$	0.0551	Average R&D-Sales ratio (Compustat)	0.102	0.133
Advertising efficiency	θ	0.7589	Average R&D-ADV ratio (Compustat)	26.579	26.212
External R&D cost scale	$\widetilde{\chi}$	1.4163	Gibrat's coefficient (Table 1)	–0.0331	–0.0327
R&D scalability	σ	–0.9743	R&D intensity coefficient (Table 1)	–0.1053	–0.1140
Advertising spillover	η	0.8768	ADV intensity coefficient (Table 1)	–0.0360	–0.0361
Returns to scale in R&D	$\psi + \sigma$	1.0257			

Table 3: Full set of calibrated parameters, and model fit, in the baseline calibration.
Notes: Sales are used as the measure of firm size. The firm entry rate is computed in the model as x_e/F. The average R&D-Sales and average R&D-ADV ratios are computed as $\sum_n F\mu_n \frac{R_n}{p_n y_n}$ and $\sum_n F\mu_n \frac{R_n}{M_n}$, respectively, where $R_n \equiv \sum_{q \in \boldsymbol{q}} R_z(z) + R_x(X_n, n)$ stands for the total R&D expenditures of a firm of size n, and M_n is total advertising expenditures at the firm level (Equation (3)).

Calibration results Table 3 shows the full set of calibrated parameter values, and the results of the moment-matching exercise. The model provides a good fit to the data. Particularly, the simulation-based slopes have the correct sign and magnitude.[34] We find that, in order to explain the deviations from proportional growth and constant R&D and advertising intensities, we need a combination of both decreasing returns to scale in R&D ($\psi + \sigma > 1$) and the spillover effect in advertising ($\eta + \zeta < 1$). The implied degree of returns to scale in R&D is $\psi + \sigma = 1.0257$, which means that a firm of 10 products would find it 6.1% more expensive to grow by a given rate through external innovation than it would be for 10 firms of one product each.

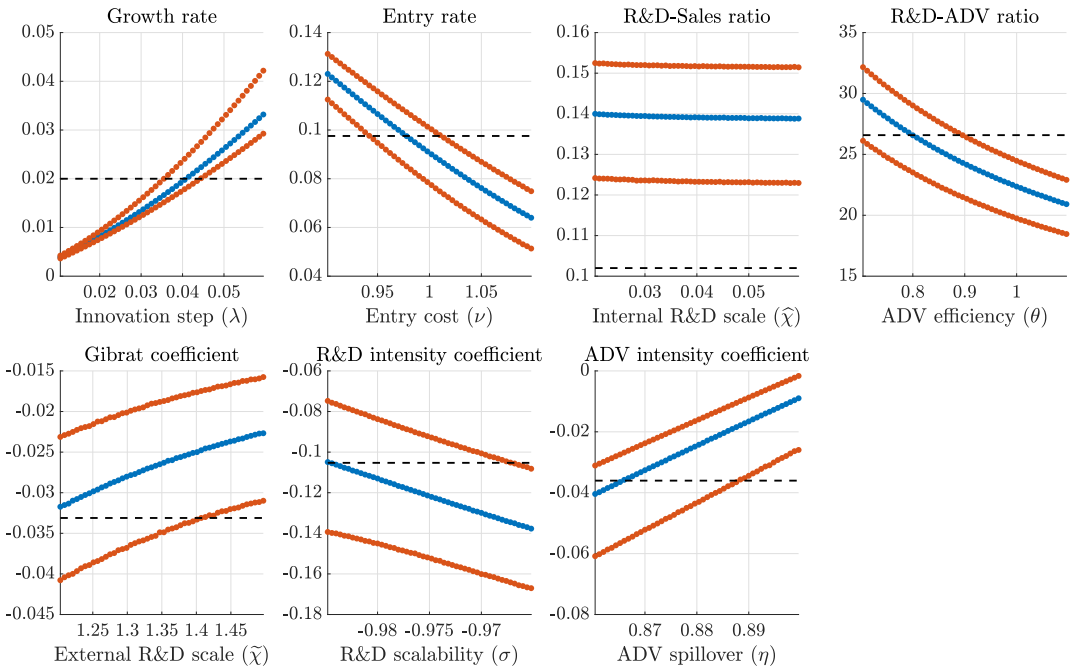

Figure 2: Identification results, based on the Daruich (2020) method.
Notes: The figure shows, for each moment-parameter pair, the median (in blue) and first and third quartiles (in red) of the distribution generated by random variation in all the six remaining parameters.

Figure 2 shows that the model is well-identified according to the method developed by Daruich (2020). The figure plots, for each moment-parameter pair, the median (blue dots) and first and third quartiles (red dots) of the distribution that is generated by allowing for the (six) remaining parameters to vary randomly over a large support, across different quantiles

[34]Note that the value for the R&D-to-advertising coefficient (column (4) in Table 1) is implied directly by the difference between the R&D and advertising coefficients.

of the identifying parameter. A moment is well-identified when this distribution is monotonic in the chosen parameter, the interquartile range is small (so the relative importance of other parameters is small), and the empirical target, which is marked by a dashed horizontal line in the figure, is close to the median of the distribution around the calibrated parameter value. By these criteria, our calibration identifies well nearly all parameters.

Validation To validate the calibration exercise, we check the model's performance on a number of non-targeted moments. The first set of moments that we consider is the dispersion of firm growth, R&D, and advertising intensities across firm size. Studying the model predictions on this front is important to provide validity to our sales variance decomposition exercise described in Section 4.2. Moreover, second-order moments of the cross-sectional distribution of firms have been a long-standing object of attention in the firm dynamics literature. It is a well-known stylized fact in the literature that not only the average, but also the variance of firm growth rates decreases with firm size (e.g. Hymer and Pashigian (1962), Evans (1987) and Klette and Kortum (2004)).

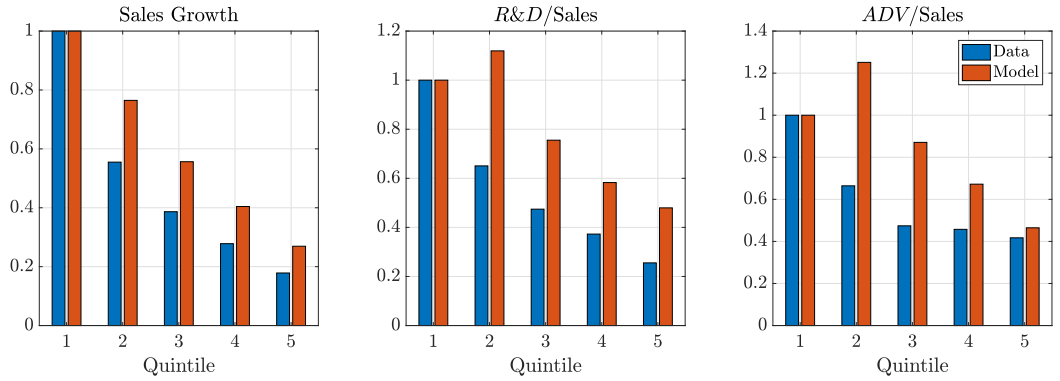

Figure 3: Standard deviation within sales quintiles, relative to first quintile (normalized to one): Model vs. Data.
Notes: Firms are ranked in sales quintiles according to their normalized level of sales (sales as a ratio of average sales in the same year). The standard deviation in the first quintile is normalized to one.

On the left panel of Figure 3, we compare the standard deviation in sales within each quintile of the firm size distribution, both in the calibrated model and in the data. Our model fits the relationship between the variance of firm growth and firm size well, even though it was not calibrated to fit this dimension of the data. In addition, the central and right panels of Figure 3 report the relationship between the standard deviation of R&D

and advertising intensity with firm size, showing that there is more dispersion in R&D and advertising intensity at the lower end of the firm size distribution. Our model predicts an overall decrease in these variance which is in line with the data.[35]

Second, an important implication of the mechanism in the model is that more advertising-intensive firms exhibit stronger deviations from Gibrat's law. As we saw in Section 2, there are significant differences in the strength of the Gibrat's law deviation across the advertising intensity distribution in the data. In Figure 4, we show that the model provides a good fit for the predicted deviations, by advertising intensity deciles.

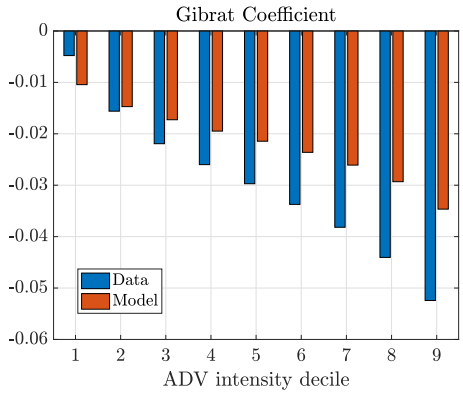

Figure 4: Gibrat's law deviation by decile of the advertising intensity cross-sectional distribution: Model vs. Data.

Finally, since firm growth comes from R&D, advertising, and their interaction in the model, it is important to check whether the model generates a consistent relationship between these variables. Table 4 shows that the correlation coefficients between R&D intensity and sales growth, as well as between advertising intensity and firm growth, are both in line with the data. In this table, we also check whether the model can generate the correct persistence in the growth process of firms and, as this persistence is tightly linked to the probability of moving along the size space, the persistence in the R&D and advertising processes. As seen in Table 4, the model predicts within-firm levels of serial correlation which are comparable to those in the data. The persistence in the R&D-to-advertising expenditures ratio is also

[35]The only exception is on the second quintile of the R&D and advertising intensity dispersion, where we find an increase in dispersion relative to the first quintile. This is due to the coarseness of the size (n) state space. In the model, dispersion in sales comes both from dispersion in total quality (a continuous variable) and size (a discrete variable). Therefore, when distributing firms in sales quintiles, at the lower end of the distribution it may happen that we group high-quality, low-n firms that have high R&D and advertising expenses, together with relatively lower-quality firms with lower expenditures but larger sizes.

well-captured by the model.

	Data	**Model**
$corr$(R&D intensity, firm growth)	0.1468	0.2191
$corr$(ADV intensity, firm growth)	0.0985	0.1281
$autocorr$(R&D intensity)	0.9233	0.8375
$autocorr$(ADV intensity)	0.8784	0.7958
$autocorr$(R&D/ADV ratio)	0.9220	0.9170

Table 4: Correlation and Autocorrelation Coefficients: Model vs. Data.
Notes: R&D and advertising intensity in logarithm.

4.2 Imperfect Scaling and Sales Dispersion

Using the calibrated model, we now explore the role of our advertising channel on two prominent features of the data: imperfect scaling in firm growth and R&D expenses, and cross-sectional dispersion in the distribution of firm sales.

First, we seek to understand which fraction of the deviation from Gibrat's law can be explained by the advertising channel vis-a-vis technological differences in R&D. To this end, we compare the predicted deviations from perfect scaling in growth and R&D in the baseline calibration, with the predictions of an otherwise identical parameterization of the model in which we impose $\sigma + \psi = 1$, implying that all weak scaling must come from the advertising spillover alone. Table 5 shows the results of this exercise. The table shows that the Gibrat's law coefficient is reduced by half (in absolute value) when we impose constant returns in R&D, meaning that our novel advertising channel explains about half of the observed deviation in Gibrat's law. Moreover, the advertising channel can explain about 60% of the deviation from constant R&D intensity. In other words, according to our model, the majority of the deviation from perfect scaling in R&D investment is actually not due to the R&D technology, but to the interaction between R&D and advertising at the firm level.

For our second exercise, we explore the sources of cross-sectional dispersion in firm sales implied by our model. A recent and growing literature, surveyed in the Introduction, explores the sources of firm heterogeneity that explain the dispersion in the firm size distribution observed in the data. The literature has identified four main drivers of cross-sectional dispersion: marginal costs, markups, product scope, and product quality (or "appeal"). Hottman *et al.*

Regression coefficient	$\boldsymbol{\sigma+\psi<1}$ (DRTS)	$\boldsymbol{\sigma+\psi=1}$ (CRTS)	% explained by ADV channel
Gibrat's law coefficient	–0.0327	–0.0163	49.74%
R&D intensity coefficient	–0.1140	–0.0694	60.85%

Table 5: Contributions of the advertising channel to the deviations from proportional growth and constant R&D intensity.
Notes: Comparison between models with decreasing returns to scale (DRTS) and constant returns to scale (CRTS) in the R&D technology, with parameters otherwise set to their calibrated values (Table 3).

(2016) find that most of the variation in firm sales (76%) comes from appeal. In our model, differences in appeal come from differences in intrinsic and extrinsic quality, induced by R&D and advertising expenditures. Using our model, we can decompose the cross-sectional dispersion in sales in order to understand how differences in intrinsic and extrinsic quality across firms contribute to generating sales heterogeneity.

Consider a firm with intrinsic quality portfolio $\boldsymbol{q}$ and n products. Using the static equilibrium conditions, sales as a share of GDP are $\frac{Sales}{Y} = \frac{1-\beta}{1+\Phi^*}\left(\frac{1}{\bar{Q}}\sum_{q\in\boldsymbol{q}} q + \gamma n^{\frac{\eta}{1-\zeta}}\right)$. Note that sales are additively separable in two components, the first one related to average intrinsic quality of the firm's portfolio relative to the economy, and the second related to extrinsic quality through the advertising margin (recall that $\gamma = 0$ in a model with no advertising). Using a variance decomposition as in Eaton *et al.* (2004) and Hottman *et al.* (2016), we can write:

$$\frac{Cov\left(\frac{1-\beta}{1+\Phi^*}\frac{1}{\bar{Q}}\sum_{q\in\boldsymbol{q}} q, \frac{Sales}{Y}\right)}{Var\left(\frac{Sales}{Y}\right)} + \frac{Cov\left(\frac{1-\beta}{1+\Phi^*}\gamma n^{\frac{\eta}{1-\zeta}}, \frac{Sales}{Y}\right)}{Var\left(\frac{Sales}{Y}\right)} = 1 \tag{14}$$

Cross-sectional sales dispersion is accounted for by R&D-induced intrinsic quality differences (first term), and advertising-induced extrinsic quality differences (second term). A similar decomposition can be done at the product level. On each product j, the firm sells $Sales_j = (1+\Phi^*)^{\beta-1}\left(q_j + \gamma a_j \bar{Q} n^{\frac{\eta}{1-\zeta}}\right)$, where $a_j \equiv \frac{q_j^{\frac{1}{1-\zeta}}}{\sum_{q\in\boldsymbol{q}} q^{\frac{1}{1-\zeta}}}$. The decomposition now reads:

$$\frac{Cov\left(\frac{1-\beta}{1+\Phi^*}\frac{q_j}{\bar{Q}}, \frac{Sales_j}{Y}\right)}{Var\left(\frac{Sales_j}{Y}\right)} + \frac{Cov\left(\frac{1-\beta}{1+\Phi^*}\gamma a_j n^{\frac{\eta}{1-\zeta}}, \frac{Sales_j}{Y}\right)}{Var\left(\frac{Sales_j}{Y}\right)} = 1 \tag{15}$$

where, once again, the first (respectively, second) term can be attributed to intrinsic

(extrinsic) quality differences. Comparing Equation (14) and (15), the additional source of dispersion at the product-level relative to firm-level differences comes from the composition of quality portfolios. Decomposition (15) captures cross-sectional dispersion in a_j's, which measures the relative intrinsic quality of each product in relation to the firm's portfolio.

In our calibrated model, intrinsic quality differences explain a little more than two thirds (70.7%) of sales dispersion. The remainder is explained by extrinsic quality differences across firms, induced by their advertising decisions. Similar results hold at the firm-product level (with 72.0% of dispersion due to intrinsic quality differences). All in all, these numbers show that advertising plays an important role in explaining cross-sectional dispersion among firms.

4.3 Effects of Advertising on Economic Growth

Next, we study the effects that advertising has on economic growth through its interaction with R&D. For this, we implement a decrease in the advertising cost (equivalently, an increase in θ). We find, in our calibrated model, that R&D and advertising act as substitutes at the firm level, implying that more effective advertising can have a detrimental effect on growth by decreasing the rate of innovation in the economy.[36] Interestingly, even when R&D and advertising are substitutes, the effect of advertising on growth is ambiguous. To see this, we decompose the economic growth rate by combining (8) and (9) to obtain:

$$g = \lambda^E x_e + z\lambda^I + \lambda^E \sum_{n=1}^{+\infty} F\mu_n n x_n \tag{16}$$

The three additive terms in g correspond to the contribution to growth by entrants, incumbents performing internal improvements, and incumbents performing external innovations, respectively. In addition, there is a compositional effect coming from shifts in the distribution of firm, $F\mu_n$. Since firms of different sizes have different R&D investment behavior, this compositional channel might also affect the rate of growth of the economy.

We plot these different effects against advertising efficiency (θ) in Figure 5. On the one

[36] This is ultimately a quantitative result. Indeed, in Online Appendix E we show that R&D and advertising can be either complements or substitutes, depending on parameters. We also show that the parameters driving this margin in the calibrated model are those related to external R&D ($\widetilde{\chi}$, ν, and σ), and the advertising spillover parameter η.

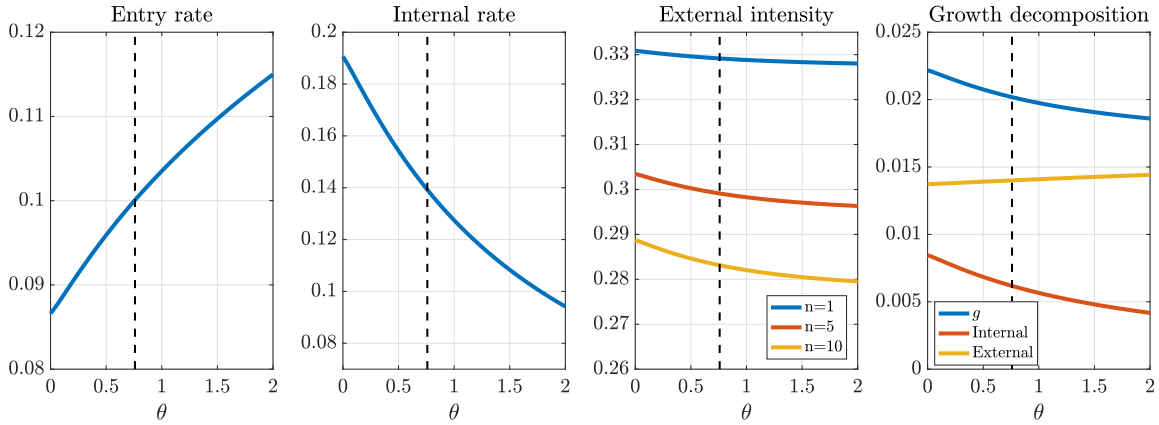

Figure 5: Growth decomposition, for different values of advertising efficiency (θ), in the calibrated economy.
Notes: The first three panels plot x_e/F (Entry rate), z (Internal rate), and x_n (External intensity) for $n = 1, 5, 10$. The fourth panel plots $g = Internal + External$, where $Internal = z\lambda^I$ and $External = \lambda^E\big(x_e + \sum_{n=1}^{+\infty} F\mu_n n x_n\big)$. The calibrated value of θ is marked by the dashed vertical line.

hand, when advertising becomes cheaper (higher θ), the value of becoming an incumbent firm increases, which fosters entry of new firms. The increased R&D investment by entrants has a positive effect on innovation and economic growth. It also increases the rate of creative destruction. On the other hand, a higher creative destruction rate implies that firms are now more likely to lose one of their products by being displaced by another firm. As a result, firms have smaller incentives to perform internal R&D. This decrease in internal R&D investment depresses economic growth. Moreover, incumbent firms also decrease their external R&D intensity when advertising becomes more efficient. This decrease is even more pronounced for larger firms. Finally, there is a compositional effect operating through changes in the firm size distribution. As a result of higher entry rate and lower external innovation by incumbent, the firm size distribution shifts to the left (Figure 6). This places more mass on the part of the firm distribution that is most R&D intensive, which tends to foster growth.

Overall, the decrease in internal and external R&D investment more than offset the increase in the entry rate and the compositional shift in the firm size distribution, and the growth rate decreases when advertising efficiency goes up. In our calibrated economy, we find that these detrimental effects can be substantial. In particular, growth would increase by 0.22 percentage points (from a growth rate of 2% to 2.22%) if advertising was to be shut down completely (if θ was set to zero).[37] This is in spite of there being a larger share of

[37]Since our model is calibrated to firms performing R&D and advertising, representing 76.4% of total R&D

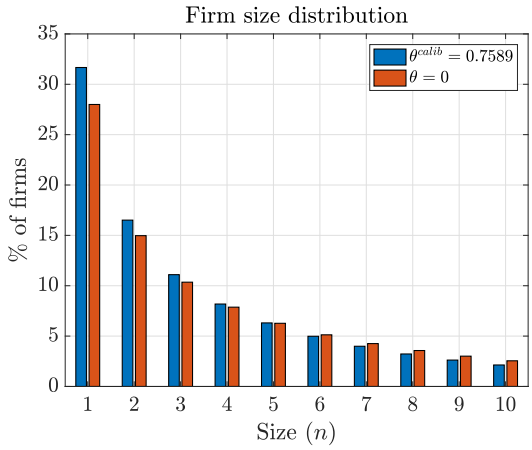

Figure 6: Change in firm size distribution as a function of advertising efficiency (θ).

small and more innovative firms in an economy with advertising. For instance, the share of firms with 5 products of less falls from 73.77% to 67.46% when advertising is shut down. Moreover, in Appendix C we show that this growth effect of advertising offsets potentially beneficial level effects, and overall leads to a decrease in welfare.

Empirical validation of the R&D-Advertising substitution We have shown that, while our model could potentially generate complementarity between R&D and advertising (see Online Appendix E), the calibration predicts substitutability. To validate this key quantitative result, we provide empirical evidence for this crowding out between advertising and R&D.

To test for the existence of substitution between R&D and advertising expenditures, we use exogenous variation in the cost of R&D across time and space. Starting with Minnesota in 1982, several states introduced R&D tax incentives to stimulate innovation. These tax incentives are typically in the form of tax credits, corresponding to a certain share of R&D expenditures, which can be deducted from the state corporate income tax. The inception date of the tax varies across states, and the tax credit rate varies across states and over time within each state. By the year 2009, 32 states offered such credits with rates varying between 2% and 20%. Therefore, we can exploit exogenous variation in the relative price of R&D over time and across states.

expenditures in Compustat (see discussion in Section 2), these quantitative results, as well as those in Section 5 and Appendix C, should be interpreted as upper bounds to the aggregate effects for the whole economy.

We use annual data from Compustat on R&D, advertising, sales, and our measures of firm age and financial constraints from Section 2. We extend our original sample to the period 1950-2009, to cover periods before and after the introduction of subsidies. We proxy the location of the firm R&D activity by the headquarters location from Compustat. We focus our analysis on firms reporting positive advertising and R&D expenditures. Data on tax credits are obtained from Wilson (2009) and Falato and Sim (2014).[38]

To provide evidence for substitution between R&D and advertising, we show that a decrease in the relative price of R&D (through an increase in R&D tax credits) leads to a significant decrease in advertising intensity. We perform the following firm-level regressions:

$$\log\left(\frac{Adv_{ijs,t}}{Sales_{ijs,t}}\right) = \alpha_0 + \beta\, TaxCredit_{s,t} + \mathbf{X}'_{ijs,t}\boldsymbol{\gamma} + \alpha_j + \alpha_s + \alpha_t + u_{ijs,t} \tag{17}$$

for firm i in industry j and state s at time t, where α_j, α_s and α_t control for industry, state, and time fixed effects respectively, $TaxCredit_{s,t}$ is a measure of R&D tax credit, and $\mathbf{X}_{ijs,t}$ is a vector of control variables including sales, the age of the firm, firm leverage, state corporate tax rates, and selling, general and administrative expense (SG&A).[39]

Regression results are reported in Table 6. Columns (1) to (4) use different measures for the tax credit rate and R&D relative cost in order to take into account the magnitude of the credit.[40] Column (1) shows a significantly negative relationship between advertising intensity and the relative price of R&D using changes in the statutory tax credit. In particular, a one-percentage-point increase in the statutory state R&D tax credit is associated with a 1.99% decrease in advertising expenditures. In column (2), we use a tax-adjusted credit rate. Column (3) uses an alternative measure of the marginal effective R&D tax credit which accounts for the different definitions of the R&D expenditures which are eligible for tax credit as well as the horizon over which the tax credit is calculated. For all these cases, a decrease in the relative cost of R&D leads to a decrease in advertising intensity, supporting our result that R&D and advertising are substitutes at the firm level. Finally, in column (4), we use

[38] We use data for all 50 U.S. states as well as for the District of Columbia. When states have several credit brackets, we use the top marginal rate.

[39] SG&A includes operation expenses not directly related to production such as accounting, advertising, delivery, directors' remuneration, engineering, marketing, legal expenses, and R&D.

[40] A full description of these different measures is reported in Online Appendix H.

	(1) $\log\left(\frac{Adv}{Sales}\right)$	(2) $\log\left(\frac{Adv}{Sales}\right)$	(3) $\log\left(\frac{Adv}{Sales}\right)$	(4) $\log\left(\frac{Adv}{Sales}\right)$
State credit rate	–1.988			
	(0.585)			
Tax-adjusted state rate		–1.999		
		(0.636)		
Effective state rate			–2.125	
			(0.591)	
R&D user cost				2.100
				(0.654)
Controls	✓	✓	✓	✓
Time FE	✓	✓	✓	✓
Industry FE	✓	✓	✓	✓
State FE	✓	✓	✓	✓
Observations	29232	29232	26992	26992
R^2	0.42	0.42	0.42	0.42

Table 6: Effect of R&D subsidies on advertising intensity at the firm level.
Notes: Data from Compustat from 1950 to 2009, Wilson (2009), and Falato and Sim (2014). For a description of the different tax measures, see Online Appendix H. Controls include firm-level log sales, age, leverage, and log SG&A expenses, as well as the state tax. Firm age is measured as the elapsed time since the first observation in the data. Standard errors are clustered at the state level (in parentheses).

one additional state-level measure of the cost of R&D, developed by Bloom *et al.* (2002): the R&D user cost. Column (4) of Table 6 shows that higher R&D user costs are also associated with higher levels of advertising intensity, as expected if R&D and advertising are substitutes.

To confirm these findings, we run a series of robustness checks. The full description of these checks and the corresponding tables can be found in Appendix D. First, we show that our results are robust to the inclusion of firm fixed effects, state- and industry-specific time trends. Second, we use patent data to construct two alternative proxies for the location of the firm's R&D activity, namely the location of the inventors and of the patent assignee. All these results confirm the substitutability between R&D and advertising obtained in our baseline regression and support the prediction of our model. Finally, we show that a lower cost of R&D is associated with higher levels of Selling, General and Administrative (SG&A) expenses net of R&D and advertising. This suggests that, while R&D and advertising are substitutes at the firm level, R&D is a complement to intangible expenditures *other* than advertising. This provides further evidence that advertising and R&D might interact at the firm level in a unique way relative to other intangibles.

5 Policy Implications

In the above discussion, we have seen that advertising can be detrimental to growth and welfare. In this section, we study the policy implications of our model. Specifically, we analyze two types of policies. First, we explore the effects of a tax on advertising.[41] Second, we evaluate the effectiveness of R&D subsidies in the presence of advertising.

5.1 Advertising Tax

Consider a tax on advertising expenditure along the BGP. The firm's total advertising expenditure is now $(1+\xi)m_j$ on each product j, where $\xi \geq 0$ is the tax rate. Namely, for every unit of expenditure in advertising, the firm must pay a tax ξ. Proceeds of the tax are rebated lump-sum to households.

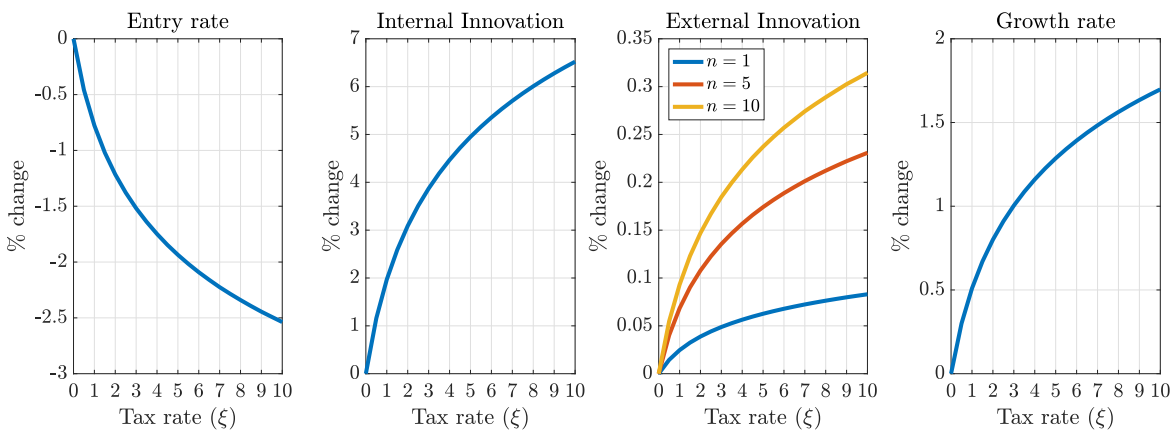

Figure 7: Percentage change (with respect to $\xi = 0$) in growth, entry, creative destruction and internal innovation rates of the calibrated economy, for different sizes of the advertising tax rate.

Figure 7 plots, for different levels of the tax rate, the percentage change (with respect to $\xi = 0$) in the entry rate, internal, and external innovation intensities, and the growth rate. Note that the advertising tax is equivalent to a decrease in advertising efficiency, which unambiguously increases the growth rate of the economy in the calibrated model.[42] In theory, an infinite tax on advertising (equivalent to setting $\theta = 0$) would generate a 0.22 percentage-point increase in economic growth. However, we show that an extremely high tax

[41] The idea of taxing advertising is not new (see Solow (1968)). For completeness, in Appendix C we study both subsidies and taxes on advertising in the calibrated model.

[42] In particular, $\theta^{post-tax} \equiv \theta^{pre-tax}(1+\xi)^{-\zeta}$, as per Equation (4).

rate on advertising would be required to achieve non-negligible gains in terms of growth. For instance, as seen in Figure 7, as little as a 1% increase in the growth rate of the economy (from 2% to 2.02%) would require a tax rate of about 300%.

The main reason for this result is that the elasticity of sales to advertising expenditure is low ($\zeta = 0.1$), which implies that firms' response in innovation efforts to the change in the advertising cost from higher tax rates is small for relatively low tax rates: firms very modestly substitute advertising spending for R&D spending, and overall the economy's growth rate responds very little. In sum, even though the advertising sector is detrimental to growth, very high levels of taxation would be required to achieve significant growth gains.

5.2 R&D Subsidies

Next, we turn to the effects of R&D subsidies. A study of this type of industrial policy in the context of an endogenous growth model of firm dynamics is not new to our work. For example, Akcigit (2009) and Acemoglu *et al.* (2018) find that R&D subsidies increase growth, particularly when targeted toward entrants and small incumbent innovative-intensive firms. This policy scheme is typically welfare-improving because it has the potential of correcting for the inefficiencies stemming from R&D (namely, that firms fail to appropriate the full social value of their innovations, as they will be passed on to future producers).[43] In our calibrated economy, advertising poses a second source of distortions: by altering innovation decisions, it may impede the economy from reaching its full growth potential.

Counterfactual model To understand the role that the advertising-innovation interaction plays, we compare two subsidized economies. The first economy corresponds to our baseline calibration of Section 4.1. The second economy is a recalibrated model in which advertising is shut down ($\theta = 0$), and the weak scalability in firm growth and R&D intensity is fully attributed to the R&D technology parameters (ψ and σ). In this latter specification, we have two fewer parameters to calibrate, and 5 as opposed to 7 moments to target (as we drop the moments related to advertising).

[43] Jones and Williams (2000), Alvarez-Pelaez and Groth (2005) and Bloom *et al.* (2013) show that the inefficiency in Schumpeterian models arising from a business-stealing effect is typically dominated by positive externalities.

				ADV Model		No ADV Model	
Parameter		**Target**	**Data**	**Param.**	**Moment**	**Param.**	**Moment**
Innovation step	λ	Average growth rate	0.020	0.0444	0.020	0.0220	0.020
Entry cost	ν	Firm entry rate	0.098	0.9825	0.100	1.1646	0.097
Internal R&D scale	$\widehat{\chi}$	R&D-Sales ratio	0.102	0.0551	0.133	0.0106	0.107
Advertising efficiency	θ	R&D-ADV ratio	26.579	0.7589	26.212	.	.
External R&D scale	$\widetilde{\chi}$	Gibrat's coefficient	–0.0331	1.4163	–0.0327	2.6891	–0.0252
R&D scalability	σ	R&D intensity coef.	–0.1053	–0.9743	–0.1140	–0.9388	–0.1299
Advertising spillover	η	ADV intensity coef.	–0.0360	0.8768	–0.0361	.	.
Returns to scale in R&D ($\psi + \sigma$)				1.0257		1.0612	

Table 7: Internally calibrated parameters, and model fit, in the baseline calibration with advertising and decreasing returns to scale in R&D ("ADV Model") and an alternative calibration with no advertising and decreasing returns to scale in R&D ("No ADV Model"). *Notes:* See Table 3.

Table 7 reports the internally-identified parameter values for the calibrated models with and without advertising, and the fit of the two models. As expected, absent the advertising channel, we need stronger decreasing returns to scale in R&D in order to match the deviations from constant growth and R&D intensity ($\psi + \sigma = 1.0612$, versus $\psi + \sigma = 1.0257$ in the baseline model). In the recalibrated model, a 10-product firm would find it 15.1% more expensive to grow by a given rate than it would be for 10 firms of one product each. In the baseline calibration, this number was 6.1%. The model without advertising has a harder time quantitatively explaining both deviations from constant R&D intensity and Gibrat's law jointly. In contrast, the baseline calibration with the advertising spillover and decreasing returns in R&D is not only able to closely match both of these deviations, but also the advertising regression coefficient.

Figure 8 shows the relationship between dispersion in firm growth and R&D intensity with firm size, in both models and the data. The model with advertising matches the evolution of the standard deviations across sales quintiles more closely than the model without advertising. Even though the latter model correctly predicts a decline in growth dispersion across firm size, the rate of this decline is lower than that in the data and the baseline model. We find a similar pattern regarding dispersion in R&D intensity. All in all, the model with advertising does a better job in matching second-order moments regarding firm growth and R&D intensity in the cross-section of firms.

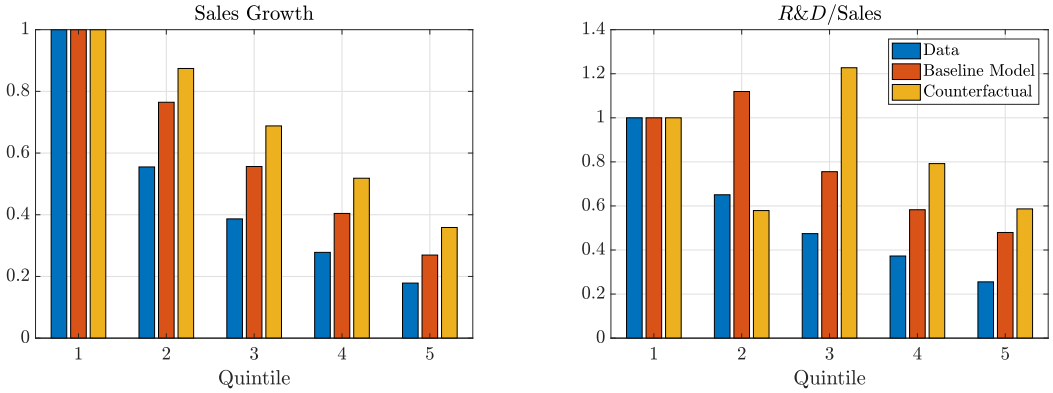

Figure 8: Standard deviations within sales quintiles, relative to first quintile (normalized to one): Baseline and Counterfactual Models vs. Data.
Notes: Firms are ranked in sales quintiles according to their normalized level of sales (sales as a ratio of average sales in the same year). The standard deviation in the first quintile is normalized to one.

Subsidy results With these two models at hand, we assume that a time- and size-invariant subsidy $s \in (0,1)$ is given to each type of firm conducting R&D. This subsidy is financed by a lump-sum tax on households. The R&D cost functions now read $R_e(x_e) = (1-s)\nu\bar{Q}x_e$, $R_z(z_j) = (1-s)\widehat{\chi}q_j z_j^{\widehat{\psi}}$ and $R_x(X_n, n) = (1-s)\widetilde{\chi}\bar{Q}X_n^{\widetilde{\psi}}n^{\sigma}$.

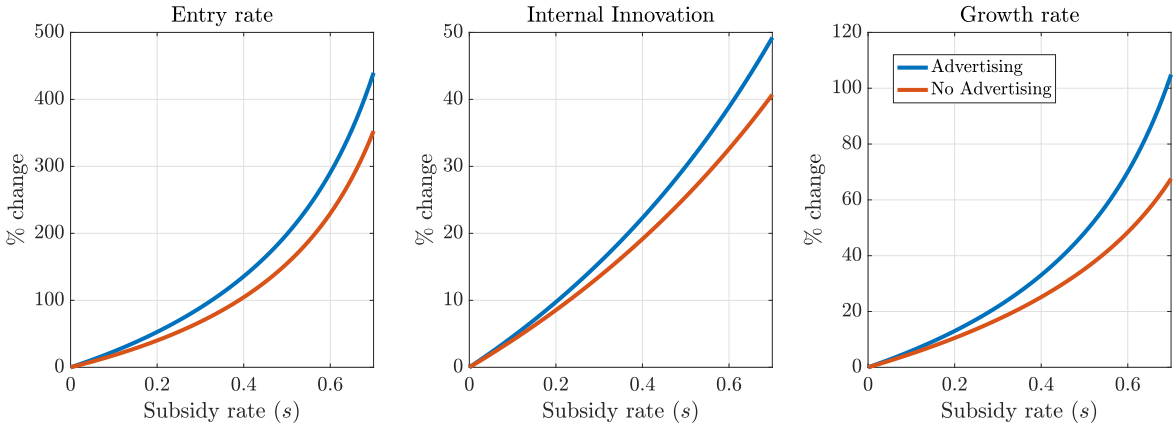

Figure 9: Percentage change, with respect to the BGP solution with $s = 0$, in the entry rate (x_e/F), the internal innovation rate (z), and the growth rate (g), in both calibrated economies. Blue line: baseline economy with both advertising and decreasing returns in R&D; red line: economy with no advertising and with decreasing returns to scale in R&D.

Figure 9 shows the percentage change, with respect to the no-subsidy allocation ($s = 0$), in growth, entry and internal innovation rates, under both calibrated economies and for different levels of the subsidy. In line with previous studies, we find that the subsidy increases growth. This is because innovation becomes cheaper, which fosters entry and raises R&D expenditure for all firms. More importantly, we find that the subsidy has a bigger impact on growth in

the economy with advertising. For instance, in the economy with advertising, a 10% subsidy increases the growth rate to $g = 2.14\%$, while in the economy without advertising a subsidy of the same size increases it to $g = 2.09\%$, i.e. a differential of 0.05 percentage points. This differential increases with the subsidy. For instance, a subsidy of 50% increases the growth rate by 0.3 percentage points more in the economy with advertising ($g = 2.99\%$ versus $g = 2.69\%$). The reason for this result is twofold. On the one hand, in the economy with advertising the subsidy has a much larger positive impact on entry, as smaller firms benefit more from the now cheaper R&D investment because of the advertising spillover channel. On the other hand, the impact of the subsidy on the internal innovation rate is also stronger. Both of these contribute to growth disproportionately more than they do in the economy without advertising.

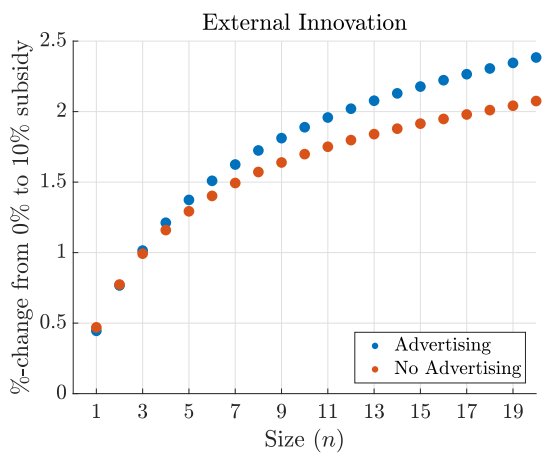

Figure 10: Percentage change in external innovation intensity for an increase from $s = 0\%$ to $s = 10\%$, across firm sizes, under both calibrated economies.

Another noteworthy effect can be seen in Figure 10, which plots the change in the external innovation rate for different sizes for a change from $s = 0\%$ to $s = 10\%$. When the spillover effect in advertising is active, firms are more responsive to the subsidy in terms of external R&D. This is particularly true for larger firms as the difference in external R&D response to a given change in subsidies gets larger with firm size. This is because in the economy without advertising, there are stronger decreasing returns to scale. This implies that larger firms are even less efficient in conducting R&D, which dampens their response to the subsidy. Finally, because of the substitutability between R&D and advertising, higher R&D subsidies are associated with lower advertising expenditures. For instance, a 10% (respectively, 50%)

subsidy decreases the average firm-level advertising expenditure by 14% (respectively, 46.7%).

Overall, our results suggest that identifying the source of technical efficiency in producing innovation across firm size has very relevant implications for the impact of industrial policy.

6 Conclusion

This paper proposes a model of firm dynamics and endogenous growth through product innovations that explicitly incorporates advertising decisions by firms. In modeling advertising, we have been inspired by observations from the empirical marketing literature, specifically the existence of advertising spillovers across goods. In our calibrated model, smaller firms benefit more in terms of decreased advertising costs from brand extensions. In equilibrium, these firms become relatively more concerned with conducting innovative activities, as they benefit more from advertising additional product lines due to the spillover effect. This mechanism generates the empirical observation that R&D intensity is decreasing with firm size even in the absence of differences in R&D technology across firm size.

In the calibrated model, the advertising-R&D interaction is responsible for a large share of the deviation from perfect scaling in firm growth (49.7%) and R&D intensity (60.8%). Advertising also explains a substantial share of cross-sectional dispersion in sales (29.3% at the firm level, and 28% at the product level). Moreover, we find that advertising is detrimental to economic growth because it crowds out innovation investment within the firm. Key to this argument is that R&D and advertising are substitutes, which we confirm empirically by exploiting exogenous variation in the cost of innovation arising from changes in the tax treatment of R&D in the U.S. Based on these results, we have proposed a set of growth-enhancing industrial policies, and we have seen that the effects of R&D subsidies depend critically on the interaction between the two types of intangibles.

Identifying the sources of heterogeneity in firm growth remains an important question. Further understanding the consequences of R&D size dependence and studying the role of other types of intangible investments on innovation, firm dynamics, and the aggregate economy, are interesting avenues for future research.

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Advertising, Innovation, and Economic Growth

by Laurent Cavenaile and Pau Roldan-Blanco

Appendix

A Summary Statistics

	ADV and R&D firms		All firms	
	Mean	St. dev.	Mean	St. dev.
Sales (million U.S.D.)	2577.6	12334.7	1846.2	10372.7
Age (years)	14.6	12.8	15.2	13.1
Leverage	0.278	31.2	1.603	234.6
R&D (million U.S.D.)	115.1	585.0	67.7	409.9
ADV (million U.S.D.)	74.3	406.6	56.0	304.3
R&D/Sales	0.102	0.136	0.109*	0.156*
ADV/Sales	0.037	0.067	0.040*	0.072*
R&D/ADV	26.6	231.5		
Share of firms doing R&D		48.0%		
Share of total sales from firms doing R&D		63.5%		
Share of firms doing ADV		46.8%		
Share of total sales from firms doing ADV		59.4%		
Share of R&D firms also doing ADV		63.3%		
Share of sales of R&D firms from firms also doing ADV		65.1%		
Share of R&D by firms also doing ADV		76.4%		
Share of R&D in our sample		52.6%		

Table A.1: Descriptive statistics. Compustat data from 1980 to 2015. The table reports means and standard deviations for our sample of firms performing R&D and advertising, as well as for all non-financial firms in Compustat. Leverage is computed as the ratio of total debt (long term and current liabilities) to equity. Reported shares are computed for firm/year observations with respect to all non-financial firms in our sample.
* Excludes outliers with corresponding ratios above 100%.

B Derivations and Proofs

B.1 Value Functions

Consider discretizing the time space in small steps of size Δt. The value function for a typical incumbent firm with intrinsic quality portfolio $\boldsymbol{q}$ and n products at time t is:

$$
\begin{aligned}
V_{n,t}(\boldsymbol{q}) = \max_{x_t, \{z_{j,t}\}} \Bigg\{ & \sum_{q_j \in \boldsymbol{q}} \Big(\widetilde{\pi}_t q_j - \widehat{\chi} z_{j,t}^{\widehat{\psi}} q_j \Big) \Delta t - \widetilde{\chi} n^{\sigma + \widetilde{\psi}} x_t^{\widetilde{\psi}} \bar{Q}_t \Delta t + \gamma_t \bar{Q}_t n^{\frac{\eta}{1-\zeta}} \\
& + e^{-r_{t+\Delta t}\Delta t} \Bigg(\sum_{q_j \in \boldsymbol{q}} \Big[z_{j,t} \Delta t + o(\Delta t) \Big] V_{n,t+\Delta t} \Big(\boldsymbol{q} \backslash_- \{q_j\} \cup_+ \{q_j(1+\lambda^I)\} \Big) \\
& \qquad + \sum_{q_j \in \boldsymbol{q}} \Big[\tau_t \Delta t + o(\Delta t) \Big] V_{n-1,t+\Delta t} \Big(\boldsymbol{q} \backslash_- \{q_j\} \Big) \\
& \qquad + \Big[n x_t \Delta t + o(\Delta t) \Big] \int_0^1 V_{n+1,t+\Delta t} \Big(\boldsymbol{q} \cup_+ \{q_j(1+\lambda^E)\} \Big) \mathrm{d}j \\
& \qquad + \Big[1 - \sum_{q_j \in \boldsymbol{q}} z_{j,t} \Delta t - \sum_{q_j \in \boldsymbol{q}} \tau_t \Delta t - n x_t \Delta t - o(\Delta t) \Big] V_{n,t+\Delta t}(\boldsymbol{q}) \Bigg) \Bigg\} + o(\Delta t)
\end{aligned}
$$

where $o(\Delta t)$ has the property $\lim_{\Delta \to 0} \frac{o(\Delta t)}{\Delta t} = 0$. Here, we use the fact that, for each Poisson arrival rate $k \in \{z, x, \tau\}$, the term $k\Delta t + o(\Delta t)$ (respectively, $1 - k\Delta t - o(\Delta t)$) approximates the probability of exactly one Poisson event (respectively, zero Poisson events) within an interval of short length (i.e., for small $\Delta > 0$). The probability of two or more events is equal to $o(\Delta t)$ in the limit as $\Delta \to 0$. To obtain Equation (6), simply subtract $e^{-r_{t+\Delta t}\Delta t} V_{n,t}(\boldsymbol{q})$ from both sides, divide every term by Δt, and take the continuous-time limit as $\Delta \to 0$ (using the fact that $\lim_{\Delta \to 0} \frac{o(\Delta t)}{\Delta t} = 0$ and $\lim_{\Delta \to 0} \frac{1 - e^{-r_{t+\Delta t}\Delta t}}{\Delta t} = r_t$).

For entrants, the value function is:

$$
\begin{aligned}
V_{0,t} = \max_{x_{e,t} > 0} \Bigg\{ & - \nu x_e \bar{Q} \Delta t + e^{-r_{t+\Delta t}\Delta t} \Bigg(\Big[x_{e,t} \Delta t + o(\Delta t) \Big] \mathbb{E}_j \Big\{ V_{1,t+\Delta t}(\{q_j(1+\lambda^E)\}) \Big\} \\
& + \Big[1 - x_{e,t} \Delta t - o(\Delta t) \Big] V_{0,t+\Delta t} \Bigg) \Bigg\} + o(\Delta t)
\end{aligned}
$$

where $V_{0,t}$ denotes firm value at $(n, \boldsymbol{q}) = (0, \emptyset)$. To obtain Equation (7), once again subtract $e^{-r_{t+\Delta t}\Delta t} V_{0,t}$ from both sides, divide through by Δt, and take the limit as $\Delta \to 0$.

B.2 Proof of Proposition 1

Assume that $z_{j,t} = z_t$, $\forall j \in [0,1]$, a result that we prove independently in Proposition 3. At any time t, aggregate quality is given by:

$$
\begin{aligned}
\bar{Q}_{t+\Delta t} = & \underbrace{\Big[\tau_t \Delta t + o(\Delta t) \Big] (1+\lambda^E) \bar{Q}_t}_{\text{One external innovation}} + \underbrace{\Big[z_t \Delta t + o(\Delta t) \Big] (1+\lambda^I) \bar{Q}_t}_{\text{One internal innovation}} \\
& + \underbrace{\Big[1 - \tau_t \Delta t - z_t \Delta t - o(\Delta t) \Big] \bar{Q}_t}_{\text{No innovations}} + \underbrace{o(\Delta t)}_{\substack{\text{2 or more} \\ \text{innovations}}}
\end{aligned}
$$

Subtracting $\bar{Q}_t$ from both sides, dividing through by Δt, taking the limit as $\Delta \to 0$ and using that $\lim_{\Delta \to 0} \frac{o(\Delta t)}{\Delta t} = 0$, gives $\dot{\bar{Q}}_t = \tau \lambda^E \bar{Q}_t + z \lambda^I \bar{Q}_t$. Therefore,

$$g = \tau\lambda^E + z\lambda^I$$

as we sought to show. □

B.3 Proof of Proposition 2

Let μ_n denote the equilibrium share of incumbent firms that own $n \geq 1$ product lines, such that $\mu_n \in [0,1]$, $\forall n$, and $\sum_{n=1}^{+\infty} \mu_n = 1$. The invariant distribution must satisfy the following flow equations:

# products	*Inflows*	*Outflows*
$n = 0:$	$F\mu_1\tau$	$= x_e$
$n = 1:$	$F\mu_2 2\tau + x_e$	$= F\mu_1(x_1 + \tau)$
$n \geq 2:$	$F\mu_{n+1}(n+1)\tau + F\mu_{n-1}(n-1)x_{n-1}$	$= F\mu_n n(x_n + \tau)$

The left-hand (or right-hand) side of these equalities describes the mass of firms that enters into (or exits out of) the state identified by the first column. We prove that Equation (10) is the solution to the above system of flow equations by use of mathematical induction. Specializing the formula to $n = 1$, and using the convention that $\prod_{i=1}^{0} x_i = 1$, we have:

$$\mu_1 = \frac{x_e}{F\tau}$$

which is true by the first flow equation. For $n = 2$, Equation (10) gives:

$$\mu_2 = \frac{x_e x_1}{2F\tau^2}$$

which can be written as $2F\tau^2\mu_2 = x_1 x_e$. Adding $x_e\tau$ to both sides an dividing through by $F\tau$ we obtain $\mu_2 2\tau + \frac{x_e}{F} = \frac{x_e}{F}\left(\frac{x_1}{\tau} + 1\right)$. Rearranging, we obtain exactly the flow equation for $n = 2$. Having shown that Equation (10) holds for $n = 1$ and $n = 2$, it is enough to show that it holds for $n + 1$, assuming it does for n and $n - 1$, where $n \geq 2$ is arbitrary. Accordingly, suppose $\mu_{n-1} = \frac{x_e}{F}\frac{\prod_{i=1}^{n-2} x_i}{(n-1)\tau^{n-1}}$ and $\mu_n = \frac{x_e}{F}\frac{\prod_{i=1}^{n-1} x_i}{n\tau^n}$. Plugging these into the third flow equation, we have

$$F\mu_{n+1}(n+1)\tau + x_e\frac{\prod_{i=1}^{n-2} x_i}{\tau^{n-1}}x_{n-1} = x_e\frac{\prod_{i=1}^{n-1} x_i}{\tau^n}(x_n + \tau)$$

Simplifying,

$$\begin{aligned}
F\mu_{n+1}(n+1)\tau &= x_e\left(\frac{\prod_{i=1}^{n-1} x_i}{\tau^n}(x_n + \tau) - \frac{\prod_{i=1}^{n-2} x_i}{\tau^{n-1}}x_{n-1}\right) \\
&= x_e\left(\frac{\prod_{i=1}^{n} x_i}{\tau^n} + \tau\frac{\prod_{i=1}^{n-1} x_i}{\tau^n} - \frac{\prod_{i=1}^{n-1} x_i}{\tau^{n-1}}\right) \\
&= x_e\frac{\prod_{i=1}^{n} x_i}{\tau^n}
\end{aligned}$$

which implies that $\mu_{n+1} = \frac{x_e}{F}\frac{\prod_{i=1}^{n} x_i}{(n+1)\tau^{n+1}}$, what we wanted to show. □

B.4 Proof of Proposition 3

We find Γ and $\{\Upsilon_n\}_{n=1}^{+\infty}$ using the method of undetermined coefficients. Plugging the guess $V_n(\boldsymbol{q}) = \Gamma \sum_{q_j \in \boldsymbol{q}} q_j + \Upsilon_n \bar{Q}$ into (6), we get that:

$$r\Gamma \sum_{q_j \in \boldsymbol{q}} q_j + r\Upsilon_n \bar{Q} = \max_{\substack{x \in [0,\bar{x}] \\ \{z_j \in [0,\bar{z}]\}_{\mathcal{J}_f}}} \left\{ \sum_{q_j \in \boldsymbol{q}} \left[z_j \Gamma \lambda^I q_j + \widetilde{\pi} q_j + \tau \Big((\Upsilon_{n-1} - \Upsilon_n)\bar{Q} - \Gamma q_j \Big) - \widehat{\chi} z_j^{\widehat{\psi}} q_j \right] \right.$$
$$\left. + nx \Big(\Gamma \bar{Q}(1+\lambda^E) + (\Upsilon_{n+1} - \Upsilon_n)\bar{Q} \Big) - \widetilde{\chi} n^{\sigma+\widetilde{\psi}} x^{\widetilde{\psi}} \bar{Q} + \gamma \bar{Q} n^{\frac{\eta}{1-\zeta}} \right\} + \Upsilon_n \bar{Q} g$$

Equating the terms with q_j and $\bar{Q}$, we can decouple the innovation problems into:

$$(q_j): \qquad r\Gamma = \max_{z_j} \left\{ \widetilde{\pi} + z_j \Gamma \lambda^I - \tau\Gamma - \widehat{\chi} z_j^{\widehat{\psi}} \right\}$$
$$(\bar{Q}): \qquad (r-g)\Upsilon_n = \max_{x_n} \left\{ (\Upsilon_{n-1} - \Upsilon_n) n\tau + n x_n \Big(\Gamma(1+\lambda^E) + \Upsilon_{n+1} - \Upsilon_n \Big) - \widetilde{\chi} n^{\sigma+\widetilde{\psi}} x_n^{\widetilde{\psi}} + \gamma n^{\frac{\eta}{1-\zeta}} \right\}$$

The first-order conditions of each problem are:

$$z_j = \left(\frac{\Gamma \lambda^I}{\widehat{\psi}\widehat{\chi}} \right)^{\frac{1}{\widehat{\psi}-1}} \qquad \text{and} \qquad x_n = n^{\frac{1-\sigma-\widetilde{\psi}}{\widetilde{\psi}-1}} \left(\frac{\Gamma(1+\lambda^E) + \Upsilon_{n+1} - \Upsilon_n}{\widetilde{\psi}\widetilde{\chi}} \right)^{\frac{1}{\widetilde{\psi}-1}}$$

respectively. Assuming that there is positive entry in equilibrium ($x_e > 0$), we can exploit the free-entry condition $V_0 = 0$ in (7) to get that:

$$\Gamma = \frac{\nu - \Upsilon_1}{1+\lambda^E}$$

This means that the optimal internal R&D investment by incumbents is:

$$z_j = \left(\frac{\lambda^I(\nu - \Upsilon_1)}{\widehat{\psi}\widehat{\chi}(1+\lambda^E)} \right)^{\frac{1}{\widehat{\psi}-1}}$$

so $z_j = z, \forall j \in [0,1]$. Back into the optimality condition for z, we can obtain the implied rate of creative destruction:

$$\tau = \frac{1}{\nu - \Upsilon_1} \left[\widetilde{\pi} - \widehat{\chi} \left(\frac{\lambda^I(\nu - \Upsilon_1)}{\widehat{\chi}\widehat{\psi}(1+\lambda^E)} \right)^{\frac{\widehat{\psi}}{\widehat{\psi}-1}} \right] - \frac{\rho}{1+\lambda^E}$$

where we have used that $g = r - \rho$ from the Euler equation, and $g = \tau\lambda^E + z\lambda^I$. It remains to find an expression for Υ_n. Using free-entry, we know:

$$x_n = n^{\frac{1-\sigma-\widetilde{\psi}}{\widetilde{\psi}-1}} \left(\frac{\nu - \Upsilon_1 + \Upsilon_{n+1} - \Upsilon_n}{\widetilde{\psi}\widetilde{\chi}} \right)^{\frac{1}{\widetilde{\psi}-1}}$$

Back into the second maximization problem, we get the second-order difference equation:

$$(r-g)\Upsilon_n = (\Upsilon_{n-1} - \Upsilon_n)n\tau + \gamma n^{\frac{\eta}{1-\zeta}} + \widetilde{\chi}(\widetilde{\psi}-1)n^{-\frac{\sigma}{\widetilde{\psi}-1}}\left[\frac{\nu - \Upsilon_1 + \Upsilon_{n+1} - \Upsilon_n}{\widetilde{\psi}\widetilde{\chi}}\right]^{\frac{\widetilde{\psi}}{\widetilde{\psi}-1}}$$

Solving for Υ_{n+1} gives (12). Using $V_0 = 0$ and the guess $V_n = \Gamma \sum_j q_j + \Upsilon_n \bar{Q}$, it is clear that the boundary condition for this difference equation must then be $\Upsilon_0 = 0$. □

Finally, let us show that with constant returns to scale in R&D and without advertising, the model exhibits no deviation from constant R&D intensity and Gibrat's law (as in Klette and Kortum (2004)). In a model with no advertising, we have $\theta = 0$, implying $\gamma = 0$. Using Equation (11), we can write Equation (12) as:

$$\Upsilon_{n+1} = \Upsilon_n - \nu + \Upsilon_1 + \widetilde{\vartheta}\left(\rho\Upsilon_n n^{\frac{\sigma}{\widetilde{\psi}-1}} - (\Upsilon_{n-1} - \Upsilon_n)\tau n^{\frac{\sigma+\widetilde{\psi}-1}{\widetilde{\psi}-1}}\right)^{\frac{\widetilde{\psi}-1}{\widetilde{\psi}}}$$

With constant return to scale in R&D (i.e. $\widetilde{\psi} + \sigma = 1$), this equation further reduces to:

$$\Upsilon_{n+1} = \Upsilon_n - \nu + \Upsilon_1 + \widetilde{\vartheta}\left(\frac{\rho}{n}\Upsilon_n - (\Upsilon_{n-1} - \Upsilon_n)\tau\right)^{\frac{\widetilde{\psi}-1}{\widetilde{\psi}}}$$

Next, we guess and verify that $\Upsilon_n = n\Upsilon$, where Υ is a constant. Solving for Υ gives:

$$\Upsilon = \frac{1}{\rho+\tau}\left(\frac{\nu}{\widetilde{\vartheta}}\right)^{\frac{\widetilde{\psi}}{\widetilde{\psi}-1}}$$

By Equation (13b), this implies that external innovation intensity is constant across firm size:

$$x = \left(\frac{\nu}{\widetilde{\psi}\widetilde{\chi}}\right)^{\frac{1}{\widetilde{\psi}-1}}$$

Since internal R&D scales perfectly with size, total R&D intensity at the firm level is constant in firm size. As R&D is the only source of firm growth in this case, Gibrat's law holds.

C Welfare Effects of Advertising

In Section 4.3, we argued that advertising is detrimental to growth within the calibrated model. In this section, we explore the consequences for welfare. There exists a long-lasting debate in Economics about the welfare implications of advertising. This literature has studied the effects of advertising on welfare through mainly two potential channels: informative advertising and taste-shifting advertising.

- Models of informative advertising are theories in which advertising can be used to remove frictions in markets with imperfect information by providing relevant information about a product quality, or simply about the existence of the product (e.g. Nelson (1974), Butters (1977), and Grossman and Shapiro (1984)). In this context, advertising could be welfare improving. On the other hand, Becker and Murphy (1993) argue that advertising could simply be used to alter consumer preferences by creating favorable associations to the good that is being advertised. Kihlstrom and Riordan (1984) and Milgrom and Roberts (1986) further argue that expenditures in uninformative advertising could still signal a product quality and hence provide ex-post information.

- Another strand of the literature considers advertising as a pure taste shifter by manipulating consumer preferences (e.g. Dixit and Norman (1978), Becker and Murphy (1993) and Benhabib and Bisin (2002, 2011)). Molinari and Turino (2015) consider the case of purely combative advertising, in which advertising does not affect consumer preferences in equilibrium but firms nevertheless advertise to maintain their market shares. In this case, advertising expenditures are a pure waste of resources. When advertising directly affects tastes, a relevant question is whether welfare should be compared using ex-ante or ex-post preferences. In addition, advertising could distort prices and monopoly power, in which case advertising could be welfare decreasing even when measured by ex-post preferences (e,g, Dixit and Norman (1978), Benhabib and Bisin (2002)).

In our model, we consider yet another side effect of advertising when quantifying its welfare implications: its effect on R&D investment, innovation, and economic growth. We can rewrite the household utility in equilibrium as:

$$U(C_0, g) = \underbrace{\ln(C_0)/\rho}_{\substack{\text{Level}\\ \text{effect}}} + \underbrace{g/\rho^2}_{\substack{\text{Growth}\\ \text{effect}}}$$

where C_0 is proportional to $\bar{Q}_0 = 1$ (a normalization). The first term in the ex-post welfare decomposition accounts for a "level effect" of advertising: by conducting advertising, firms expand demand contemporaneously, which increases the perceived utility derived from aggregate consumption. This is because advertising ultimately constitutes a demand shifter for the firm by effectively changing consumer's preferences. Additionally, advertising critically shapes the dynamic incentives to conduct R&D and, therefore, has an indirect effect on growth. This is captured by the "growth effect" term.[44]

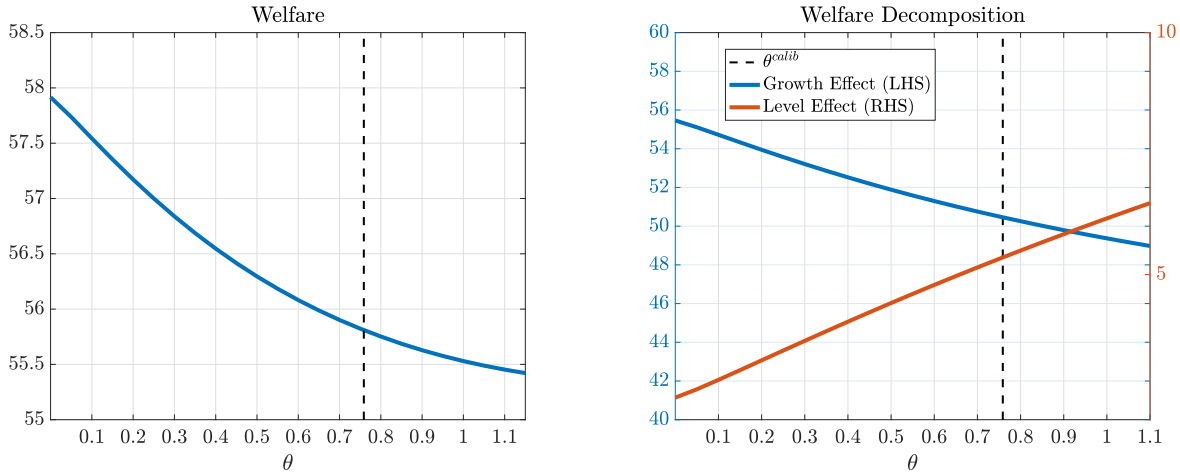

Figure C.1: Welfare decomposition, for different values of advertising efficiency (θ). The calibrated value of θ is marked by the dashed vertical line.

To study the effect of advertising, we compare ex-post welfare in the calibrated economy across different advertising efficiency levels (θ). Figure C.1 plots welfare and its level-growth decomposition

[44] In Online Appendix G.2.3 we present a version of the model in which advertising does not increase the value of consumption in equilibrium. Making welfare comparisons in that case would be equivalent to comparing utility using ex-ante preferences. In that case, welfare is unambiguously decreasing in advertising efficiency (θ).

as a function of θ. The level effect is unambiguously positive. However, as advertising decreases growth in our calibration, the final effect is ambiguous. Using our calibrated parameters, we find that welfare is reduced relative to a world with no advertising ($\theta = 0$) because the growth effects dominates. According to this result, a policy in which the advertising sector is taxed can be welfare-improving for the economy. We analyze this implication in detail in Section 5.1. For completeness, however, in Figure C.2 we explore the welfare effects of both taxes ($\xi > 0$) and subsidies ($\xi < 0$) on advertising, where the tax rate ξ is defined as in Section 5.1. We see that welfare increases with lower subsidies and positive taxes.

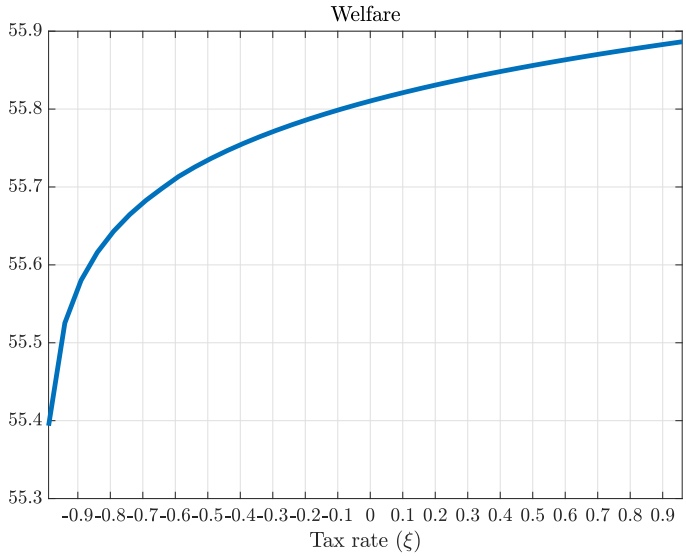

Figure C.2: Welfare for different values of the advertising subsidy/tax rate (ξ). Negative (positive) values of ξ correspond to a subsidy (tax).

D Robustness Checks

D.1 Key Facts

In this appendix, we introduce a series of robustness checks on our main empirical findings.

- First, as our baseline regressions control for industry and time fixed effects, they exploit variation between firms within the same industry in a given year. One may therefore be concerned that the effect is driven by unobserved characteristics correlated across firms. To alleviate this concern, Table D.1 introduces firm-level fixed effects into the four regressions. Moreover, one might be concerned about biases due to time-varying factors affecting the composition of industries. Indeed, over our period of analysis (1980-2015), the U.S. economy underwent several structural shifts which altered the composition and relative weight of different sectors. Prominent examples are the decline in the manufacturing share of total value added, or the productivity slowdown. Similarly, there may have been changes in the geographical composition of industries, perhaps due to changes in legislation or regional trends and business cycles. To the extent that advertising activity may be concentrated across sectors or geographical areas, our coefficients may capture spurious correlations arising from these or other structural breaks. To alleviate these concerns, Table D.1 also controls for state and industry time trends. All the coefficients of interest remain statistically significant and with the same sign, albeit with larger magnitudes.
- Second, we experiment with the treatment of outliers. In the baseline regression, we drop firms with year-on-year sales of more than 1,000%. Table D.2 shows that the Gibrat's law deviation remains significant when we implement instead a 2% (column (1)) and 5% (column (2)) winsorization on the distribution of sales growth. Column (3) uses an alternative definition for sales growth which adjusts for extreme values.[45] Once again, qualitatively our results are unaffected.
- Third, while leverage is used as a proxy of financial frictions at the firm level in our baseline regression, we may experiment with other measures as well (see Table D.3). In Panel A, we control for firm-level investment rates, measured as the change in property plant and equipment over assets, in the specification with industry- and state-specific trends, and firm fixed effects. In addition, Panel B shows results using the Kaplan-Zingales index of financial constraints instead of leverage or the investment rate.[46] Neither of these alternative measures change the sign or significance of our results.
- Fourth, in Table D.4 we show that the sign and significance of the four key coefficients of interest is unaltered when using two popular measures of firm size other than sales: total assets and total employment.
- Finally, Table D.5 shows that our finding that firms and industries with higher advertising intensity levels exhibit stronger deviations from Gibrat's law is robust to the inclusion of firm fixed effects and state and industry time trends.

[45] Specifically, the year-on-year growth in firm sales y_{it} is computed as $\frac{y_{it}-y_{i,t-1}}{0.5(y_{it}+y_{i,t-1})}$ instead of $\frac{y_{it}}{y_{i,t-1}}-1$.
[46] The Kaplan-Zingales index is computed using Compustat variables as described in Lamont *et al.* (2001).

	(1) $\frac{\Delta Sales}{Sales}$	(2) $\log\left(\frac{R\&D}{Sales}\right)$	(3) $\log\left(\frac{Adv}{Sales}\right)$	(4) $\log\left(\frac{R\&D}{Adv}\right)$
$\log(Sales)$	–0.397	–0.281	–0.164	–0.117
	(0.0193)	(0.0158)	(0.0221)	(0.0236)
Firm Age	–0.338	–0.121	0.829	-0.950
	(0.0692)	(0.0919)	(0.159)	(0.158)
Leverage	–0.0000414	–0.0000303	–0.0001000	0.0000697
	(0.0000963)	(0.0000878)	(0.000106)	(0.000105)
Time FE	✓	✓	✓	✓
Firm FE	✓	✓	✓	✓
State Time Trend	✓	✓	✓	✓
Industry Time Trend	✓	✓	✓	✓
Observations	23370	23370	23370	23370
R^2	0.26	0.18	0.19	0.18

Table D.1: Firm level regressions (Facts #1 - #4) with firm fixed effect, state and industry time trends.
Notes: Standard errors are clustered by firm (in parentheses).

	(1) $\frac{\Delta Sales}{Sales}$ (2% wins.)	(2) $\frac{\Delta Sales}{Sales}$ (5% wins.)	(3) $\frac{\Delta Sales}{0.5(Sales+Sales_{-1})}$
$\log(Sales)$	–0.559	–0.314	–0.228
	(0.0179)	(0.00805)	(0.00720)
Firm Age	–0.414	–0.221	–0.139
	(0.104)	(0.0518)	(0.0382)
Leverage	–0.00000660	0.0000385	0.0000221
	(0.0000797)	(0.0000465)	(0.0000368)
Time FE	✓	✓	✓
Firm FE	✓	✓	✓
Industry Time Trend	✓	✓	✓
State Time Trend	✓	✓	✓
Observations	24331	24331	24402
R^2	0.37	0.33	0.26

Table D.2: Gibrat's law regressions with alternative measures of growth.
Notes: Standard errors are clustered by firm (in parentheses).

Panel A. Investment rate	(1) $\frac{\Delta Sales}{Sales}$	(2) $\log\left(\frac{R\&D}{Sales}\right)$	(3) $\log\left(\frac{Adv}{Sales}\right)$	(4) $\log\left(\frac{R\&D}{Adv}\right)$
$\log(Sales)$	–0.339	–0.283	–0.172	–0.111
	(0.0182)	(0.0180)	(0.0245)	(0.0268)
Firm Age	0.0927	0.0569	–0.0106	0.0674
	(0.0148)	(0.0154)	(0.0236)	(0.0279)
Investment Rate	0.173	0.0438	0.108	–0.0642
	(0.0855)	(0.0280)	(0.0666)	(0.0681)
Time FE	✓	✓	✓	✓
Firm FE	✓	✓	✓	✓
State Time Trend	✓	✓	✓	✓
Industry Time Trend	✓	✓	✓	✓
Observations	21945	21945	21945	21945
R^2	0.21	0.18	0.20	0.19

Panel B. KZ index	(1) $\frac{\Delta Sales}{Sales}$	(2) $\log\left(\frac{R\&D}{Sales}\right)$	(3) $\log\left(\frac{Adv}{Sales}\right)$	(4) $\log\left(\frac{R\&D}{Adv}\right)$
$\log(Sales)$	–0.319	-0.293	–0.184	–0.108
	(0.0164)	(0.0187)	(0.0260)	(0.0284)
Firm Age	–0.307	–0.181	0.816	–0.997
	(0.0609)	(0.101)	(0.187)	(0.172)
Kaplan-Zingales	0.0000281	0.00000137	–0.0000335	0.0000349
	(0.0000226)	(0.0000361)	(0.0000451)	(0.0000312)
Time FE	✓	✓	✓	✓
Firm FE	✓	✓	✓	✓
State Time Trend	✓	✓	✓	✓
Industry Time Trend	✓	✓	✓	✓
Observations	20163	20163	20163	20163
R^2	0.20	0.19	0.21	0.19

Table D.3: Firm level regressions (Facts #1 - #4), with investment rate and the Kaplan-Zingales index as altyernative measures of financial constraints, with firm fixed effects, state and industry time trends.
Notes: Standard errors are clustered by firm (in parentheses).

Panel A. Assets	(1) $\frac{\Delta Assets}{Assets}$	(2) $\log\left(\frac{R\&D}{Assets}\right)$	(3) $\log\left(\frac{Adv}{Assets}\right)$	(4) $\log\left(\frac{R\&D}{Adv}\right)$
$\log(Assets)$	–0.854	–0.334	–0.245	–0.0886
	(0.133)	(0.0127)	(0.0178)	(0.0192)
Controls	✓	✓	✓	✓
Time FE	✓	✓	✓	✓
Firm FE	✓	✓	✓	✓
State Time Trend	✓	✓	✓	✓
Industry Time Trend	✓	✓	✓	✓
Observations	20613	23370	23370	23370
R^2	0.15	0.22	0.23	0.18

Panel B. Employment	(1) $\frac{\Delta Emp.}{Emp.}$	(2) $\log\left(\frac{R\&D}{Emp.}\right)$	(3) $\log\left(\frac{Adv}{Emp.}\right)$	(4) $\log\left(\frac{R\&D}{Adv}\right)$
$\log(Emp.)$	–0.340	–0.188	–0.106	–0.0813
	(0.0810)	(0.0206)	(0.0254)	(0.0276)
Controls	✓	✓	✓	✓
Time FE	✓	✓	✓	✓
Firm FE	✓	✓	✓	✓
State Time Trend	✓	✓	✓	✓
Industry Time Trend	✓	✓	✓	✓
Observations	21923	22157	22157	22157
R^2	0.09	0.36	0.15	0.18

Table D.4: Firm level regressions with assets and employment as the measure of firm size (Fact #1 - #4).
Notes: Controls include firm age and leverage. Standard errors are clustered by firm (in parentheses).

	(1) $\frac{\Delta Sales}{Sales}$	(2) $\frac{\Delta Sales}{Sales}$	(3) $\frac{\Delta Sales}{Sales}$
$\log(Sales)$	–0.488	–0.294	–0.311
	(0.0271)	(0.0125)	(0.00826)
$\log(Sales) \times \log\left(\frac{Adv}{Sales}\right)$	–0.0262		
	(0.00428)		
$\log(Sales)\times$ Share of Adv. Firms		–0.122	
		(0.0365)	
$\log(Sales)\times$ Adv. Intensity			–0.403
			(0.128)
Controls	✓	✓	✓
Time FE	✓	✓	✓
Firm FE	✓	✓	✓
State Time Trend	✓	✓	✓
Industry Time Trend	✓	✓	✓
Observations	23370	148252	148252
R^2	0.27	0.16	0.16

Table D.5: Gibrat's law deviation as a function of firm-level advertising intensity (column (1)) and industry-level measures of advertising use, with firm fixed effects and state and industry time trends.
Notes: Controls include firm-level age and leverage. Additionally, column (1) controls for log advertising intensity at the firm level. Standard errors are clustered by firm (in parentheses).

	Group 1: ADV=0 R&D=0	**Group 2:** ADV=0 R&D>0	**Group 3:** ADV>0 R&D=0	**Group 4:** ADV>0 R&D>0
	$\frac{\Delta Sales}{Sales}$	$\frac{\Delta Sales}{Sales}$	$\frac{\Delta Sales}{Sales}$	$\frac{\Delta Sales}{Sales}$
$\log(Sales)$	–0.285	–0.323	–0.342	–0.397
	(0.0111)	(0.0142)	(0.0195)	(0.0193)
Firm Age	–0.0420	0.0739	0.0733	–0.338
	(0.0254)	(0.0596)	(0.0147)	(0.0692)
Leverage	–0.00000351	–0.0000486	–0.00000747	–0.0000414
	(0.00000268)	(0.0000445)	(0.0000139)	(0.0000963)
Time FE	✓	✓	✓	✓
Firm FE	✓	✓	✓	✓
State Time Trend	✓	✓	✓	✓
Industry Time Trend	✓	✓	✓	✓
Observations	58120	34470	26515	23370
R^2	0.13	0.17	0.19	0.26

Table D.6: Gibrat's law regressions for four groups of firms: no-advertising and no-R&D firms (Group 1); no-advertising and R&D firms (Group 2); advertising and no-R&D firms (Group 3); and advertising and R&D firms (Group 4).
Notes: Standard errors are clustered by firm (in parentheses).

D.2 R&D Subsidies

This appendix shows robustness results for the empirical regressions on the R&D-advertising substitution:

- First, in our baseline regression we control for time fixed effects, as well as time-invariant industry and state characteristics, and therefore we exploit variation in the average response across firms within an industry and a given state. To demonstrate that this response does not just reflect possible unobserved and correlated characteristics between firms, in Table D.7 we show that our coefficients remain qualitatively unaffected when we introduce firm fixed effects into the analysis. Significance levels are reduced for the statutory and tax-adjusted rates. However, relative to our preferred measures of the tax rate (the effective state rate and R&D user cost), results remain strongly significant.
- Second, we run robustness checks on firm location. State R&D subsidies can only be claimed for R&D that is conducted in the given state. However, firms may conduct R&D activities in different states than the headquarter state. Since we do not have a direct measure of where the innovative activity takes place, we use two additional proxies for the location of the firm's R&D activity: the location of the inventors of the firm's patents, and that of the patent assignee. Using data from the Harvard Patent Network Dataverse, first we identify the location of the inventor by assigning to each patent the mode of its inventors' location. Next, using information about assignees from the NBER's Patent Database Project, we assign patents to a firm and match these to our sample of firms in Compustat.[47] Finally, we assign a location to each firm, which is determined by the mode of its patent locations. Panel A of Table D.8 shows the results using this definition of location. Panel B shows results when we instead assign the location provided by the patent database itself, which does not always coincide with the inventor-based location. In both cases, we confirm the substitutability between R&D and advertising in the baseline specification. Importantly, results are now significant for all the different tax measures, even when we control for firm-level fixed effects.
- Finally, Table D.9 shows that a lower R&D cost is associated with higher Other Expenses (OE), computed as Selling, General and Administrative expenses (SG&A) net of Advertising and R&D expenses. This suggests that, while R&D and advertising are substitutes at the firm level, R&D is complement to other types of intangibles.

[47] Harvard Patent Network Dataverse available at http://dataverse.harvard.edu/dataverse/patent. NBER's Patent Database Project available at http://sites.google.com/site/patentdataproject/Home/downloads.

	(1) $\log\left(\frac{Adv}{Sales}\right)$	(2) $\log\left(\frac{Adv}{Sales}\right)$	(3) $\log\left(\frac{Adv}{Sales}\right)$	(4) $\log\left(\frac{Adv}{Sales}\right)$
State credit rate	–0.208			
	(0.400)			
Tax-adjusted state rate		–0.165		
		(0.400)		
Effective state rate			–1.101	
			(0.485)	
R&D user cost				1.240
				(0.522)
Controls	✓	✓	✓	✓
Time FE	✓	✓	✓	✓
State FE	✓	✓	✓	✓
Firm FE	✓	✓	✓	✓
Observations	29387	29387	27137	27137
R^2	0.30	0.30	0.29	0.29

Table D.7: Effect of R&D subsidies on advertising intensity at the firm level.
Notes: Robustness check with firm fixed effects. For a description of the different tax measures, see Online Appendix H. Standard errors are clustered by state (in parentheses).

	(1) $\log\left(\frac{Adv}{Sales}\right)$	(2) $\log\left(\frac{Adv}{Sales}\right)$	(3) $\log\left(\frac{Adv}{Sales}\right)$	(4) $\log\left(\frac{Adv}{Sales}\right)$	(5) $\log\left(\frac{Adv}{Sales}\right)$	(6) $\log\left(\frac{Adv}{Sales}\right)$	(7) $\log\left(\frac{Adv}{Sales}\right)$	(8) $\log\left(\frac{Adv}{Sales}\right)$
Panel A. Based on patent assignee location								
State credit rate	–1.058				–0.805			
	(0.635)				(0.397)			
Tax-adjusted state rate		–1.047				–0.794		
		(0.668)				(0.412)		
Effective state rate			–1.414				–1.342	
			(0.662)				(0.540)	
R&D user cost				1.318				0.984
				(0.676)				(0.599)
Observations	19821	19821	18459	18459	19879	19879	18517	18517
R^2	0.46	0.46	0.47	0.47	0.31	0.31	0.30	0.30
Panel B. Based on inventor location								
State credit rate	–1.169				–0.835			
	(0.579)				(0.274)			
Tax-adjusted state rate		–1.154				–0.809		
		(0.620)				(0.280)		
Effective state rate			–1.461				–1.352	
			(0.578)				(0.457)	
R&D user cost				1.453				1.091
				(0.711)				(0.567)
Observations	19079	19079	17784	17784	19132	19132	17837	17837
R^2	0.47	0.47	0.47	0.47	0.30	0.30	0.30	0.30
Controls	✓	✓	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓	✓	✓
Industry FE	✓	✓	✓	✓				
State FE	✓	✓	✓	✓				
Firm FE					✓	✓	✓	✓

Table D.8: Effect of R&D subsidies on advertising intensity at the firm level.
Notes: Regressions based on patent assignee location (Panel A) or inventor location (Panel B). For a description of the different tax measures, see Online Appendix H. Standard errors are clustered by state (in parentheses).

	(1) $\log\left(\frac{OE}{Sales}\right)$	(2) $\log\left(\frac{OE}{Sales}\right)$	(3) $\log\left(\frac{OE}{Sales}\right)$	(4) $\log\left(\frac{OE}{Sales}\right)$	(5) $\log\left(\frac{OE}{Sales}\right)$	(6) $\log\left(\frac{OE}{Sales}\right)$	(7) $\log\left(\frac{OE}{Sales}\right)$	(8) $\log\left(\frac{OE}{Sales}\right)$
State credit rate	0.636				0.601			
	(0.186)				(0.235)			
Tax-adjusted state rate		0.638				0.604		
		(0.203)				(0.243)		
Effective state rate			0.813				0.802	
			(0.258)				(0.364)	
R&D user cost				–0.910				–0.840
				(0.272)				(0.416)
Controls	✓	✓	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓	✓	✓
Industry FE	✓	✓	✓	✓				
State FE	✓	✓	✓	✓				
Firm FE					✓	✓	✓	✓
Observations	29158	29158	26921	26921	29750	29750	27484	27484
R^2	0.56	0.56	0.57	0.57	0.38	0.38	0.36	0.39

Table D.9: Effect of R&D subsidies on intensity of Other Expenses (OE), computed as Selling, General and Administrative expenses (SG&A) net of R&D and Advertising expenses, at the firm level.
Notes: For a description of the different tax measures, see Online Appendix H. Standard errors are clustered by state (in parentheses).

Advertising, Innovation, and Economic Growth

by Laurent Cavenaile and Pau Roldan-Blanco

Online Appendix

E Comparative Statics

In this Appendix:

1. We show that our model can be solved with increasing returns to scale in R&D technology.
2. We show that the model can deliver both substitutability and complementarity between R&D and advertising.
3. We show which parameters drive the substitution/complementarity margin in the calibrated model.

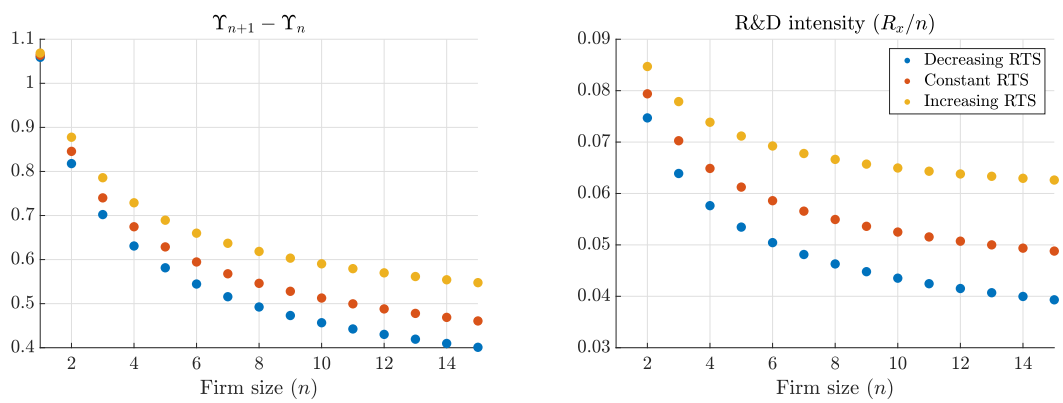

Figure E.1: Comparative statics for R&D intensity across n, for $\sigma + \widetilde{\psi} > 1$ (decreasing returns), $\sigma + \widetilde{\psi} = 1$ (constant returns), and $\sigma + \widetilde{\psi} < 1$ (increasing returns).

First, we show that the result that small firms are more innovation-intensive does not hinge on the relative degree of scalability in the returns of different types of innovation technologies. In Figure E.1 we show three solutions of the model, using decreasing, constant, and increasing returns to scale in external R&D, respectively. The left panel shows the marginal value from acquiring an additional product ($\Upsilon_{n+1} - \Upsilon_n$), and the right panel shows the optimal R&D intensity, as functions of firm size. Decreasing returns lower innovation incentives for all sizes with respect to constant returns, but make it even more profitable for small firms to invest more intensely into R&D (i.e, the R_x/n line is steeper). Symmetrically, having increasing returns in innovation increases optimal intensity for all n. Interestingly, so long as the degree of increasing returns is not too strong, smaller firms may still optimally choose to invest relatively more in R&D in order to reap the benefits of advertising spillovers, as is the case in the figure. This occurs even though the marginal benefit from innovation is lower than in the other two cases (so that the R_x/n line flattens but it is still decreasing).

Next, we show that the model can deliver both substitutability and complementarity between R&D and advertising. Figure E.2 depicts, for a numerical example, the change in external R&D across firm

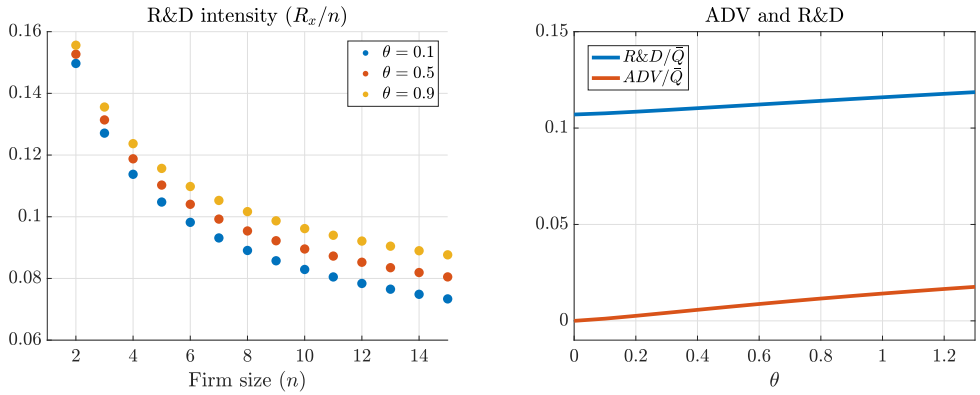

Figure E.2: External R&D intensity in BGP (left panel), and R&D and advertising expenditures in BGP, for different values of advertising efficiency, θ (right panel). All variables are normalized by $\bar{Q}$.

size for different levels of advertising efficiency as well as total incumbent R&D and advertising.[48] In this example, and contrary to the calibrated model, R&D and advertising are complements: an increase in advertising efficiency is associated with larger investment in both advertising and R&D.

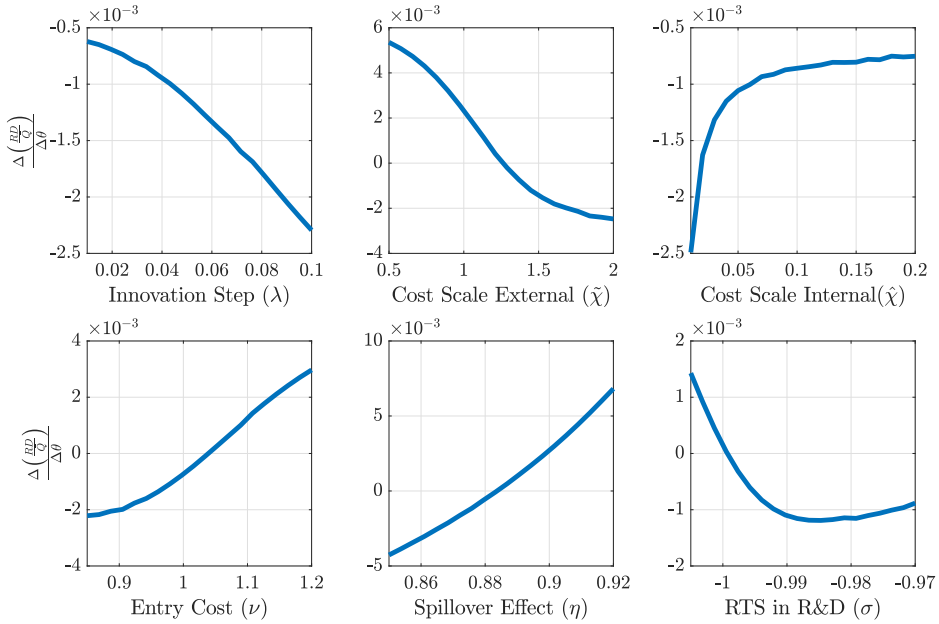

Figure E.3: Change in incumbent R&D, for a small change in θ, evaluated at the calibrated θ. The figure shows this across different parameters, with all the remaining parameters being fixed at their calibrated values from Table 3.

To understand which parameters drive this margin in the calibrated version of the model, in Figure E.3 we plot the change in incumbent R&D (normalized by $\bar{Q}$) for a small change in θ away from its calibrated value (a proxy for the derivative of R&D with respect to θ). We plot this change for different values of each one of the internally calibrated parameters in some neighborhood around

[48]The parameter values used in this example are: $\rho = 0.02$, $\widehat{\psi} = \widetilde{\psi} = 2, \sigma = -0.85$, $\lambda^E = \lambda^I = 0.05$, $\widehat{\chi} = 1$, $\widetilde{\chi} = 5$, $\nu = 2$, $\beta = 0.2$, $\eta = 0.9$ and $\zeta = 0.2$.

their calibrated value, where we fix all the remaining parameters to their calibrated values (from Table 3). When this derivative is negative, an increase in θ (i.e. a decrease in the cost of advertising) leads to a decrease in R&D, so R&D and advertising are substitutes. When the derivative is positive, R&D and advertising are complements.

The figure shows that the key parameters that control for this margin are the cost scale parameter in the external R&D technology ($\widetilde{\chi}$), the entry cost (ν), the advertising spillover parameter (η), and the parameter controlling the degree of returns to scale in R&D (σ). Note, however, that we obtain substitution for all values of the innovation step (λ), as well as the cost scale parameter in the internal R&D technology ($\widehat{\chi}$). Intuitively, the parameters related to the external R&D technology are drivers of the substitution/complementarity margin because the R&D-advertising interaction in the model operates through external innovations. In particular, we obtain that the model delivers complementarity when external R&D is relatively cheaper (low enough values of $\widetilde{\chi}$), and when it scales weakly with firm size (low enough values of σ). Additionally, as firm entry in the model occurs through external R&D, we also obtain complementarity when entry is relatively cheaper (low enough values of ν). Finally, a high enough spillover in advertising (η) also generates complementarity, everything else equal.

F The BGP Algorithm

The following describes the steps of the BGP algorithm:

1. Guess a number $\Phi^* > 0$.
2. Compute Υ_1 by imposing $\Upsilon_2 = 2\Upsilon_1$ in Equation (12) at $n = 1$.
   (a) Compute g from (8), τ implied by (9), Γ from (11), and z from (13a).
   (b) Compute $\{\Upsilon_n\}_{n=2}^{+\infty}$ iterating (12) forward from Υ_1 and $\Upsilon_0 = 0$, $\{x_n\}_{n=1}^{+\infty}$ from (13b), and F/x_e using that $\sum_{n=1}^{N} \mu_n = 1$, where μ_n comes from (10).
3. Verify convergence of the firm size distribution. If there is no convergence by iteration $k \in \mathbb{N}$, go back to step 2 with the new guess for Υ_1 equal to the solution of (12) at $n = 1$ when $\Upsilon_2 = 2\Upsilon_1 - \varepsilon_k$, for a small $\varepsilon_k > 0$.
4. Compute Φ^* as the solution to (5), and compare it to the initial guess. If it is too far, go back to step 1 using this solution as the new guess.

In step 2, we compute the maximum value for Υ_1 such that Υ_n can be weakly concave (i.e, $(\Upsilon_{n+1} - \Upsilon_n)$ decreasing). In particular, we force Υ_n to be straight line from $n = 1$ to $n = 2$ (note $\Upsilon_2 - \Upsilon_1 = \Upsilon_1 - \Upsilon_0$). If μ_n does not converge, it must be because $(\Upsilon_n - \Upsilon_{n-1})$ has not settled to a flat line as n has approached N, which means that the guess for Υ_1 was incorrect. Then, we iterate on new guesses for Υ_1 to allow for more concavity on the Υ_n sequence (indeed, note that in any iteration $k \geq 1$ we always start the Υ_n sequence at a Υ_1 such that $\Upsilon_2 - \Upsilon_1 < \Upsilon_1 - \Upsilon_0$). In step 3, we bisect the new guess by a factor of ten on each new iteration, i.e. $\varepsilon_{k+1} = \varepsilon_k/10$. Finally, in step 4, we drop the old Φ^* guess in case of no convergence, and use the resulting fixed-point as the new guess.

G Model Extensions

G.1 Patents and Major Innovations

Besides differences in R&D intensity across size, the literature has emphasized other motives why smaller firms might be relatively more efficient in conducting innovations. One such literature has focused on patent behavior. Two key findings are noteworthy:

Fact 1. Entrants and smaller firms typically produce relatively more major and radical innovations (when the quality of a patent is based on the number of external citations that it receives).

Fact 2. These firms also tend to patent relatively more on average.[49]

In this section, we show that a simple extension of our baseline model with advertising can deliver both of these additional facts even in the absence of decreasing returns to scale in R&D.

Let us assume that each innovation creates a new patent that potentially cites other patents that exist at the time the new patent is introduced. Following Akcigit and Kerr (2018), each innovation belongs to a technological cluster, and there exist two types of technological advances: *follow-up* and *major advances*. A major advance in a production line creates a whole new cluster of innovation, while a follow-up innovation belongs to the same cluster as the patent that it improves upon. Each patent within a technological cluster is assumed to cite all the previous innovations in the same cluster with some positive probability. Consequently, major advances in the model do not cite any other existing patent and potentially receive citations from follow-up innovations within the same cluster. Once a new major innovation creates a new technological cluster, the cluster that it replaces receives no more citations. Likewise, follow-up innovations can also receive citations from subsequent follow-up innovations in the same cluster. However, on average, major advances receive more citations than follow-up innovations.

Formally, we extend the baseline model by allowing external innovations to result in major technological advances with some probability. Whereas internal R&D can only result in a follow-up innovation, with step size $\lambda^I > 0$, external innovation can either be a follow-up within an existing technological cluster, or be a major advance and create a new technological cluster altogether. Let ω denote the probability with which a successful external innovation leads to a major technological advance. In this case, the step size for quality improvement is equal to $\lambda^H > \lambda^I$. With the remaining probability $(1-\omega)$, the successful external innovation is a follow-up, which leads to a step size $\lambda^L \in (0, \lambda^H]$. In sum, for any product line j and a small interval $\Delta t > 0$, intrinsic quality is given by:

$$q_{j,t+\Delta t} = q_{jt} + \begin{cases} \lambda^H q_{jt} & \text{w.prob. } \omega\tau_t\Delta t + o(\Delta t) & \text{[External, major advance]} \\ \lambda^L q_{jt} & \text{w.prob. } (1-\omega)\tau_t\Delta t + o(\Delta t) & \text{[External, follow-up]} \\ \lambda^I q_{jt} & \text{w.prob. } z_{jt}\Delta t + o(\Delta t) & \text{[Internal]} \\ 0 & \text{w.prob. } 1 - \tau_t\Delta t - z_{jt}\Delta t - o(\Delta t) & \text{[No innovations]} \end{cases}$$

[49] Both facts have been documented by Akcigit and Kerr (2018). On Fact 1, they show that the fraction of a firm's patent in the top patent quality decile is decreasing with firm size. On fact 2, they show that there is a negative correlation between firm size (measured by employment) and patent intensity (measured as the ratio of patents to employment).

Therefore, the average quality improvement from a successful external innovation is equal to $\lambda^E \equiv \omega\lambda^H + (1-\omega)\lambda^L$. The growth and creative destruction rates of the economy are still given by (8) and (9), respectively. Additionally, the model now delivers Fact 1: smaller firms, while exhibiting a higher R&D intensity because of the advertising spillover effect, also produce relatively more major innovations.

Next, we show that the extended model can deliver the prediction that smaller firms tend to patent relatively more on average, and that their patents are of higher quality (when quality is measured by the number of external citation that it receives). For this, we can characterize expected patent citations. Let us assume that the probability that the n-th follow-up innovation cites all relevant past patents is κ^n, where $0 < \kappa < 1$. This generates a decline in the relative citation rate as a technological cluster ages. The expected number of citations received by major as well as follow-up innovations for major technological advances is:

$$\mathbb{E}[cit^M] \equiv \underbrace{\frac{\tau\omega}{\tau+z}\cdot 0}_{\substack{\text{External and major}\\ \text{innovation}}} + \underbrace{\Lambda\left\{\kappa + \Lambda\left[\kappa^2 + \Lambda\left(\kappa^3 + \dots\right)\right]\right\}}_{\text{Follow-up innovations}} = \sum_{j=1}^{\infty}(\kappa\Lambda)^j \tag{G.1}$$

In the first term, $\frac{\tau\omega}{\tau+z}$ is the probability that a successful innovation in a given product line is external and major, which creates a new cluster and does not add a citation within the existing cluster. In the second term, we have defined:

$$\Lambda \equiv \frac{\tau(1-\omega)+z}{\tau+z}$$

as the probability of a follow-up innovation, coming either from an external or an internal innovation. The probability that such an innovation yields a single citation is κ, and therefore the probability of the n-th follow-up yielding a citation is $(\kappa\Lambda)^n$. The expected number of citations is then the sum of all such probabilities. Since $\kappa\Lambda < 1$, Equation (G.1) can be expressed as:

$$\mathbb{E}[cit^M] = \frac{\kappa\Lambda}{1-\kappa\Lambda}$$

A similar derivation shows that the expected number of citations for the n-th follow-up innovation in a given technological cluster, denoted $\mathbb{E}[cit_n^F]$, is given by:

$$\mathbb{E}[cit_n^F] = \frac{\kappa^{n+1}\Lambda}{1-\kappa\Lambda}$$

Therefore, $\mathbb{E}[cit_n^F] < \mathbb{E}[cit^M]$ for any number of follow-ups, $n \in \mathbb{N}$. Since in the model with advertising smaller firms invest relatively more in external R&D, these firms also hold relatively more patents, and these patents are of higher quality on average (as measured by the number of external citations). In sum, our model with advertising can generate the observed negative correlation between firm size and the fraction of top quality patents in the firm's patent portfolio, even when there exist non-decreasing returns to scale in the R&D technology.

G.2 Alternative Advertising Functions

The main mechanism presented in the paper relies on one key assumption: advertising acts as a *demand shifter* in a way that is comparable to the effect of increased intrinsic quality from innovation.

This is a necessary (though not sufficient) condition for the substitution between R&D and advertising that we obtain in our calibration.

The purpose of this section is to propose different alternative ways to model advertising, and show that they also lead to advertising being a demand shifter. Thus, the results obtained in this paper would qualitatively hold under the different alternative models presented below.

G.2.1 Goodwill Accumulation

In our baseline model, we assume that the advertising decision is static. Advertising expenditures affect current demand but have no long-lasting effect on consumer demand. An alternative way of modeling advertising and its effects on demand which is often used in the literature is to assume that advertising expenditures accumulate over time to increase a brand equity (so-called goodwill). Goodwill in turns acts as a demand shifter. The evolution of goodwill is:

$$\dot{G}_j = d_j - \delta G_j \tag{G.2}$$

where $\delta \in [0, 1]$ controls the rate at which goodwill depreciates. Allowing for goodwill accumulation would not qualitatively change the results derived in our baseline model. This said, the marketing literature shows that the depreciation rate is relatively high, with the effect of goodwill on sales almost entirely vanishing after one year (see for instance Assmus *et al.* (1984) and Dubé *et al.* (2005)). With our calibration at the yearly frequency, modeling advertising as a static decision is not likely to be a major concern. In addition, the introduction of goodwill in the model only affects the decision of firms of different ages. Since the focus of our paper is on firm behavior across firm size, not modeling the evolution of goodwill over firm age is not a major concern for our purposes.[50]

In the goodwill model, the final good is now defined as:

$$Y_t = \frac{1}{1-\beta} \int_0^1 q_{j,t}^{\beta} (1 + G_{j,t})^{\beta} y_{j,t}^{1-\beta} \, \mathrm{d}j$$

The inverse demand function for good j is given by:

$$p_{j,t} = q_{j,t}^{\beta} (1 + G_{j,t})^{\beta} y_{j,t}^{-\beta}$$

Advertising goodwill ($G_{j,t}$) is thus a demand shifter. Intermediate good producers maximize their profit subject to the inverse demand function and the dynamics of goodwill.

G.2.2 Advertising in Utility

Next, we show that a slight modification of the baseline model which allows advertising to feature directly into the consumers' utility function delivers a very similar allocation, and identical qualitative predictions, as the baseline model. In this extension, there is no final good sector and the household consumes goods $j \in [0, 1]$ directly. The representative household's preferences are now represented by:

$$U = \int_0^{+\infty} e^{-\rho t} \ln\left(C_t\right) \mathrm{d}t$$

[50] Moreover, firm age does not significantly affect advertising intensity and the relative use of R&D and advertising, as seen in Table 1.

where C_t is a consumption aggregator over a mass-one continuum of quality-weighted good quantities, indexed by $j \in [0,1]$, which takes the form:

$$C_t = \frac{1}{1-\beta} \int_0^1 \widetilde{q}_{jt}^{\beta} y_{jt}^{1-\beta} \mathrm{d}j$$

with $\beta \in (0,1)$. The flow budget constraint is, therefore:

$$\dot{A}_t = r_t A_t + w_t - \int_0^1 p_{jt} y_{jt} \mathrm{d}j$$

where $A_0 \geq 0$ is given, and p_{jt} is the price of good j. Each good variety j is produced with technology:

$$y_{jt} = \bar{Q}_t l_{jt}$$

where good $j = 0$ is the numeraire (so $p_{0,t} = 1$, $\forall t$). The optimality condition for good j yields:

$$\omega_t p_{jt} = \frac{1}{C_t} \left(\frac{y_{jt}}{\widetilde{q}_{jt}} \right)^{-\beta}$$

where $\omega_t \geq 0$ is the Lagrange multiplier, solving $-\frac{\dot{\omega}_t}{\omega_t} = r_t - \rho$. Recalling that $\widetilde{q}_{jt} = q_{jt} + \phi_{jt}$, we have that the inverse demand function for goods from households firms is iso-elastic, with β being the price-elasticity. Solving the incumbent firm's problem, one can show easily that $\omega_t = \frac{1}{Y_t}$, where $Y_t \equiv \int_0^1 y_{jt} \mathrm{d}j$ denotes aggregate output in the economy. Hence, the Euler equation reads $g_t = r_t - \rho$ and the demand function becomes identical to that of the baseline model. The two models are therefore qualitatively equivalent.

G.2.3 "Wasteful" Combative Advertising

In our baseline model, advertising not only shifts demand but also has an effect on consumer utility and welfare. Nevertheless, in Appendix C we showed that in the calibrated version of the model advertising is welfare decreasing as the level effect (the increase in how consumers value their consumption) is more than offset by the negative effect of advertising on growth through the substitution between advertising and R&D at the firm level.

Next, we show that the same results can be obtained in a model in which advertising does not increase the value of consumption in equilibrium. This can be seen as a model of *combative* (or *predatory*) advertising in which the advertising efforts of each firm (partially) cancel out in equilibrium. Let us define the final good technology as:

$$Y = \frac{1}{1-\beta} \int_0^1 q_j^{\beta} (1 + d_j - \iota \Phi^*)^{\beta} y_j^{1-\beta} \, \mathrm{d}j$$

with $\iota \in [0,1]$. This implies that the effectiveness of advertising at the good level is a function of the overall level of advertising expenditures in the economy (through Φ^*, i.e. the normalized aggregate extrinsic quality in the economy). Thus, the more other firms invest in advertising, the more one firm has to invest itself in order to obtain a given return to advertising.

We obtain the following demand function:

$$y_j = q_j (1 + d_j - \iota \Phi^*) p_j^{-\frac{1}{\beta}}$$

Intermediate good firms solve their profit maximization problem subject to this demand function and taking the overall level of advertising in the economy as given. From the firm's perspective, advertising acts as a demand shifter in the same way as in our baseline model. Moreover, it is easy to derive Y_t in equilibrium as:

$$Y = \left(\frac{\bar{Q}}{w}\right)^{\frac{1-\beta}{\beta}} (1-\beta)^{\frac{1-2\beta}{\beta}} [1 + (1-\iota)\Phi^*]\bar{Q}$$

If $\iota = 1$, advertising has no direct effect on consumer's utility. It can, nevertheless, have an effect on lifetime utility through its impact on the growth rate of the economy in a way that is similar to our baseline model. If $\iota = 0$, we return to our benchmark model.

G.2.4 Informative Advertising

In our baseline, model advertising is purely persuasive in the sense that it shifts demand toward advertised goods through increased marginal utility. Alternatively, one could consider advertising as providing relevant information about the product quality (see for instance Nelson (1974), Butters (1977), Grossman and Shapiro (1984) or Milgrom and Roberts (1986)). In this case, advertising could be socially optimal as it could reduce uncertainty or improve the quality of consumer-firm match.

In this section, we propose a simple model of informative advertising with differentiated products to illustrate how advertising can act as a demand shifter. We look at a static model in which advertising is used to provide information about the quality of the goods. In particular, firms send an imperfect signal about their product quality through advertising. Consumers passively receive the information and update their prior about product quality.

Consumers maximize expected utility. The utility function is quadratic and given by:

$$U = \int_0^1 q_j y_j \ \mathrm{d}j - \frac{\alpha}{2} \int_0^1 q_j^2 y_j^2 \ \mathrm{d}j$$

Before receiving signals through advertising, consumers have a prior about the quality of each good j. This prior is normally distributed with mean μ_j and variance σ_j^2. Through advertising, firms can send an imperfect public signal (s_j) about their product quality, given by:

$$s_j = q_j^* + \omega_j$$

where q_j^* is the actual quality of the good and ω_j is a Gaussian shock with mean zero and variance σ_ω^2. As in other models of informative advertising (e.g. Erdem *et al.* (2008)), we assume that higher advertising expenditures can decrease the variance of the signal.

It is not difficult to show that the posterior distribution of product quality (after receiving the signal) follows a normal distribution with mean and variance:

$$\begin{aligned} \mu_{post} &= \frac{\mu_j/\sigma_j^2 + s_j/\sigma_\omega^2}{\sigma_j^{-2} + \sigma_\omega^{-2}} \\ \sigma_{post}^2 &= \left(\frac{1}{\sigma_j^2} + \frac{1}{\sigma_\omega^2}\right)^{-1} \end{aligned}$$

respectively. The representative consumer maximizes expected utility after receiving advertising

signals. The demand function for good j can then be written as:

$$y_j = \frac{\mu_{post} - p_j}{\alpha\left(\mu_{post}^2 + \sigma_{post}^2\right)}$$

Note that:

$$\frac{\partial \mu_{post}}{\partial \sigma_\omega^2} = -\left(\frac{\sigma_j^2}{\sigma_j^2 + \sigma_\omega^2}\right)^2 \frac{s_j - \mu_j}{\sigma_j^2} \quad \text{and} \quad \frac{\partial \sigma_{post}^2}{\partial \sigma_\omega^2} = \left(\frac{\sigma_j^2}{\sigma_j^2 + \sigma_\omega^2}\right)^2$$

This means that, by doing advertising (which lowers σ_ω^2), firms can effectively lower the posterior variance of households. Furthermore, if the signal is sufficiently optimistic relative to the household's prior (i.e. $s_j > \mu_j$), increasing advertising expenditure also increases the posterior mean. To see how this affects the final demand for the good, note:

$$\frac{\partial y_j}{\partial \sigma_\omega^2} = -\frac{\partial \sigma_{post}^2}{\partial \sigma_\omega^2}\left(\frac{\frac{s_j - \mu_j}{\sigma_j^2}(p_j + \sigma_{post}^2) + (\mu_{post} - p_j)}{\alpha(\mu_{post}^2 + \sigma_{post}^2)^2}\right)$$

When the signal is optimistic ($s_j > \mu_j$), advertising will unambiguously shift demand, as it helps household reduce the posterior variance and upgrade the posterior mean, both of which boost demand. When the signal is pessimistic ($s_j < \mu_j$), there is a trade-off: increased advertising helps reduce the posterior variance, but it also lowers the posterior mean, so the net effect depends on fundamentals. For example, when the prior is diffuse ($\sigma_j \to +\infty$), the posterior distribution mean and variance satisfy $\mu_{post} \to s_j$ and $\sigma_{post}^2 \to \sigma_\omega^2$, respectively, and advertising expenditures unambiguously shift demand. More generally, for the case of pessimistic signals, there always exists a prior variance that is diffused enough for advertising to shift demand. In these cases, advertising acts as a demand shifter as in our baseline model.

G.2.5 Advertising and the Price-Elasticity of Demand

A strand of the advertising literature has focused on the effect of advertising on the elasticity of demand (see, for instance, Molinari and Turino (2015) and Benhabib and Bisin (2002)). In this section, we present a model in which advertising can change the price-elasticity of demand. We further show that the demand shifting property of advertising is maintained so that the results from our baseline model could be obtained in such a framework as well.

Following Molinari and Turino (2015), we write:

$$Y = \left[\int_0^1 q_j^{\frac{1}{\epsilon}} \left(y_j + D(d_j)\right)^{\frac{\epsilon-1}{\epsilon}} \, \mathrm{d}j\right]^{\frac{\epsilon}{\epsilon-1}}$$

where D is a decreasing function, with $D(0) \geq 0$. The inverse demand function can be written as:

$$p_j = (q_j Y)^{\frac{1}{\epsilon}} \left[y_j + D(d_j)\right]^{-\frac{1}{\epsilon}}$$

Setting $D'(d_j) < 0$ as in Molinari and Turino (2015), we obtain that advertising acts as a positive demand shifter. Furthermore, the price-elasticity of demand is equal to:

$$\left|\frac{\partial y_j / y_j}{\partial p_j / p_j}\right| = \epsilon\left(1 + \frac{D(d_j)}{y_j}\right)$$

Thus, advertising decreases the price-elasticity of demand, *ceteris paribus*. Intuitively, by conducting advertising, firms alter the substitutability between goods, and make them more price-inelastic.

H Description of R&D Tax Credit Measures

This section describes the measures of R&D tax credits which are used in our empirical investigation of the substitution between R&D and advertising in Section 4.3. Besides the statutory credit rate, we use the following measures:

- First, the tax-adjusted credit rate takes into account the fact that the tax credit is itself subject to corporate taxation in some states. We compute the tax-adjusted credit rate for state s at time t as:

$$\text{Tax-adjusted Rate}_{st} = \text{Statutory Credit Rate}_{st} \times (1 - \sigma_{st} \times \text{Tax Rate}_{st})$$

  where σ_{st} is the share of the R&D credit which is subject to corporate taxation. When the credit is taxed, the credit rate is now not only influenced by differences in the statutory credit rate over time and across states, but also by changes in corporate taxes.

- Second, we use an alternative measure of the marginal effective R&D tax credit, proposed by Wilson (2009). This measure is available only until 2006. It acknowledges the different definitions of the R&D expenditures which are eligible for tax credit as well as the horizon over which the tax credit is calculated. In some states, all R&D expenditures can lead to a tax credit, while some other states offer a credit only to R&D expenditures above a certain base level. This threshold, in turn, may be a moving average of past R&D expenditures. For such states, the moving-average base is usually computed as the product of firm sales and the R&D-to-sales ratio over the n previous periods. For a firm with R&D expenditures above the base level, the marginal effective tax credit rate (m_{st}) is computed as:

$$m_{st} = \text{Statutory Credit Rate}_{st} \times (1 - \sigma_{st} \times \tau^e_{st}) \times \left(1 - \frac{1}{n}\sum_{k=1}^{n}(1 + r_{t+k})^{-k}\right)$$

  where r is the real interest rate, n is the number of periods over which the moving-average base is calculated, and τ^e_{st} is the effective marginal tax rate which takes into account the fact that, in some states, taxes paid to the state can be deducted from federal taxable income, and vice versa. The rate also takes into account the fact that R&D tax credit are subject to corporate taxation in some states.

  For states with base definition based on the moving-average of past R&D activity, every dollar spent on R&D today decreases the amount of R&D that qualifies for a tax credit in the future and hence reduces the effective marginal tax credit rate. In some states such as New York and Connecticut, all R&D expenditure qualifies for a tax credit (i.e., there is no moving-average definition of the base level). In these cases, the marginal effective credit rate is equal to the (tax-adjusted) statutory credit rate.

- Finally, we use one additional state-level measure of the cost of R&D: the R&D user cost. This measure was extended from Hall and Jorgenson (1967) to R&D investment by Bloom *et al.*

(2002), and computed at the U.S. state and federal levels by Wilson (2009). In particular, the user cost of R&D in state j is given by:

$$\text{R\&D User Cost}_{st} = \frac{1 - v\left(m_{st} + m_{ft}\right) - z\left(\tau^e_{st} + \tau^e_{ft}\right)}{1 - \left(\tau^e_{st} + \tau^e_{ft}\right)}\left(r_t + \delta\right)$$

where the f subscript stands for *federal*, v is the share of R&D expenditures that qualifies for preferential tax treatment, z is the present discounted value of tax depreciation allowances, and δ is the depreciation rate of R&D capital. Following Wilson (2009), we set $v = 0.5$, $z = 0.525$ and $\delta = 0.15$.