Industry Life Cycles in General Equilibrium*

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Motivation

- Technological breakthroughs are often followed by an industry life cycle.
- In affected industries → Predictable paths for entry, exit and innovation (Klepper, AER, 1996):
  1. **Initial phase:** Burst of entry, firms offering many different versions of the industry's product.
  2. **Middle phase:** Continued market growth but entry slows down, some firms exit.
  3. **Late phase:** Shakeout and consolidation, fewer product innovations, more process innovations.
- Despite this narrative, we do not have a good sense of the macroeconomic implications.
  - Need a model where life-cycles emerge endogenously via product/process innovations.
  - GE allows us to speak to policy → How should industries be regulated after a technological revolution?

What we do:

Provide a **quantitative GE model of industry life cycles**, which we use to...

(i) ... understand welfare implications

(ii) ... study industrial policy in the aftermath of technological disruptions (e.g. ICTs, AI, ...)

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Roadmap

1. Theory: Overlapping Technologies + Oligopolistic Industries + Semi-Endogenous Growth

- Key ingredients:
  1. Arrival of technological revolutions opens up new innovation possibilities within an industry.
     - After the shock, a new generation of firms have to find their product through product innovation.
     - Once they have a product, they improve its efficiency via process innovation.
     - Fixed cost of operation $\rightarrow$ Number of producing firms is endogenous and time-varying.
  2. Pricing and product/process innovation decisions feature strategic interactions.
- Equilibrium: Shakeouts emerge when product innovation ("entry") costs $\ll$ process innovation costs.
  - Low entry costs generate burst of entry after the technological revolution.
  - Once some entrants are successful at process innovation, they push others out of the industry.
  - If process innovation costs are high enough, there's time for firm numbers to grow before the shakeout.

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2. Empirics: Aim to provide systematic evidence (complementing existing case studies)

- Interpret the ICT Revolution (mid-1980s) as a shock that generated new technological opportunities.
  - Compare dynamics (entry, exit, patenting) across industries with different exposure to the shock.
- Results: More exposed industries experience burst of entry, subsequent shakeout, process innovation $\uparrow$

3. Quantitative Analysis: Study optimal policy following a technological revolution

- Calibration:
  - Hit economy w/ shock that simultaneously disrupts multiple industries ("technological revolution").
  - Calibrate parameters so that transitional dynamics after the shock match those of the ICT shock.
- Policy analysis: *[Very preliminary]*
  - Along the BGP, entry subsidies are welfare improving (entry externalities $\gg$ business-stealing).
  - In response to a technological revolution, the optimal policy reaction is to lower these subsidies.
  - **Work in progress:** optimal timing, duration, policy mix, ..., following a technological revolution.

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Related Literature

1. **Empirical literature**

   Gort and Klepper (1982), Klepper (1997), Klepper and Simons (2000), Dinlersoz and MacDonald (2009), Bos et al. (2013).

   Contribution: We have a broader focus and use the ICT Revolution for identification.

2. **Theoretical literature**

   Abernathy and Utterback (1978), Jovanovic and MacDonald (1994), Klepper (1996), Jovanovic and Tse (2010) and Beraja and Buera (2024).

   Contribution: Partial-Equilibrium models, we propose a model in GE so we can look at optimal policy.

Theory

A General-Equilibrium Model of Industry Life-Cycles

Environment

- **Preferences:** Over consumption of a single final good:

$$\max \int_0^{+\infty} e^{-\rho t} \ln(C_t)\, \mathrm{d}t \qquad \text{s.t. } \dot{A}_t \leq r_t A_t + w_t L - C_t$$

- **Final good:** Continuum of industries, and each industry is populated by $\overline{N} < +\infty$ firms:

$$Y_t = \exp\left(\int_0^1 \ln\left(Y_{i,t}\right) \mathrm{d}i\right), \qquad \text{where } Y_{i,t} = \left(\sum_{n=1}^{\overline{N}} \left(y_{in,t}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}, \ \varepsilon > 1$$

- **Technology:**
  - A firm $n = 1, \ldots, \overline{N}$ may not own a product (aka "potential entrant"), in which case $y_{in,t} = 0$.
  - If a firm owns a product (aka "incumbent"), it may produce it with productivity $q_{in,t}$ using labor $l_{in,t}$:

$$y_{in,t} = q_{in,t} l_{in,t}.$$

  - To produce, an incumbent must pay a flow operating cost ϕY_t $\rightarrow$ Endogenous number of incumbents.

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Overlapping Technologies and Breakthroughs

- In any industry, at any point in time, there are two available technologies: **Traditional** (T) and **Modern** (M).
  - $\overline{N}/2$ firms are "traditional" (i.e. use only the T technology), and $\overline{N}/2$ firms are "modern".
- An industry experiences a **"technological breakthrough"** at an (exogenous) Poisson rate $a > 0$.
- With a breakthrough ...
  - ... the T technology becomes obsolete, and all of its users disappear with it.
  - ... the old M technology becomes the new T technology.
  - ... $\overline{N}/2$ firms are born who can use a new M technology $\rightarrow$ 1 as incumbent, $\overline{N}/2 - 1$ as pot. entrants.
- Each cohort of firms is characterized by a **finite productivity ladder**:
  - For the cohort born with the k-th revolution, the ladder is:

$$\mathcal{Q}^k = \{0, q_1^k, q_2^k, \ldots, q_{j_{max}}^k\}, \quad \text{with } q_j^k = \lambda^{j-1} q_1^k, \ \lambda > 1$$

  - The M technology is superior to the T technology:

$$\mathcal{Q}^{k+1} = \gamma \mathcal{Q}^k, \quad \text{with } \gamma > 1$$

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- **Product innovation** (by potential entrants):
  - To obtain a product ("entry" event) at rate $x > 0$, firm pays cost:

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  - Upon entering, firm can produce the product with productivity at step 1 of its cohort's ladder, q_1^k.

- **Process innovation** (by incumbent firms):
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  - An innovation allows productivity to advance by one step, i.e. increases q_j^k to $q_{j+1}^k = \min\{\lambda q_j^k, q_{j_{\max}}^k\}$.
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The Life-Cycle of an Industry: Illustration

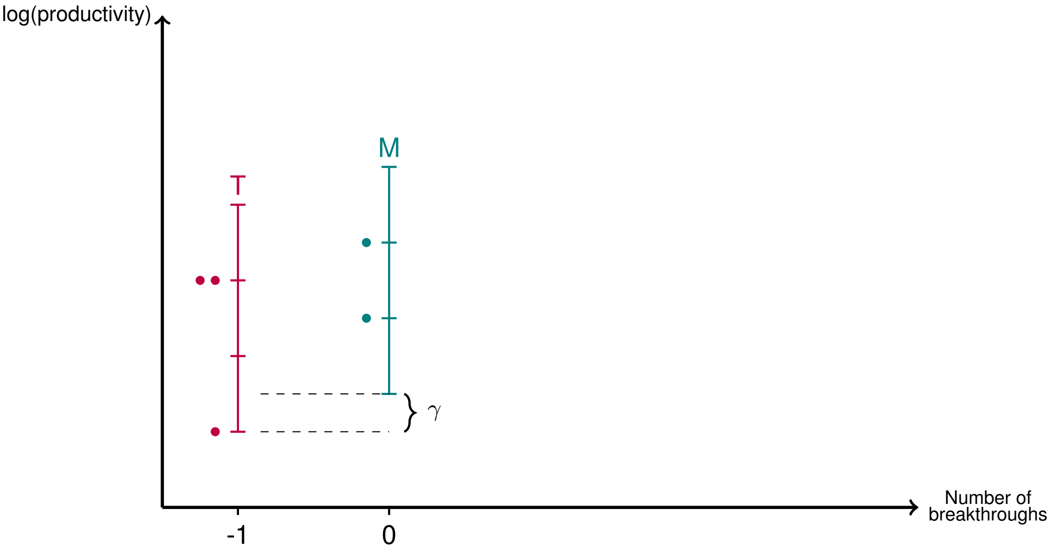

Figure: Overlapping generations of technologies within an industry as a function of the number of industry-level technological breakthroughs. *Note:* the ladders themselves need not overlap in the productivity space.

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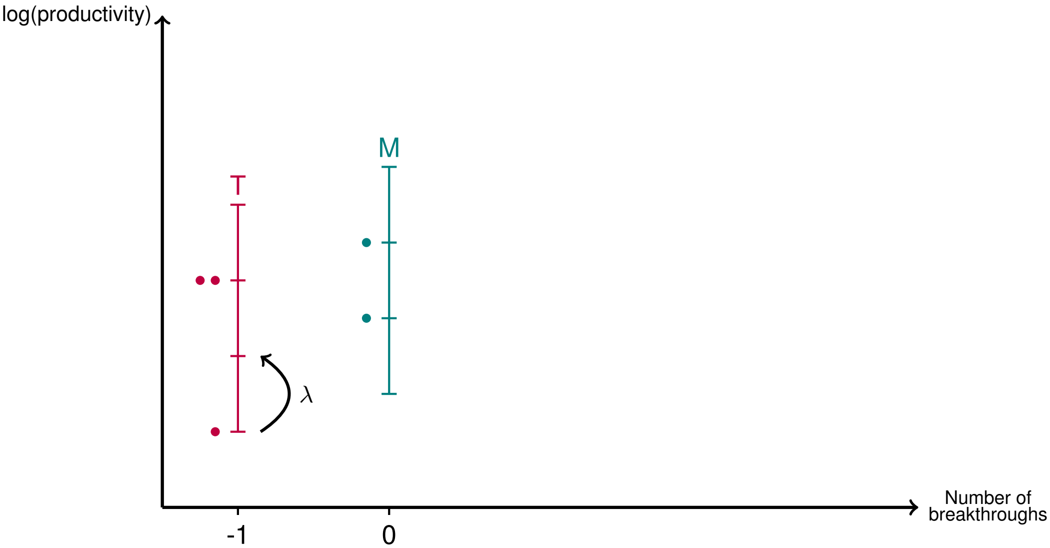

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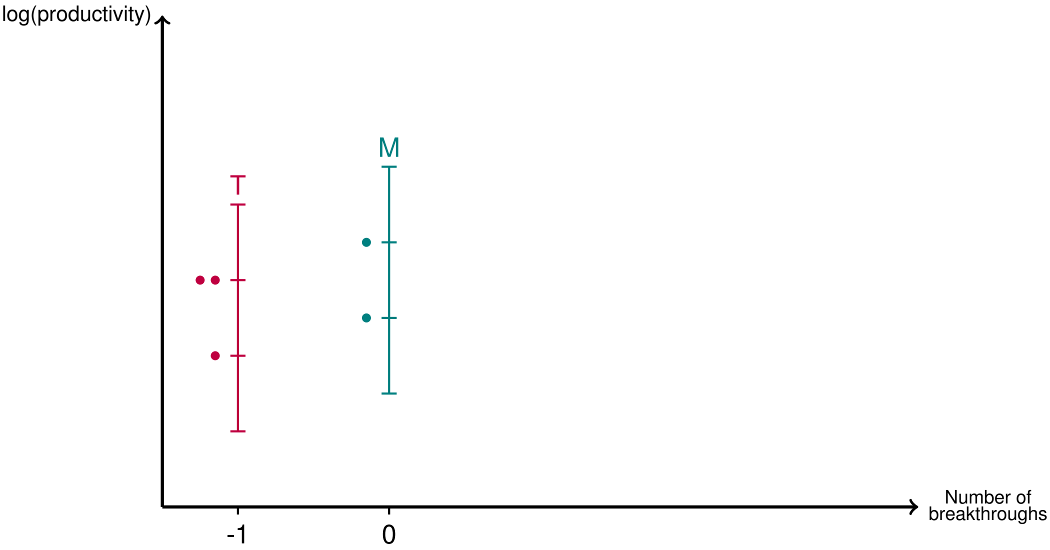

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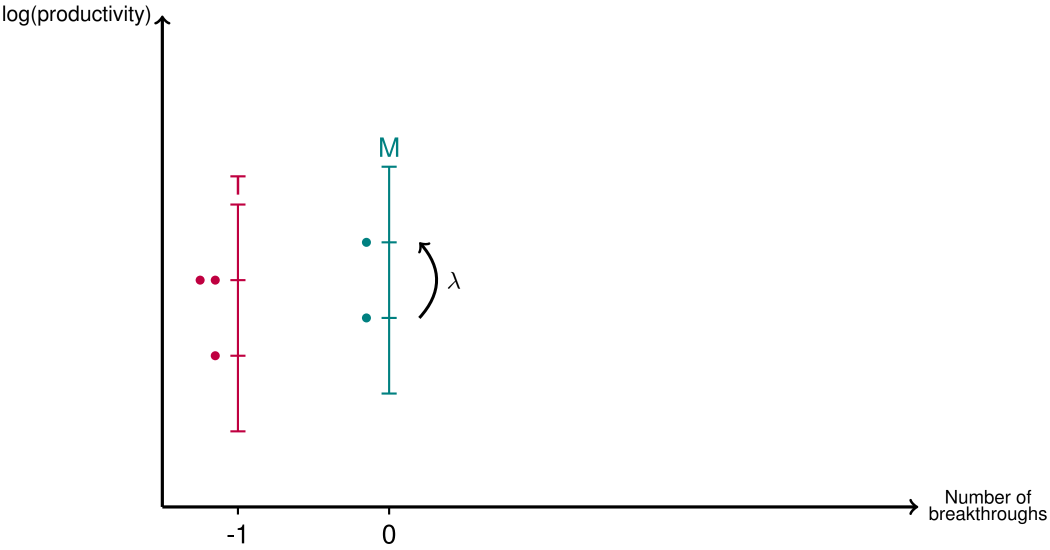

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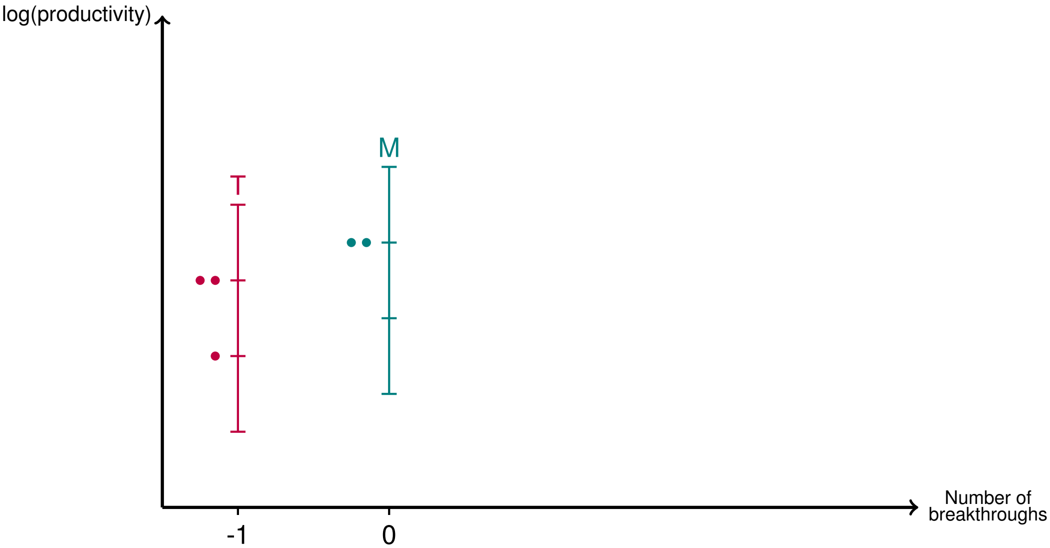

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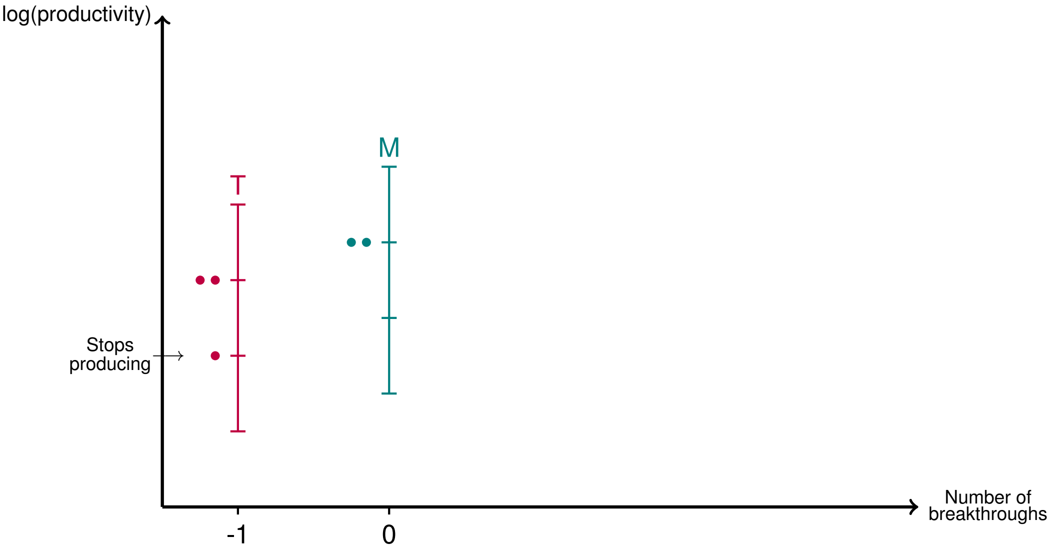

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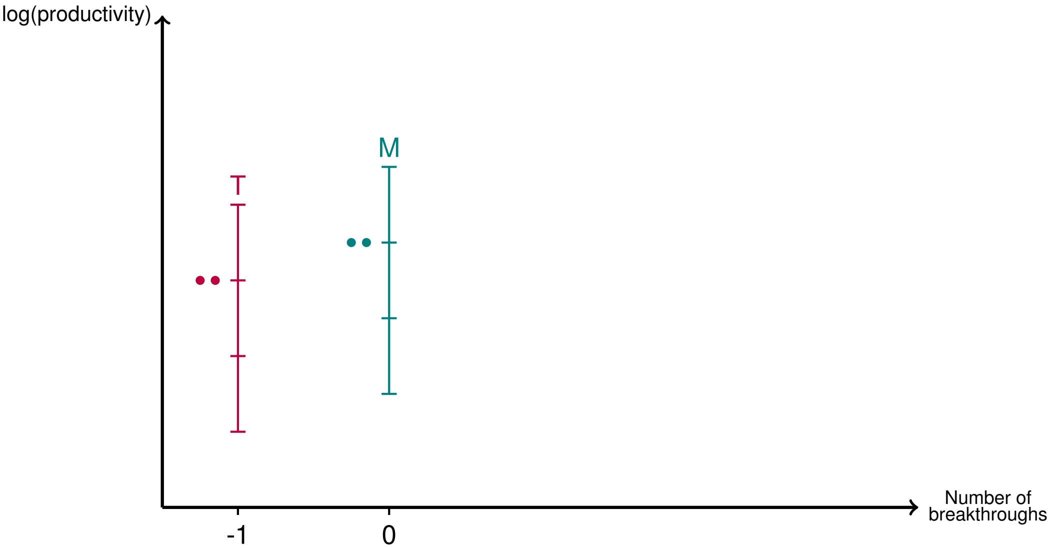

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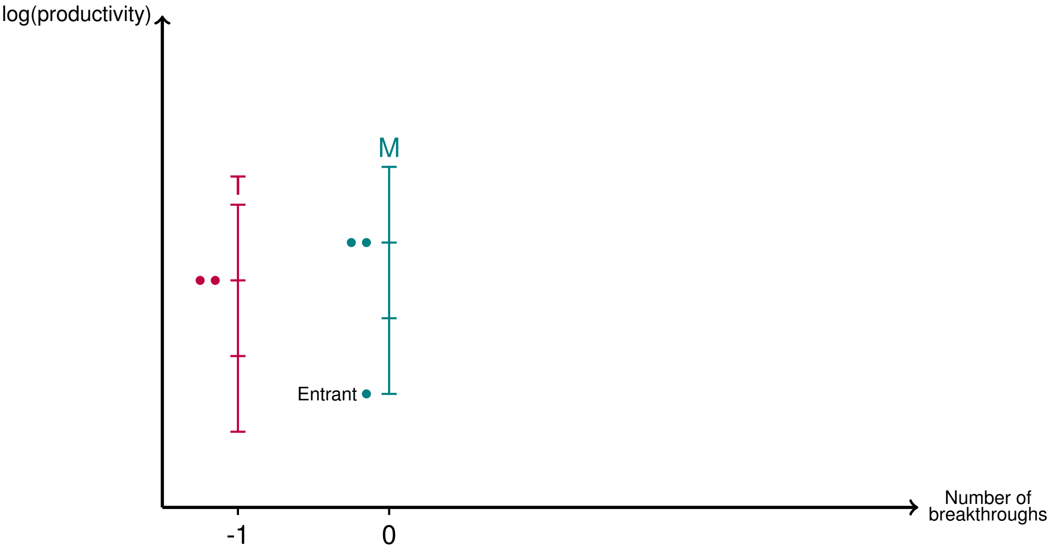

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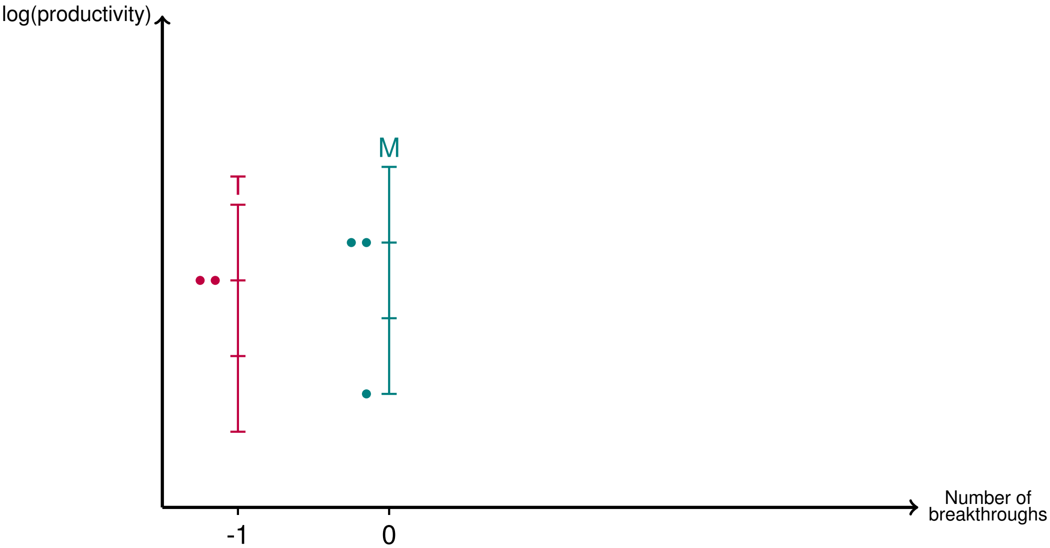

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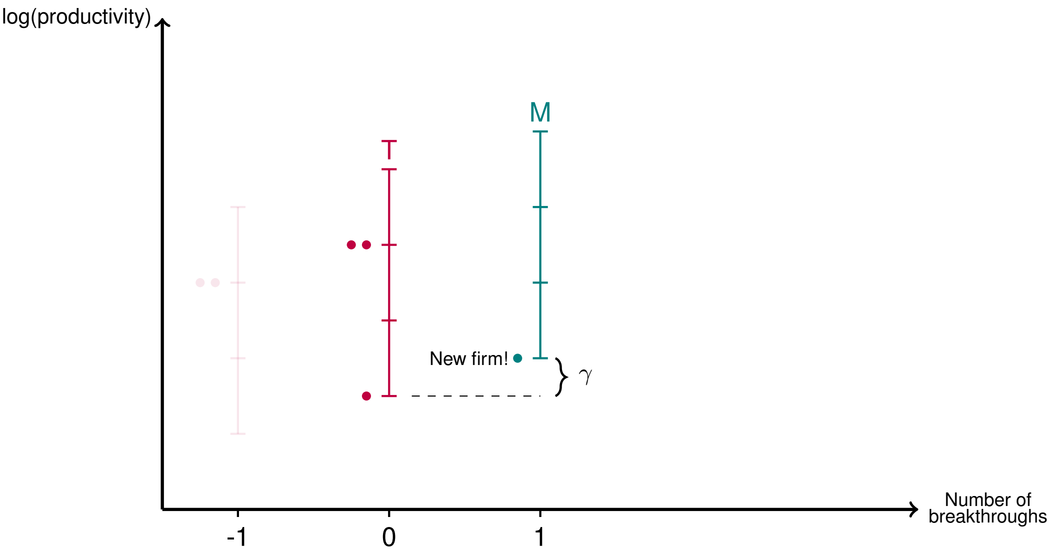

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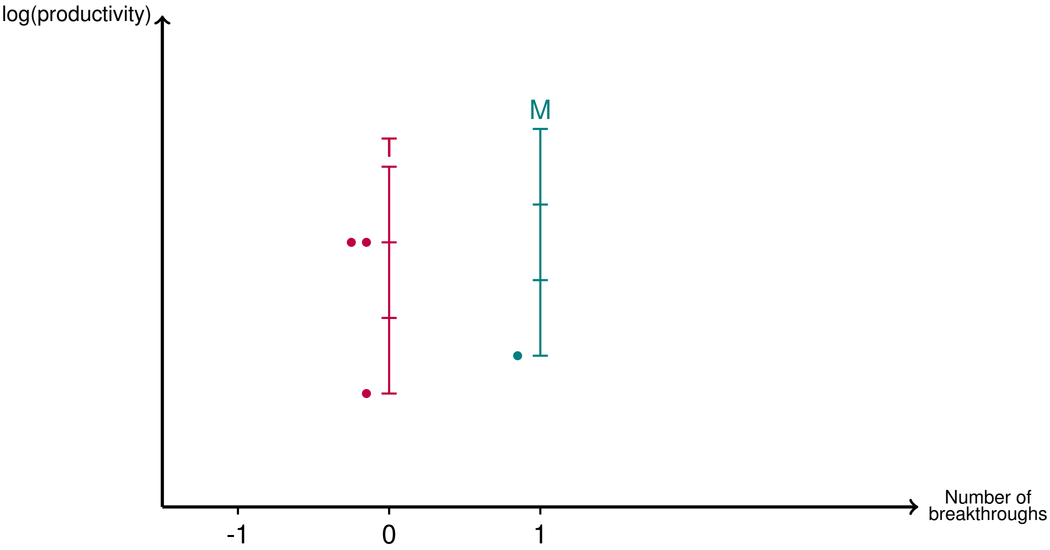

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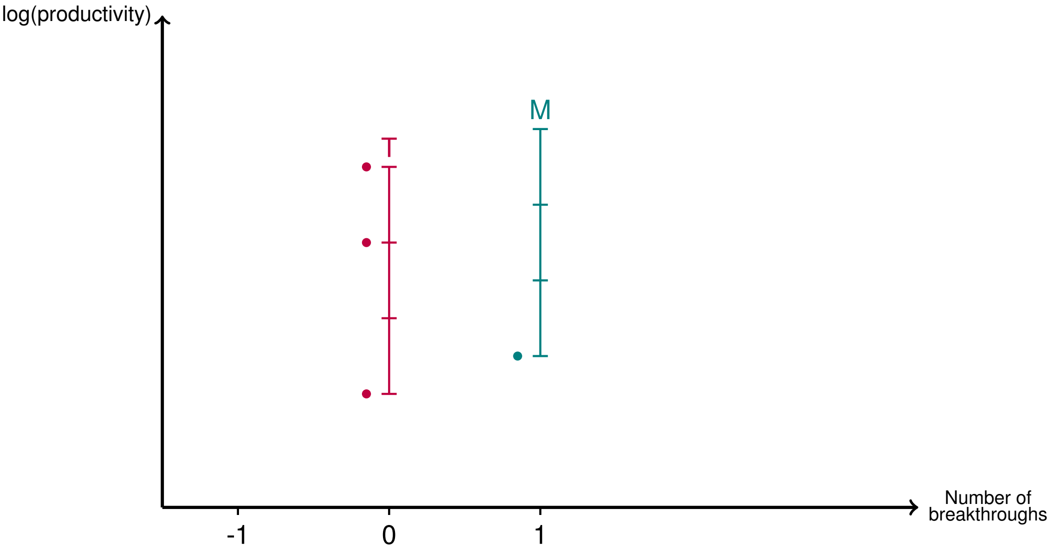

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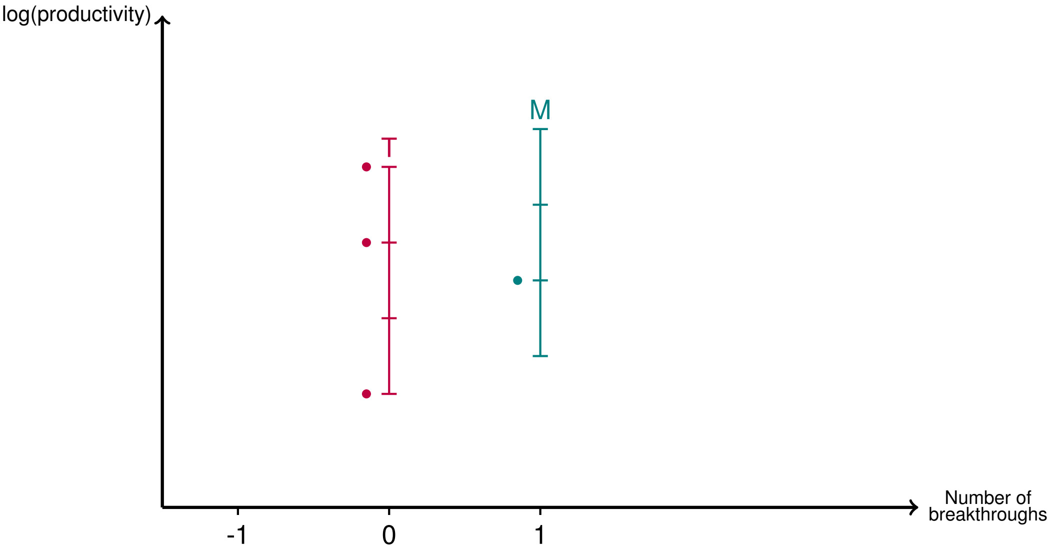

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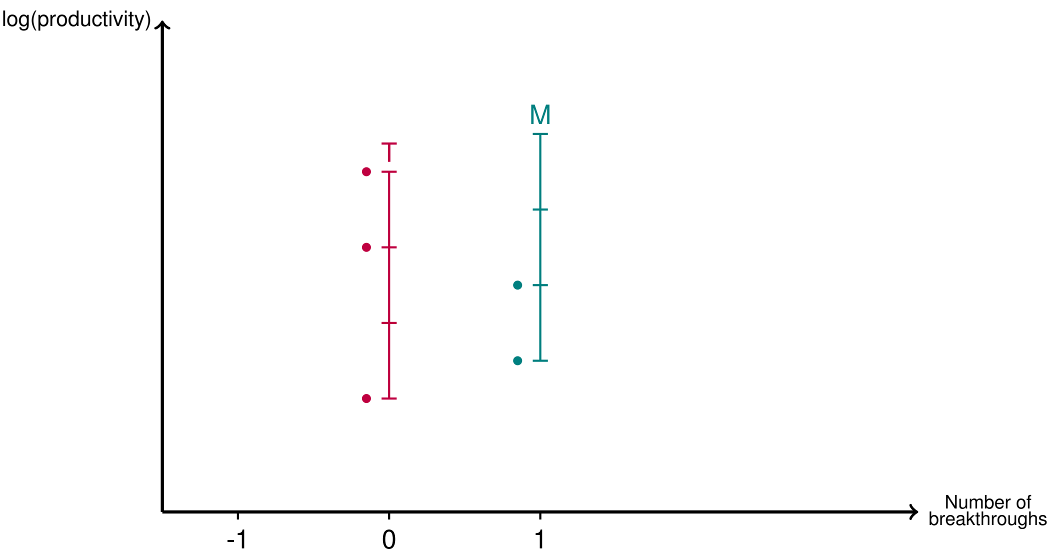

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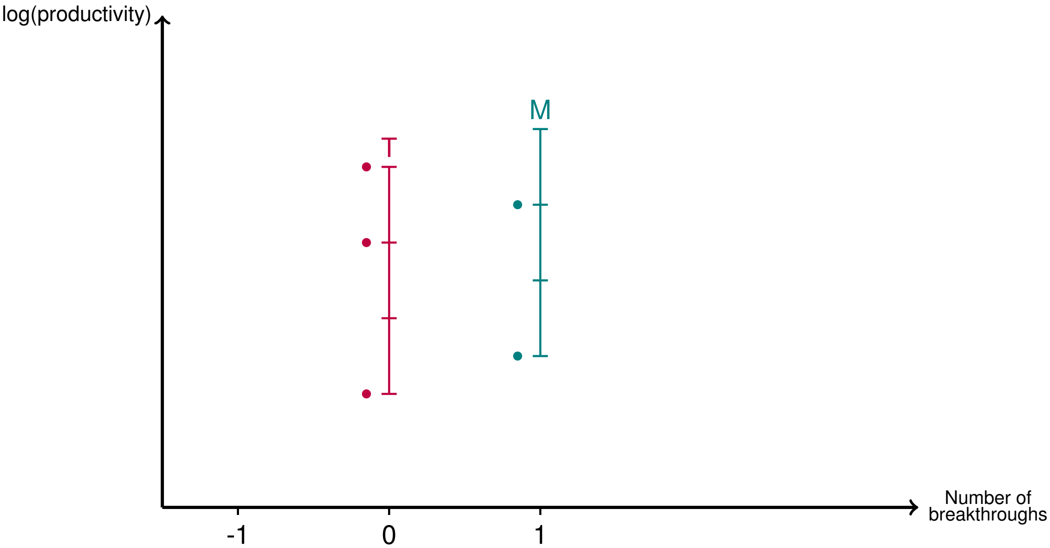

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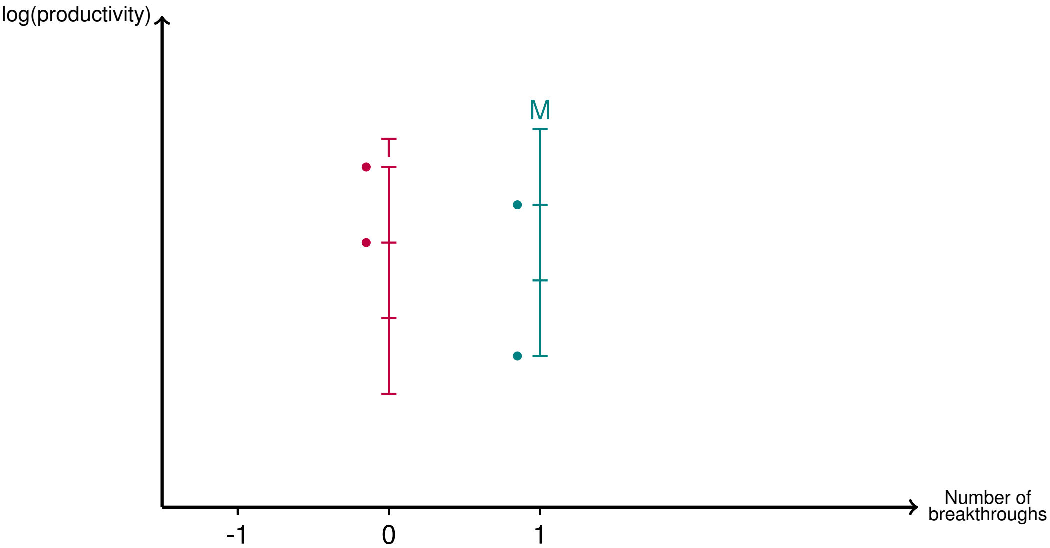

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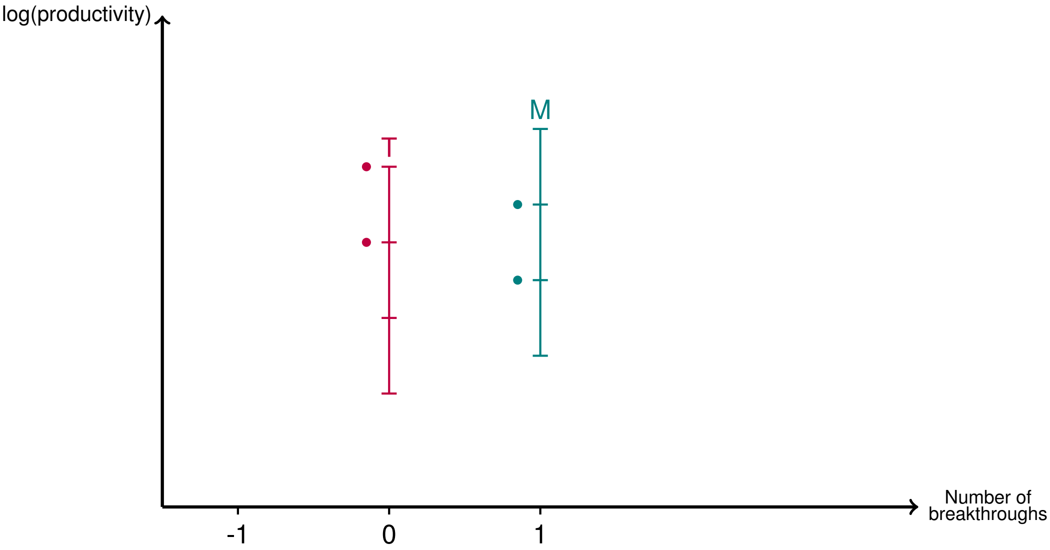

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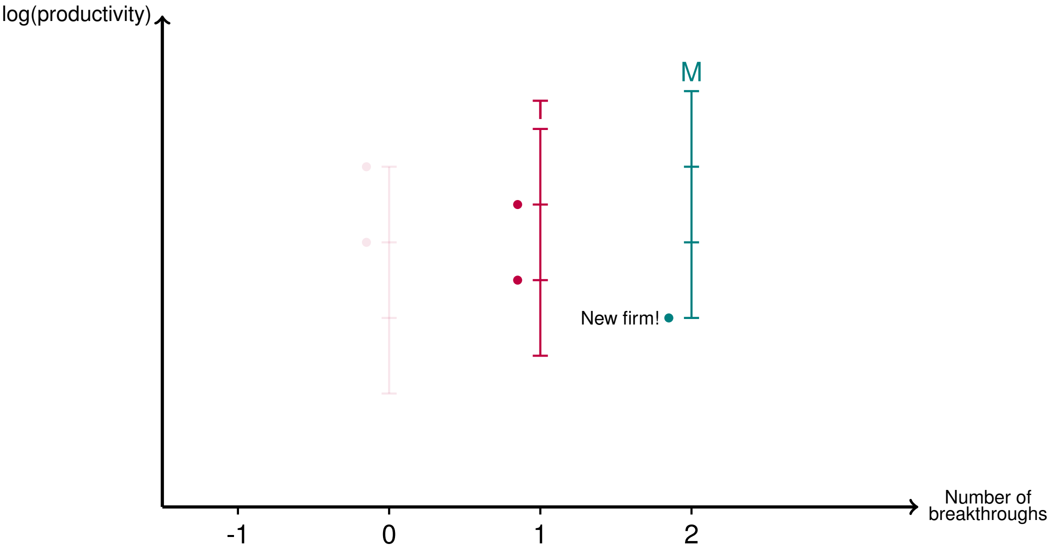

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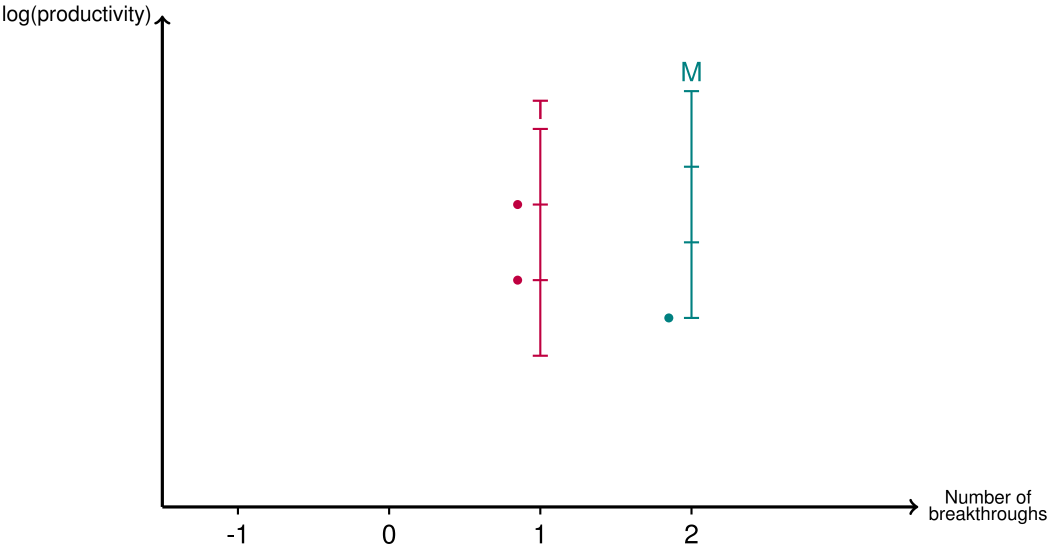

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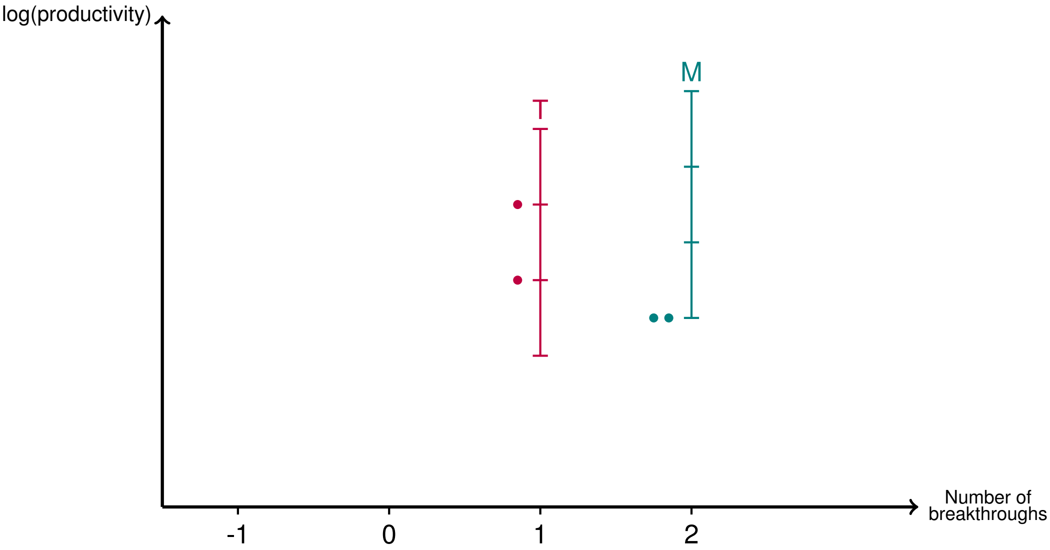

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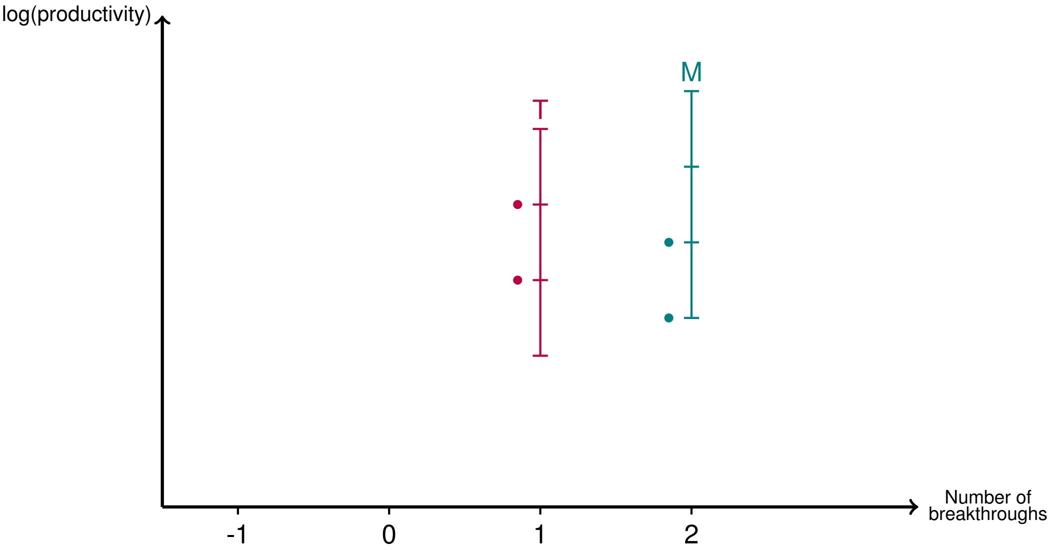

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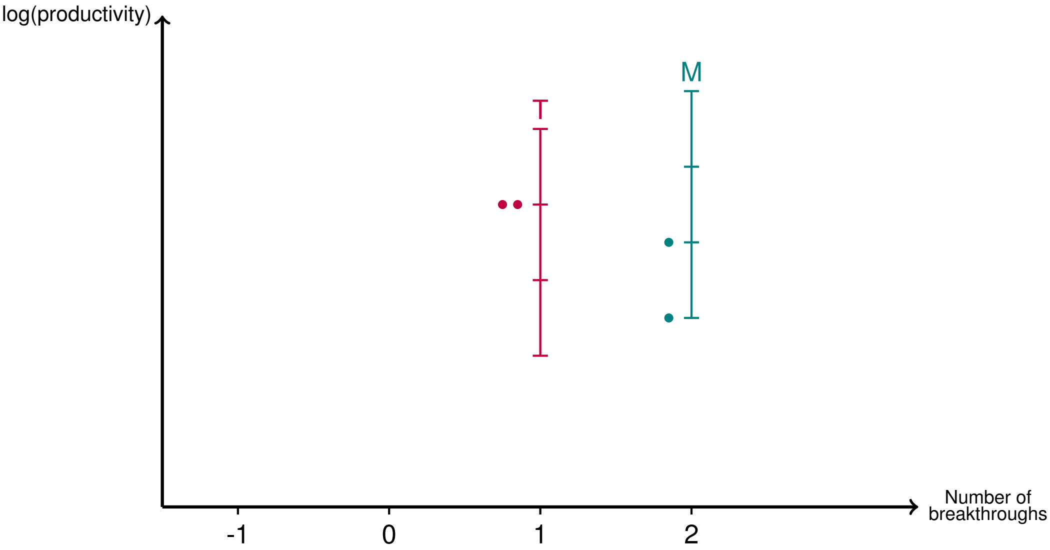

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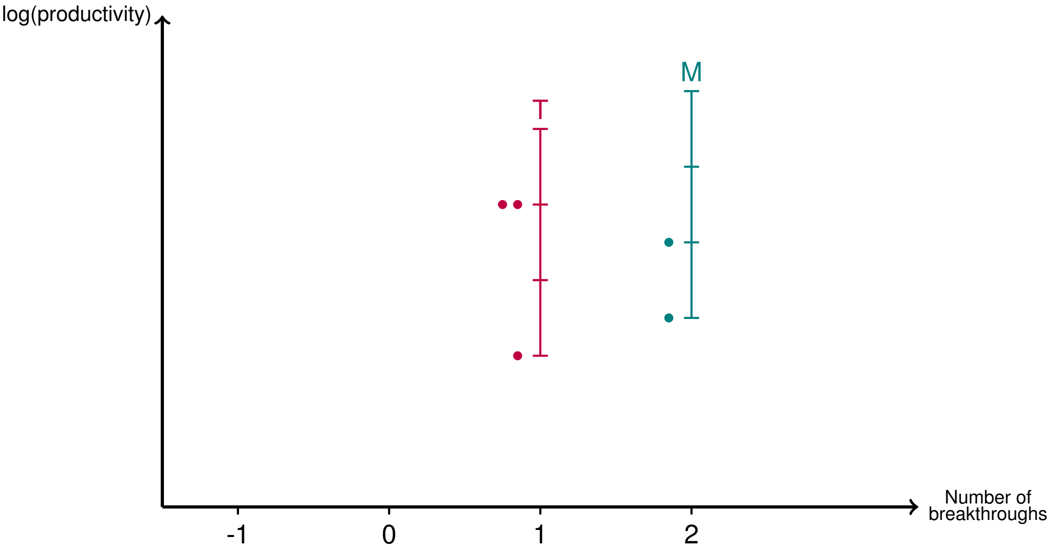

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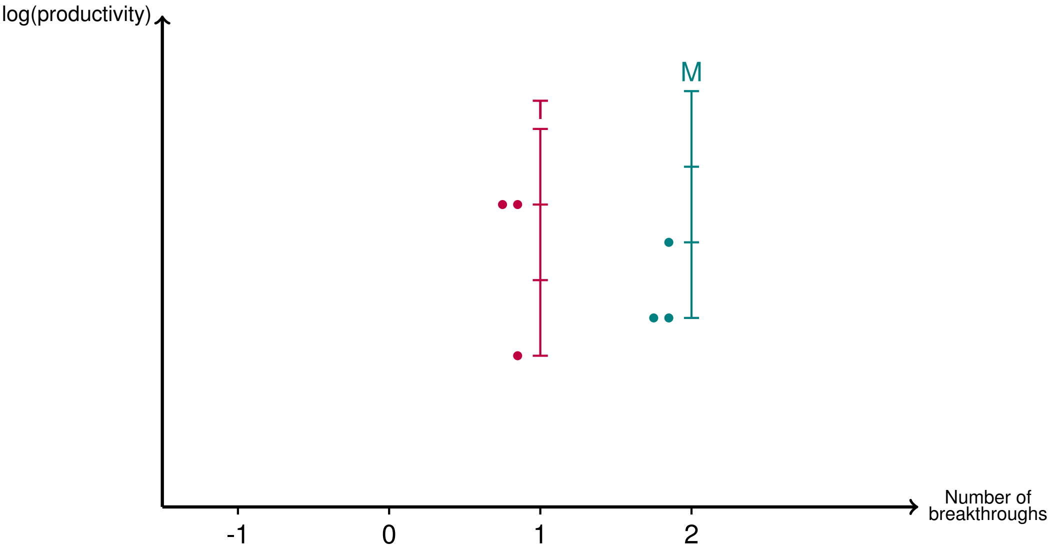

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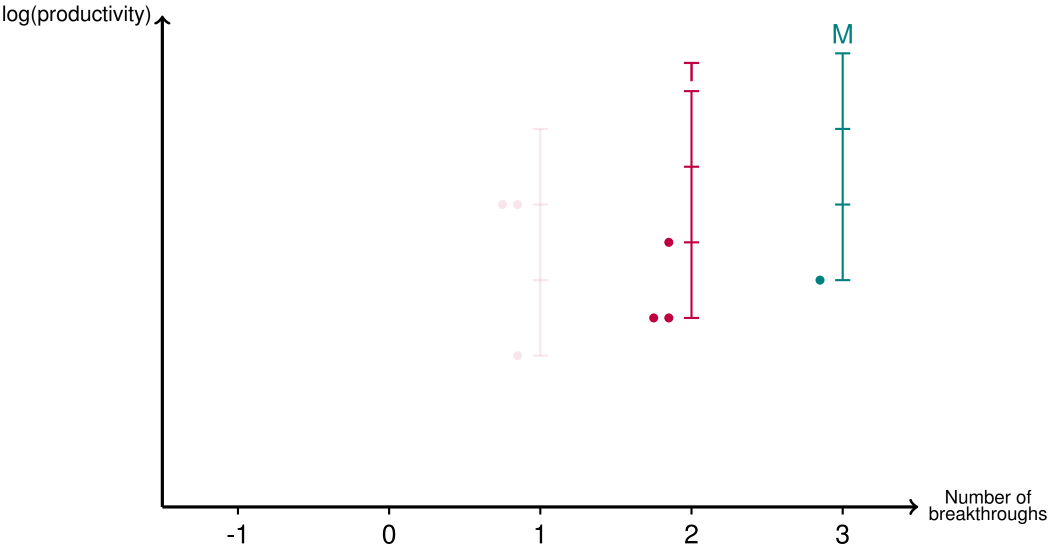

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Static Equilibrium Conditions

- Let $\mathcal{N}_{i,t} \subseteq \{1, \ldots, \overline{N}\}$ be the endogenous set of producing firms ("incumbents") in industry i at time t.

- **Markups:** Conditional on producing, firm $n \in \mathcal{N}_{i,t}$ sets a markup:

$$m_{in,t} = 1 + \frac{1}{(\varepsilon - 1)(1 - \sigma_{in,t})}, \quad \text{where} \quad \sigma_{in,t} \equiv \frac{p_{in,t} y_{in,t}}{P_{i,t} Y_{i,t}} = \left(\frac{q_{in,t}/Q_{i,t}}{m_{in,t}/M_{i,t}} \right)^{\varepsilon - 1}$$

where $M_{i,t} \equiv \left(\sum_{n \in \mathcal{N}_{i,t}} (m_{in,t})^{1-\varepsilon} \left(\frac{q_{in,t}}{Q_{i,t}} \right)^{\varepsilon - 1} \right)^{\frac{1}{1-\varepsilon}}$ and $Q_{i,t} \equiv \left(\sum_{n \in \mathcal{N}_{i,t}} (q_{in,t})^{\varepsilon - 1} \right)^{\frac{1}{\varepsilon - 1}}$.

- **Profits:** Increasing and concave in (i) firm's markup $m_{in,t}$, and (ii) relative productivity, $q_{in,t}/Q_{i,t}$. ▸ Example

$$\pi_{in,t} = \left[\left(1 - \frac{1}{m_{in,t}} \right) \sigma_{in,t} - \phi \right] Y_t$$

  - Firm needs high enough $\sigma_{in,t}$ to find it profitable to produce!

Static Equilibrium Conditions

- We have a system of $N_{i,t} \equiv |\mathcal{N}_{i,t}|$ non-linear equations in $N_{i,t}$ unknowns $\rightarrow$ Easily solved numerically.
- How to pin down N_{it}? $\rightarrow$ Multiple equilibria problem.
  - To select an equilibrium, use same criterion as Atkeson and Burstein (2008).
  - Namely, firms choose whether or not to produce by descending order of productivity, so that ...
    - ... the lowest-q producing firm makes a profit (i.e. covers the fixed cost $\phi > 0$), and
    - ... the highest-q non-producing firm would make a loss if it produced.
- **Dynamic part:** Nash equilibrium in product and process innovation policies (*next*).

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Dynamic Equilibrium Conditions

- There are three state variables for the firm:
  1. Own technology vintage $\tau = 1, 2$ ($\tau = 1$ means "T", $\tau = 2$ means "M")
  2. Own productivity step on the τ-vintage ladder, $j \in \{0, 1, 2, \ldots, j_{\max}\}$.
  3. Industry state $\mathbf{N}$: matrix giving # firms w/ some technology vintage (*rows*) and productivity step (*cols*).

- Firm chooses innovation x taking the innovation of others, $\left\{\widetilde{x}(\tau', j', \mathbf{N})\right\}$, as given.

- Firm's value: (where $\chi_0 \equiv \chi^E$, and $\chi_j \equiv \chi^I$ for $j \geq 1$)

$$\rho V\left(\tau, j, \mathbf{N}\right) = \max_x \left\{ \right\}$$

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\rho V\Big(\tau, j, \mathbf{N}\Big) = \max_x \Bigg\{ \pi(\tau, j, \mathbf{N}) - \chi_j x^{\psi} + x\Big[V\Big(\tau, \min(j+1, j_{\max}), \mathbf{N} - \mathbf{I}_{\tau,j} + \mathbf{I}_{\tau,\min(j+1,j_{\max})}\Big) - V\Big(\tau, j, \mathbf{N}\Big)\Big] + \sum_{c=1}^{\overline{N}-1} \widetilde{x}\Big(\tau'_c, j'_c, \mathbf{N}\Big)\Big[V\Big(\tau, j, \mathbf{N} - \mathbf{I}_{\tau'_c, j'_c} + \mathbf{I}_{\tau'_c, \min(j'_c+1, j_{\max})}\Big) - V\Big(\tau, j, \mathbf{N}\Big)\Big] + a\Big[\mathbf{1}_{[\tau=2]} V\Big(1, j, \begin{bmatrix} \mathbf{N}_{2,1} & \mathbf{N}_{2,2} & \mathbf{N}_{2,3} & \cdots & \mathbf{N}_{2,j_{\max}+1} \\ \overline{N}/2 - 1 & 1 & 0 & \cdots & 0 \end{bmatrix}\Big) - V\Big(\tau, j, \mathbf{N}\Big)\Big] \Bigg\}
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Optimal R&D Choices

- **Innovation choice:** for potential entrants ($j = 0$) and incumbents ($j = 1, 2, \ldots, j_{\max}$):

$$x(\tau, j, \mathbf{N}) = \left(\frac{V\left(\tau, j, \mathbf{N} - \mathbf{I}_{\tau,j} + \mathbf{I}_{\tau,\min(j+1, j_{\max})}\right) - V\left(\tau, j, \mathbf{N}\right)}{\chi_j \psi} \right)^{\frac{1}{\psi - 1}}.$$

  - In equilibrium, early innovators discourage further investment by later ones. ▸ Example

- We solve for a **symmetric Nash equilibrium** in innovation choices:
  - Each firm takes other firms' R&D choices, $\left\{\widetilde{x}(\tau', j', \mathbf{N})\right\}$, as given, and offers a best response.
  - By symmetry, in equilibrium:

$$\widetilde{x}\left(\tau', j', \mathbf{N}\right) = x\left(\tau', j', \mathbf{N}\right)$$

for all firm states $(\tau', j', \mathbf{N})$.

Aggregates

- **Aggregate productivity:** (where $h_t(s) \equiv$ Share of industries in state $s \in \{1, \ldots, \overline{S}\}$ at time t)

$$Q_t \equiv \exp\left(\sum_{s=1}^{\overline{S}} h_t(s) \ln(Q_t(s))\right), \qquad \text{where} \quad Q(s) \equiv \left(\sum_{n \in \mathcal{N}(s)} (q_n(s))^{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}}$$

- **Aggregate markup:**

$$\mathcal{M}_t \equiv \left[\sum_{s=1}^{\overline{S}} h_t(s) \left(\sum_{n \in \mathcal{N}_t(s)} \sigma_n(s) (m_n(s))^{-1}\right)\right]^{-1}$$

- **Aggregate output:**

$$Y_t = \underbrace{\mathcal{M}_t M_t^{-1}}_{\substack{\text{Misallocation} \\ \text{term } \leq 1}} Q_t L, \qquad \text{where} \quad M_t \equiv \exp\left(\sum_{s=1}^{\overline{S}} h_t(s) \ln(M_t(s))\right)$$

  - Average markup M_t drives a wedge between wages and productivity $\rightarrow \frac{w_t}{Q_t} = M_t^{-1}$.
  - Aggregate markup $\mathcal{M}_t$ equals the inverse of the labor share $\rightarrow \frac{w_t L}{Y_t} = \mathcal{M}_t^{-1}$

Transitional and Long-Run Growth

- **Aggregate productivity:** Can be decomposed into

$$Q_t = Q_t^{\text{Ladder}} \times \widehat{Q}_t$$

where

$$\underbrace{Q_t^{\text{Ladder}} \equiv \exp\left(\sum_{s=1}^{\bar{S}} h_t(s) \ln\left(q_t^{\max}(s)\right)\right)}_{\text{Highest attainable productivity, } q^{\max} \equiv \max_{k=T,M} \mathcal{Q}^k} \quad \text{and} \quad \underbrace{\widehat{Q}_t \equiv \exp\left(\sum_{s=1}^{\bar{S}} h_t(s) \ln\left(\sum_{n \in \mathcal{N}_t(s)} \left(\frac{q_{n,t}}{q_{n,t}^{\max}}(s)\right)^{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}}\right)}_{\text{Firms catching up to the top}}$$

- Sources of growth:
  - **Along the transition:** Both Q_t^{Ladder} (via breakthroughs) and $\widehat{Q}_t$ (via process innovation) grow.
  - **On the BGP:** $\dot{\widehat{Q}}_t / \widehat{Q}_t = 0$, so agg. growth is only due to frequency and magnitude of breakthroughs.

$$\frac{\dot{Y}_t}{Y_t} = \frac{\dot{Q}_t^{\text{Ladder}}}{Q_t^{\text{Ladder}}} = a \ln(\gamma)$$

Empirics

Life-Cycle Patterns After the ICT Revolution

Data

- Data sources:
  1. **US Census Business Dynamics Statistics** (BDS, 1978-2019).
     - Number of firms, entry and exit rates for NAICS 4-digit industries.
  2. **USPTO PatentsView** (1976-2019):
     - Universe of patents ultimately granted by the USPTO.
     - Set of listed Cooperative Patent Classification (CPC) technology classes (CPC classes classify patents by the technological component of the invention).
- Merging:
  - We match CPCs to 4-digit NAICS industries, using crosswalk by Lybbert and Zolas (2014).

Empirical Strategy

- **Empirical question:**
  - How do industry dynamics respond to changes in technological possibilities?
  - Endogeneity issues $\rightarrow$ Firm entry/exit/innovation are endogenous to other industry-level shifters.
- **Premises:**
  1. ICT Revolution generated heterogeneous technological opportunities across industries.
  2. Extent of these effects can be predicted by ex-ante industry characteristics (ex-ante exposure).
  3. ICT shock was largely exogenous to other idiosyncratic shifters that differently affected industries.
- **Empirical strategy:**
  1. Restrict sample to non-ICT industries only (240 4-digit NAICS industries).
     - We identify a NAICS industry as ICT using Goldschlag and Miranda (2020)'s definition. Set of ICT NAICS codes
  2. Build exposure index $\rightarrow$ Share of patents in non-ICT industries that are ICT-related in 1975-1979.
     - We identify CPC technology classes as ICT-related from list in Braguinsky et al. (2023)

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Share of ICT-Related Patents in Non-ICT Industries

- Expansion of ICT patenting mainly due to shift in patenting towards more ICT-intensive sectors.

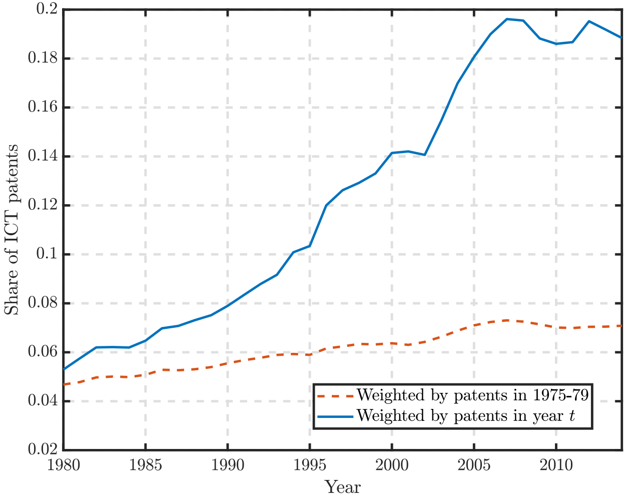

Figure: Share of patents in non-ICT industries belonging to ICT-related technology classes. The blue line weights industries by their number of patents in the current period, while the dotted orange line weights industries by their number of patents in 1975-79.

Empirical Specification

- How did the arrival of technological opportunities affect entry/exit/innovation patterns?
- Estimate a series of local projection models:

$$y_{i,t} = \alpha_{s(i),t} + \beta_t \text{ High ICT-exposure}_{i,1975-79} + \gamma_t \boldsymbol{X}_{i,1975-79} + \epsilon_{i,t}$$

where

1. $y_{i,t}$ $\rightarrow$ Outcomes for industry i in year t.
2. $\text{High ICT-exposure}_{i,1975-79}$ $\rightarrow$ Equals 1 if share of ICT patenting in initial period (1975-79) $\geq$ 10%.
3. $\boldsymbol{X}_{i,1975-79}$ $\rightarrow$ Initial number of patents (logs).
4. $\alpha_{s(i),t}$ $\rightarrow$ 2-digit sector-time fixed effect (captures sectoral trends).

- Coefficients $\{\beta_t\}_{t=1980}^{2015}$ capture effect of the ICT shock by industry's exposure to the shock.

Results: Total Industry Patenting

→ More exposed industries experience a prolonged increase in total industry patenting

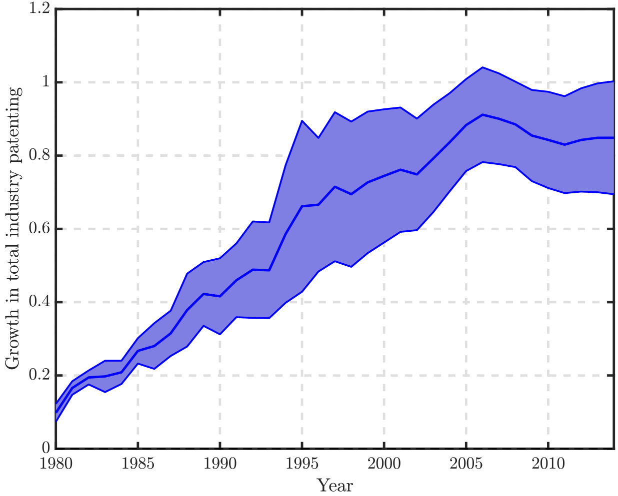

Figure: Estimates of β_t when outcome variable is log-industry patenting in year t. The error bands denote 95% confidence intervals. All regressions weighted by the number of patents in 1975-79.

Results: Net Entry

→ More exposed industries experience an increase in net entry, followed by a shakeout

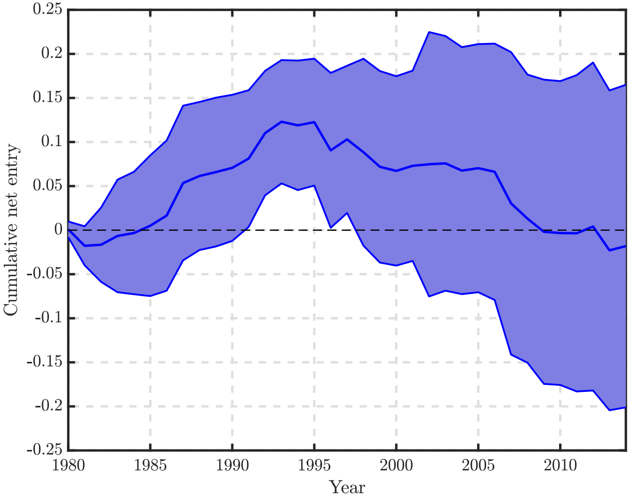

Figure: Estimates of β_t when outcome variable is cumulative net-entry (i.e. the growth rate in the total number of firms) between 1979 and year t. The error bands denote 95% confidence intervals. All regressions weighted by the number of patents in 1975-79.

Results: Process Innovation

→ More exposed industries shift from product to process innovation over time.

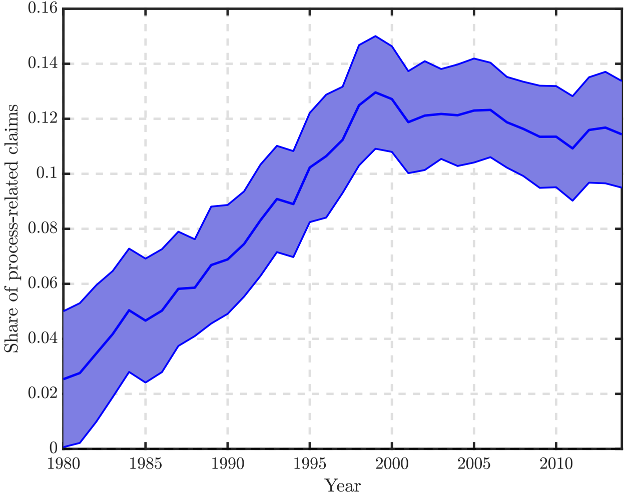

Figure: Estimates of β_t when outcome variable is the share of process-related claims (Bena and Simintzi 2023) in patents in year t. The error bands denote 95% confidence intervals. All regressions weighted by the number of patents in 1975-79.

Calibration

Bringing the Model to the Data

Calibration Strategy

- Shock the model with a **"technological revolution"**:
  - An unforseen a-shock that simultaneously disrupts 21.9% of industries (= exposed to ICT shock).
- Solve for the full transitional dynamics, and calibrate to match industry life-cycle seen in the data.
- **Result:** Low entry costs (initial burst of entry) and process inn. costs high (no rebounds after shakeout).

Param.	Value	Description	Source/Target	Model	Data
Externally calibrated					
a	1/30	Frequency of breakthroughs			
ρ	0.03	Discount rate	4% annual interest rate		
ψ	2	Innovation cost curvature	Akcigit and Kerr (2018)		
Internally calibrated					
ε	8.475	EoS within industries	Average markup	24.73%	25%
γ	1.350	Distance between ladders	Growth rate	1.00%	1.00%
χ^I	15.162	Process innovation cost	R&D share	4.73%	4.70%
χ^E	1.073	Product innovation cost	**Cum net entry:** Years to peak	10 yrs	10 yrs
ϕ	0.026	Fixed cost of production	**Cum net entry:** Top point	11.14%	12.14%
λ	1.179	Distance between rungs	**Cum net entry:** End point	0.35%	0.82%

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Transitional Dynamics after the Revolution

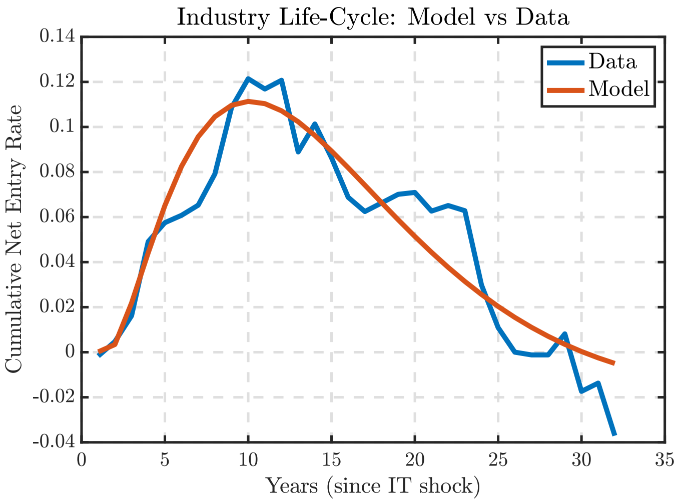

Figure: Cumulative net-entry (i.e. the growth rate in the total number of firms) in disrupted industries relative to non-disrupted industries, between year 0 (time of the shock) and year *t*. The red line is obtained from the model's transitional dynamics in response to an unforeseen technological breakthrough that simultaneously disrupts 21.9% of industries. The blue line reproduces the results from our empirical regressions.

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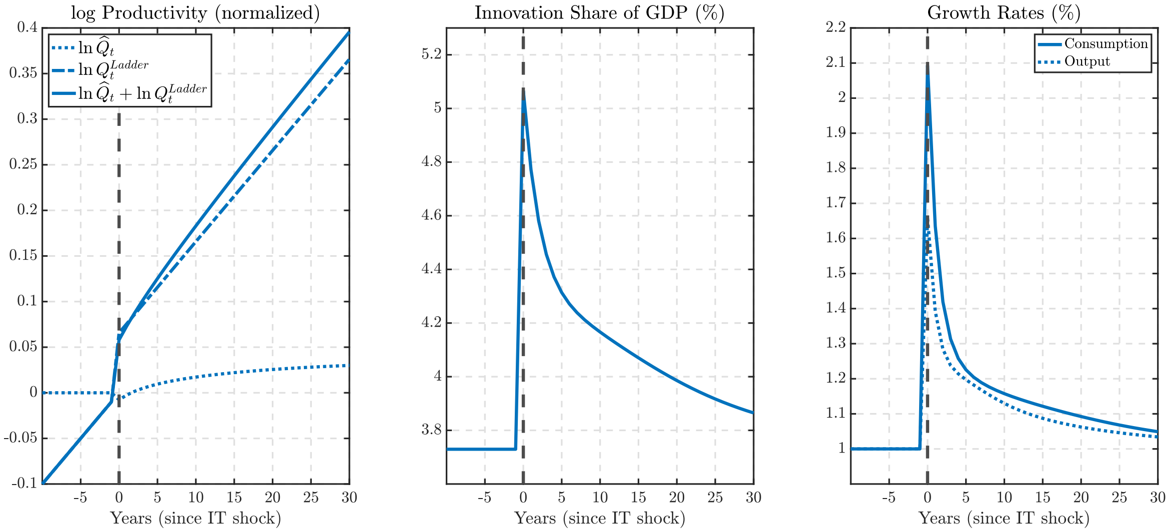

Figure: Variables of interest in the transitional dynamics computed in response to an unforeseen technological breakthrough that simultaneously disrupts 21.9% of industries.

Policy Analysis

Optimal Policy Reaction in the Aftermath of a Revolution

Externalities

- **Question:** How should a policymaker react after a technological revolution?

- Two counteracting externalities:

  1. **Entry externality:**
     - DRTS in R&D → Distribute resources among many firms is better than giving them all to one.
     - Ex-ante, this calls for maximizing number of incumbents → Subsidizing entry and/or process.
  2. **Business-stealing externality:**
     - Ex-post rent extraction might lead to over-investment in innovation.
     - This calls for taxing entry and/or process.

- In the wake of a revolution, the relative welfare impact of these two margins will change!
  - Suppose, before the shock hits, a policy was in place that balanced out these externalities.
  - Then, a revolution unexpectedly arrives, disrupting a large share of industries.
  - What is the best way to "redirect" policy to mitigate (enhance) the bad (good) effects of the revolution?

Externalities

- **Question:** How should a policymaker react after a technological revolution?
- Two counteracting externalities:
  1. **Entry externality:**
     - DRTS in R&D → Distribute resources among many firms is better than giving them all to one.
     - Ex-ante, this calls for maximizing number of incumbents → Subsidizing entry and/or process.
  2. **Business-stealing externality:**
     - Ex-post rent extraction might lead to over-investment in innovation.
     - This calls for taxing entry and/or process.
- In the wake of a revolution, the relative welfare impact of these two margins will change!
  - Suppose, before the shock hits, a policy was in place that balanced out these externalities.
  - Then, a revolution unexpectedly arrives, disrupting a large share of industries.
  - What is the best way to "redirect" policy to mitigate (enhance) the bad (good) effects of the revolution?

Welfare

- We consider a linear and constant tax/subsidy ξ on entry costs (i.e. product R&D).
- Resulting change in welfare between the two BGPs:

$$\Delta\text{Welfare} = \frac{1}{\rho}\left[\Delta \ln \begin{pmatrix} \text{Consumption} \\ \text{share of output} \end{pmatrix} + \underbrace{\Delta \ln \begin{pmatrix} \text{Aggregate} \\ \text{productivity} \end{pmatrix}}_{=\Delta \ln(\text{Aggregate Output})} \right]$$

where

1. **Consumption share:** $$\frac{C}{Y} = 1 - \sum_{s=1}^{\overline{S}} h(s)\Big[\overbrace{\phi\, N(s)}^{\text{Fixed costs}} + \overbrace{X(s)}^{\text{Innovation costs}}\Big]$$
2. **Aggregate productivity:** $$Q = \exp\left(\sum_{s=1}^{\overline{S}} h(s) \ln\left(\sum_{n\in\mathcal{N}(s)} (q_n(s))^{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}}\right)$$
3. **Misallocation wedge:** $$\mathcal{W} = \left[\sum_{s=1}^{\overline{S}} h(s)\left(\sum_{n\in\mathcal{N}(s)} \sigma_n(s)\frac{M(s)}{m_n(s)}\right)\right]^{-1}$$

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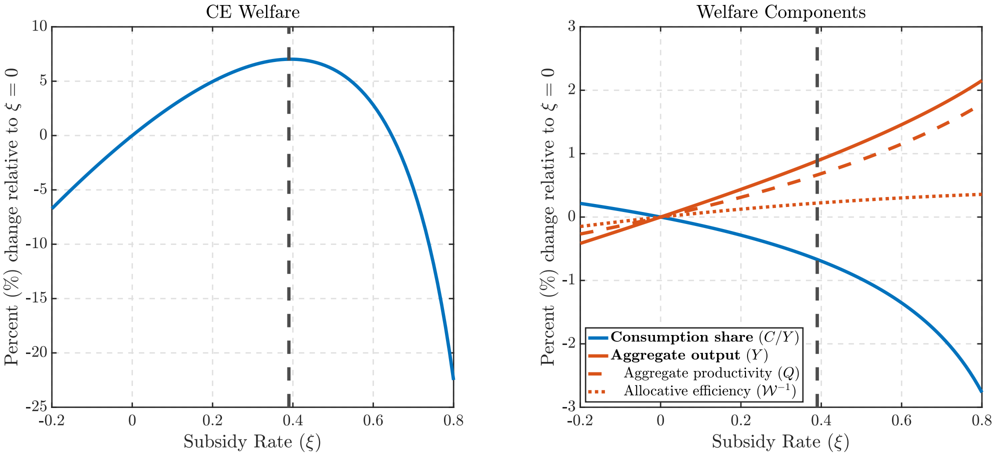
"Business-As-Usual" Optimal Policy

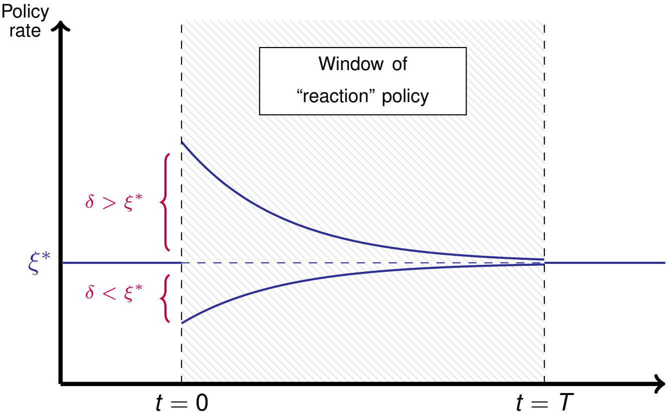
- Optimal policy ($\xi^* = 39\%$) gives 7% ↑ in CE Welfare, coming from:
  - Output ↑ 0.9%, of which 3/4 due to higher productivity ↑, and 1/4 due to better allocative efficiency.
  - Consumption share ↓ 0.67%, from innovation spending share ↑ 12% and fixed costs share ↑ 2.2%.



Optimal Policy Reaction to a Revolution

- Before $t = 0 \rightarrow$ Economy is on a BGP, and the optimal entry subsidy $\xi^* = 39\%$ is in place.
- After $t = 0 \rightarrow$ For a given T (end date for targeted policy), policymaker chooses $\delta \lessgtr 0$ such that

$$\xi_t = \xi^* + (\delta - \xi^*)\exp(-0.1t)$$



Optimal Policy Reaction to a Revolution

- **Before** the revolution hits $\rightarrow$ Too little entry, it's optimal to subsidize it ($\xi^* > 0$).
- **Right after** the revolution hits $\rightarrow$ Private incentives to entry increase a lot by themselves:
  - Business-as-usual policy is costly $\rightarrow$ R&D share of GDP is too high, firms will enter anyway!
  - So the optimal reaction is to temporarily $\downarrow$ subsidies, to avoid a short-run drop in consumption.

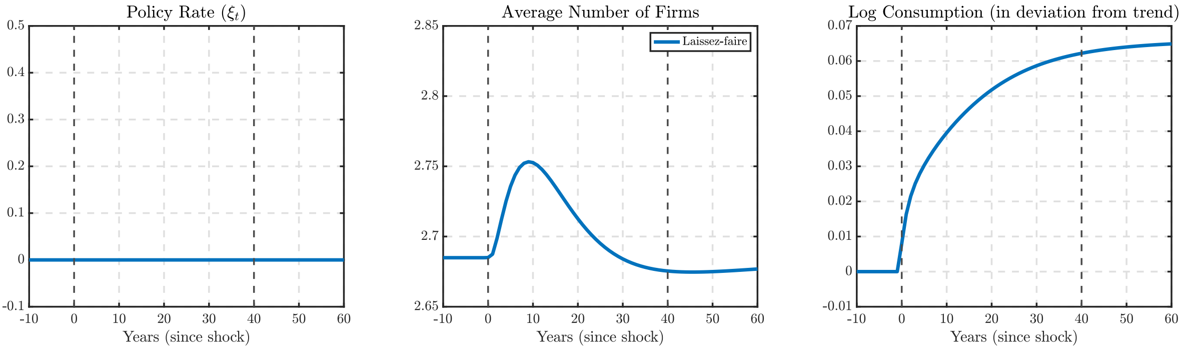


Figure: Policy responses to an unforeseen technological breakthrough that simultaneously disrupts 21.9% of industries.

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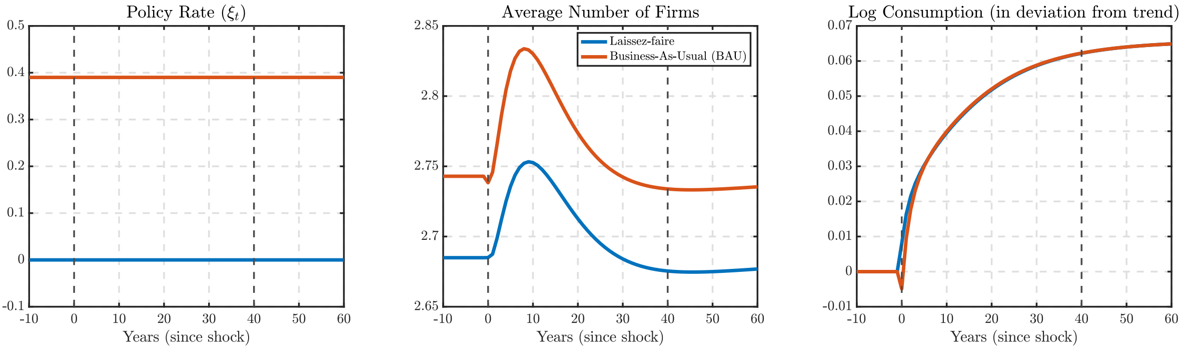

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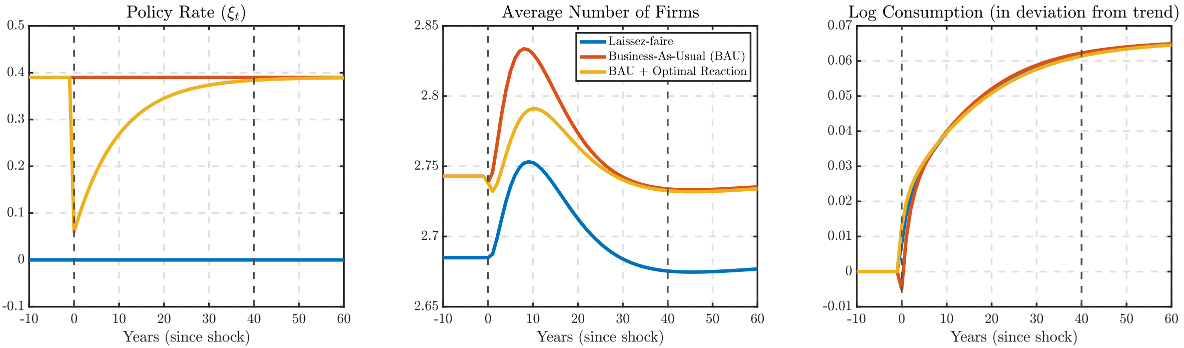

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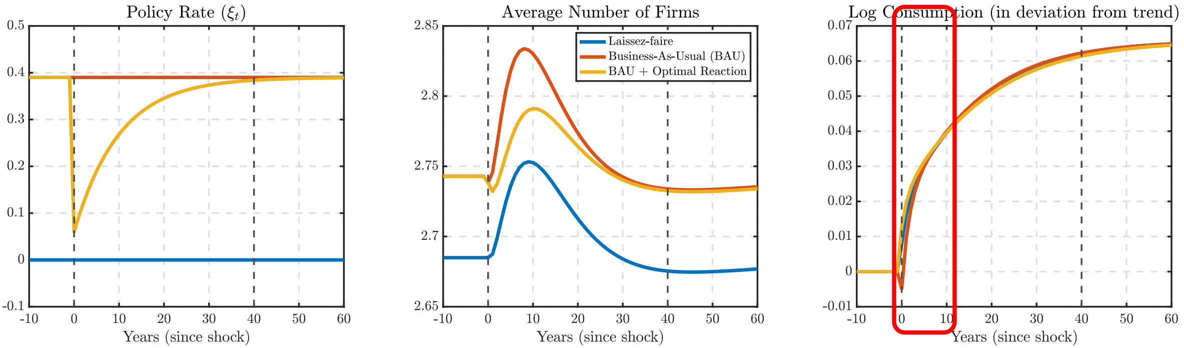

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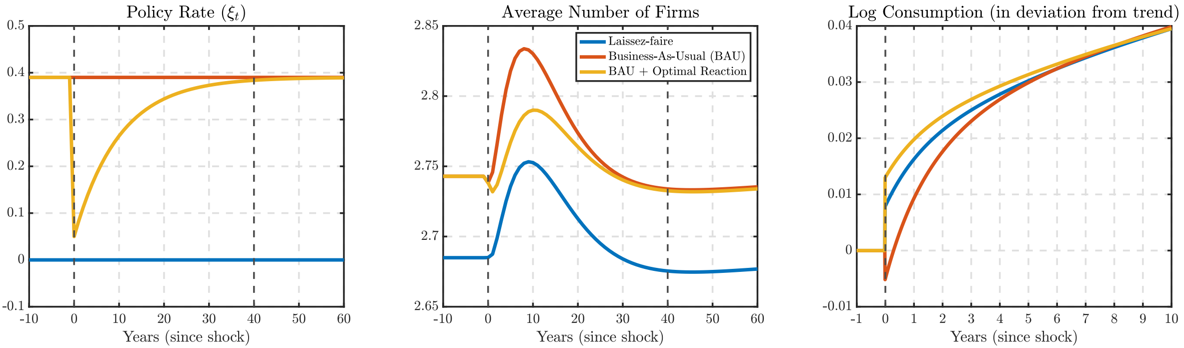

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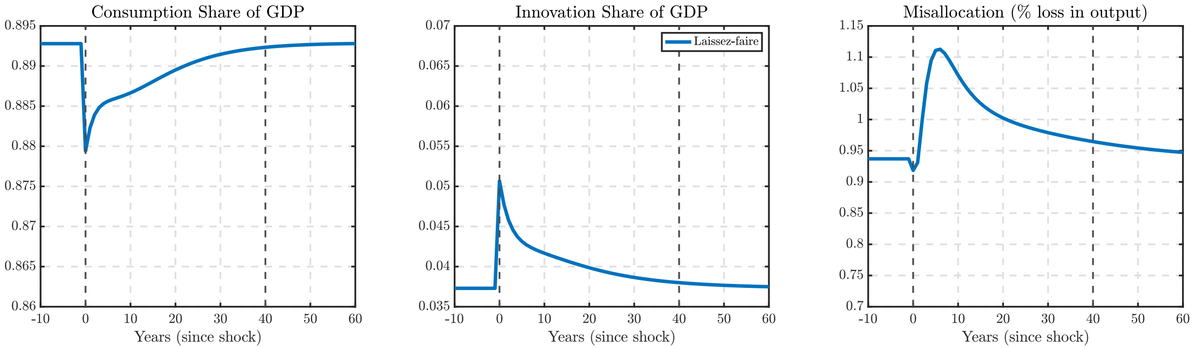

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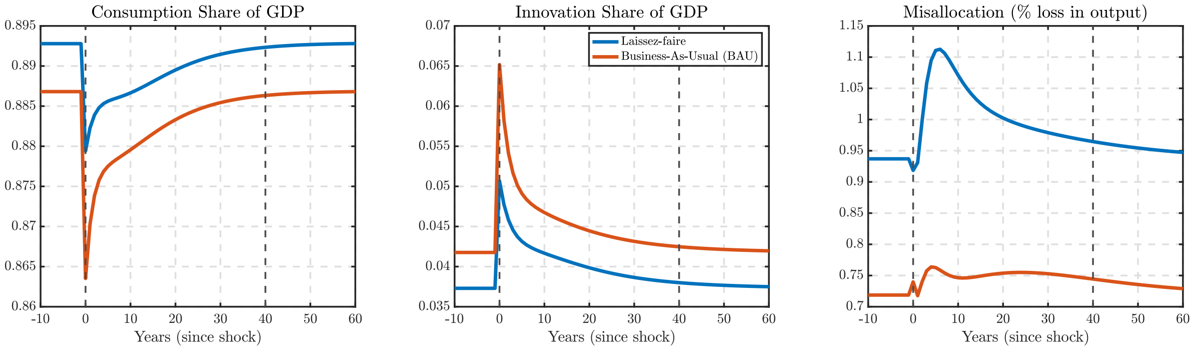

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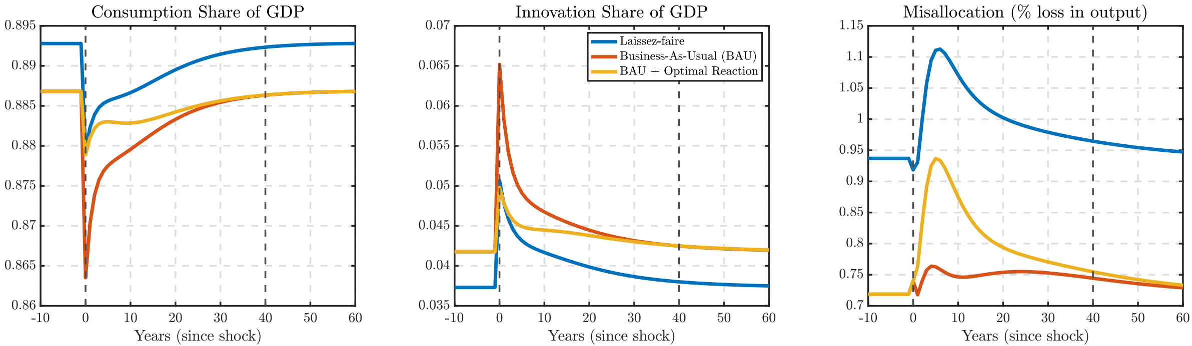

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Conclusions

- New technological opportunities trigger industry-level life cycles.
  - ICT Revolution led to an increase in entry in affected industries, followed by a shakeout.
  - These dynamics were driven by an increase in process innovations.
- New GE model to capture these dynamics.
  - Shakeout due to process innovation, early entry predicts survival (as in Klepper, 1996, 1997).
  - Preliminary results find that, in the wake of a revolution, we may want to decrease R&D subsidies.
- **What's next:**
  1. More on optimal policy $\rightarrow$ Timing, duration, policy mix, ..., following a technological revolution.
  2. Fully endogenous growth? $\rightarrow$ Technology revolutions as a by-product of firm-level innovations.

Thank you!

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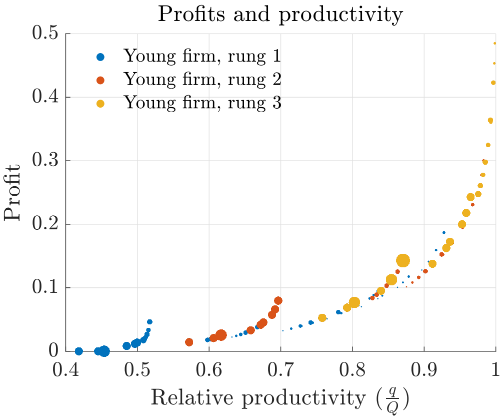
Thank you!

Appendix

Appendix: Profits and Relative Productivity: An Illustration

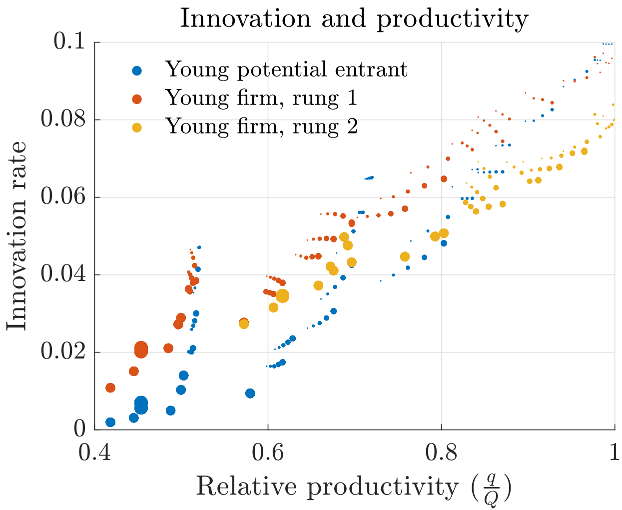
▸ Back



- Profits are generally increasing in relative productivity.
- However, the productivity distribution matters:
  - For example, one good and one bad competitor is preferable to two average competitors...
  - ...even though industry productivity is higher in the former case.

Appendix: Innovation and Relative Productivity: An Illustration

Back



- Innovation is generally increasing in relative productivity.
  - Discouragement effect when other firms innovate first.

Appendix: Set of ICT Industries

Back

Definition of ICT industry comes from Goldschlag and Miranda (2020):

- NAICS industry is identified as ICT if its share of Science, Technology, Engineering and Math (STEM) occupational employment is higher than 5 times the national average for most years.

NAICS	**Name**
3341	Computer and Peripheral Equipment Manufacturing
3342	Communications Equipment Manufacturing
3344	Semiconductor and Other Electronic Component Manufacturing
3345	Navigational, Measuring, Electromedical, and Control Instruments Manufacturing
5112	Software Publishers
5171	Wired Telecommunications Carriers
5179	Other Telecommunications
5182	Data Processing, Hosting, and Related Services
5191	Other Information Services
5415	Computer Systems Design and Related Services

Figure: Set of 4-digit NAICS industries identified as ICT-related in Braguinsky, Choi, Ding, Jo and Kim (2023), Table A2.