

# **A Theory of Dynamic Product Awareness and Targeted Advertising\***

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\* The views expressed in this presentation are the authors' and may not represent those of Banco de España or the Eurosystem.

# Motivation

- Firms have long used **advertising** to spread product awareness.
  - **Traditional methods**: radio and TV ads, billboards, door-to-door sales, ...
  - **Targeted methods**: mailing lists, customer catalogs, online ads, ...
- Advances in technology (social networks, search engines, big data, ...) have **increased efficiency** of ADV.
  - Share of digital in total ADV spending → From **4% in 2000** to **57% in 2020** (**70% expected in 2023**).
- Do these changes have an effect on **how customers are reached** and how **markets are structured**?

## This paper:

New information-based theory of product lifecycles to understand ...

- (i) ... how **expanding consumer choice sets** affect market and macro dynamics;
- (ii) ... how **better ADV technologies** affect competition, sorting, markups, misallocation, welfare.

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# Outline

## 1. Theory: A GE Model of Dynamic Product Awareness

### ■ Key features:

- 1 *Heterogeneous consumers* → (i) idiosyncratic tastes (*exogenous*); (ii) incomplete awareness sets (*endogenous*).
- 2 *Homogeneous firms* → Take advantage of limited awareness and exploit customers through markups.

### ■ Two advertising technologies:

- 1 *Traditional* → Increase consumer contact rate ⇒ Consumers find preferred product faster (↑ **consumer sorting**).
- 2 *Targeted* → Find high-valuation consumers with higher likelihood (↑ **match quality** but also ↑ **segmentation**).

## 2. Application: The Rise of Targeted Advertising (United States, 2005-2014)

### ■ Two calibrations → Match the increase in share of digital ADV (↑ targeting) in the period 2005-2014.

### ■ In the 2014 calibration → Both forms of ADV more cost-effective, but targeting now relatively cheaper.

- ... **match quality** ↑ → Higher-quality matches formed with fewer connections (customer misallocation ↓).
- ... **consumer sorting** ↓ → Awareness expands more slowly, more segmentation ⇒ market power ↑

### ■ Counterfactual: Had ADV technology not improved, welfare would have been higher despite worse sorting.

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## Related Literature

### ■ Customer Capital in Trade and Macro:

Fishman and Rob (2003), Bergemann and Välimäki (2006), Luttmer (2006), Arkolakis (2010, 2016), Dinlersoz and Yorukoglu (2012), Drozd and Nosal (2012), Gourio and Rudanko (2014), Fitzgerald, Haller and Yedid-Levi (2017), Paciello, Pozzi and Trachter (2019), Afrouzi, Drenik and Kim (2020), Roldan-Blanco and Gilbukh (2021), Ignaszak and Sedláček (2021), Einav, Klenow, Levin and Murciano-Goroff (2022).

**Contribution:** New information-based interpretation.

### ■ Intangibles and Market Power:

Aghion, Bergeaud, Boppart, Klenow and Li (2019), Cavenaile, Celik, Tian (2022), Weiss (2022), De Ridder (2022).

**Contribution:** New mechanism operating through the extensive-margin of demand.

### ■ Advertising in Economics:

Dorfman and Steiner (1954), Butters (1977), Becker and Murphy (1993), Bagwell (2007), Goeree (2008), Dinlersoz and Yorukoglu (2012), Guthmann (2020), Greenwood, Ma and Yorukoglu (2021), Rachel (2021), Cavenaile and Roldan-Blanco (2021), Argente, Fitzgerald, Moreira and Priolo (2021), Klein and Şener (2022), Baslandze, Greenwood, Marto and Moreira (2022).

**Contribution:** Focus on targeted vs non-targeted ADV, feature new GE effects.



## **Model Assumptions**

# Assumptions I: Demographics

- **Consumers:** Measure-one continuum, with preferences over a **single final good**.
- **Final good:** Assembled from a continuum mass  $M_t > 0$  (*endogenous*) of **product categories**.
  - Category  $m \in [0, M_t]$  is populated by a **finite** number  $N \in \mathbb{Z}_+$  of identical firms  $i \in \mathcal{I} \equiv \{1, 2, \dots, N\}$ .
  - A “**product**” is uniquely indexed by  $(i, m) \in \mathcal{I} \times [0, M_t]$ .
- **Product market dynamics:**
  - Each instant  $t \in \mathbb{R}_+$ , an “innovator” invests resources to find a blueprint for a **new product category**.
  - All  $N$  firms enter together upon product creation, and all exit together at rate  $\delta_M > 0$  (*obsolescence*).
- **Consumer heterogeneity:**
  - 1 ... in preferences:
    - Permanent idiosyncratic preferences over  $(i, m)$  products,  $\xi_{imj} \sim \text{Gumbel}(0, 1)$ .
    - Distribution of preferences is independent across product categories, i.e.  $\xi_{imj} \perp \xi_{im'j}, \forall m \neq m'$ .
  - 2 ... in awareness:
    - Consumer  $j$  is aware of, and can only consume from, a subset of products from each category.

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  - 2 ... in **awareness**:
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## Assumptions II: Preferences, Technology and Information

### ■ Preferences: Consumer $j \in [0, 1]$ :

$$\max \int_0^{+\infty} e^{-\rho t} \frac{C_{jt}^{1-\gamma}}{1-\gamma} dt, \quad \text{with } C_{jt} = \left[ \int_0^{M_t} \left( \sum_{i \in A_{mjt}} \bar{\Gamma} e^{\sigma \xi_{imj}} c_{imjt} \right)^{\frac{\kappa-1}{\kappa}} dm \right]^{\frac{\kappa}{\kappa-1}}$$

where

- $c_{imjt} > 0$  is the quantity purchased of product  $(i, m)$ .
- $A_{mjt} \subseteq \mathcal{I} = \{1, \dots, N\}$  is the consumer's awareness set, which evolves endogenously via ADV.

### ■ Technology: Identical firms $i \in \{1, \dots, N\}$ , use a common Cobb-Douglas technology (w/ constant TFP):

$$y_{imt} = z k_{imt}^\alpha l_{imt}^{1-\alpha}, \quad \text{with } z > 0, \alpha \in (0, 1)$$

### ■ Information:

- Firm cannot observe  $A_{mjt}$  and  $\xi_{imj}$  for any consumer  $j \in [0, 1]$ .
- *But...* they have complete information on:
  - 1 Joint distribution over  $(A_{mjt})$  sets that contain the firm and their corresponding  $(\xi_{imj})$  shifters.
  - 2 Actions of other firms within the product category  $\rightarrow$  Compete in prices (à la Bertrand).

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# Assumptions III: Advertising

## 1 Traditional advertising:

- Choose contact rate of new customers,  $\theta > 0$ , constant over time and across firms.
- Determines evolution of awareness sets ("urn-ball" without replacement).

## 2 Targeted advertising:

- Recall  $\rightarrow$  Tastes for *population* of consumers are  $\xi_{imj} \sim \text{Gumbel}(0, 1)$ .
- At age  $a \geq 0$ , tastes of consumers *who are aware* of firm  $i$  are  $\sim \text{Gumbel}(\ln(\mu_i(a)), 1)$ .
  - Firms only choose  $\mu_{0,i} \equiv \mu_i(0) \geq 1$ , at age  $a = 0$ .
  - Law of motion for targeting:

$$\ln(\mu_i(a)) = \ln(\mu_{0,i}) \left( 1 - \underbrace{s_i(a)}_{\text{Network saturation}} \right), \quad \text{where } s_i(a) \equiv \underbrace{\sum_{A \ni i} \widehat{f}(a, A)}_{\substack{\text{Proportion of awareness sets} \\ \text{that contain the firm}}}$$

- ADV costs per firm (paid in units of final good)  $\rightarrow d(\theta, \mu_0) = \nu\theta^2 + \eta(\mu_0 - 1)^2$ , with  $\nu, \eta > 0$ .



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“Urn-ball” logic  
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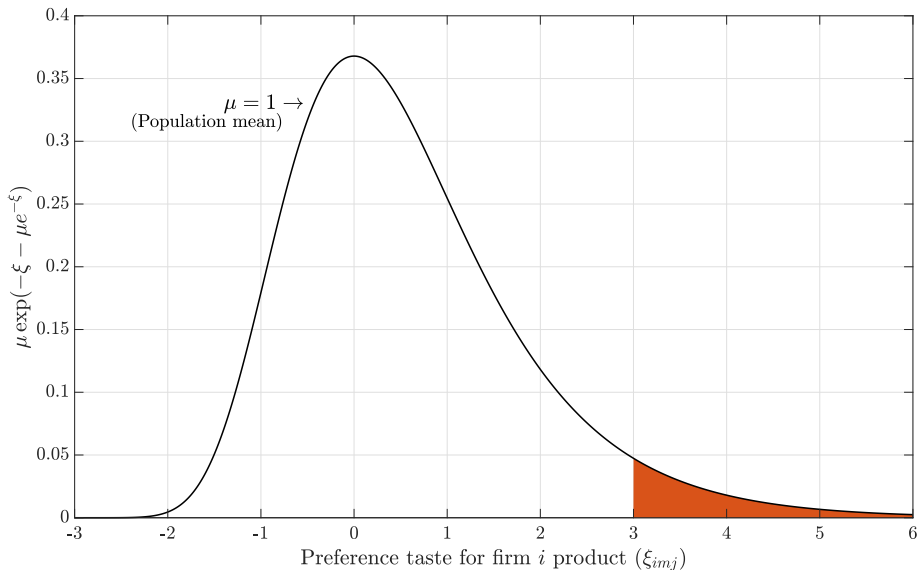
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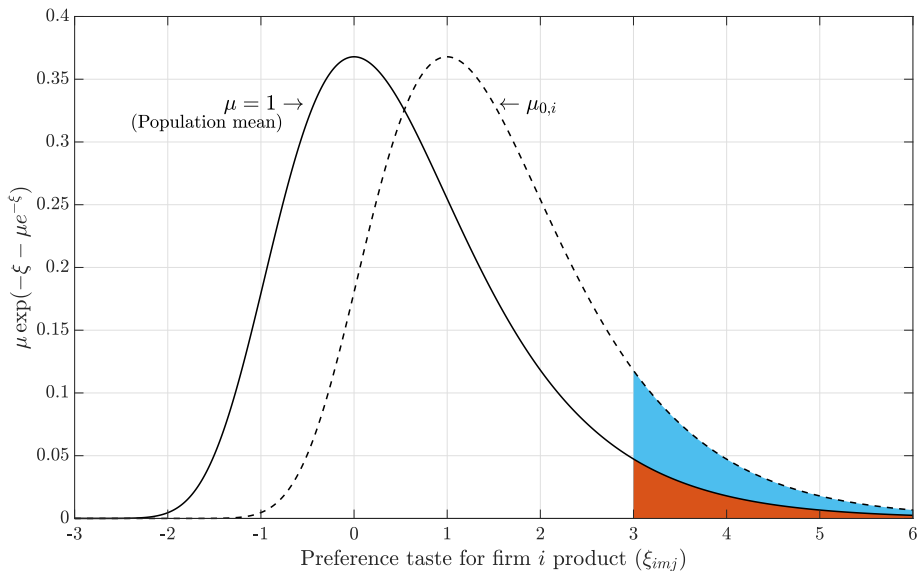
## Assumptions IV: Visualizing Targeting

- $\xi$  distribution for **population of consumers** has mean  $0 = \ln(\mu)$ , i.e. an “undistorted” Gumbel with  $\mu = 1$ .



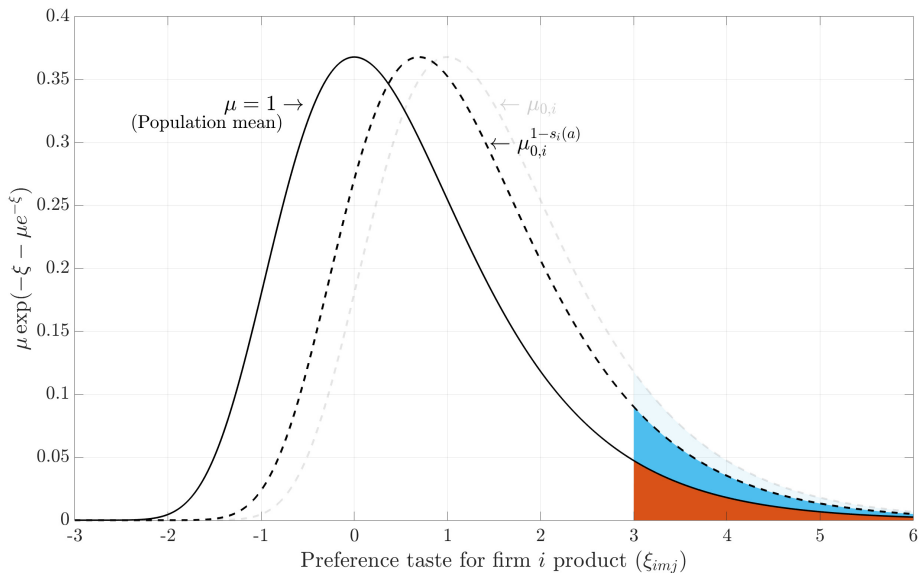
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- At age  $a = 0$ , firm gets to draw from a **distorted Gumbel** with mean  $\mu_{0,i} > 1$  (chosen once-and-for-all).



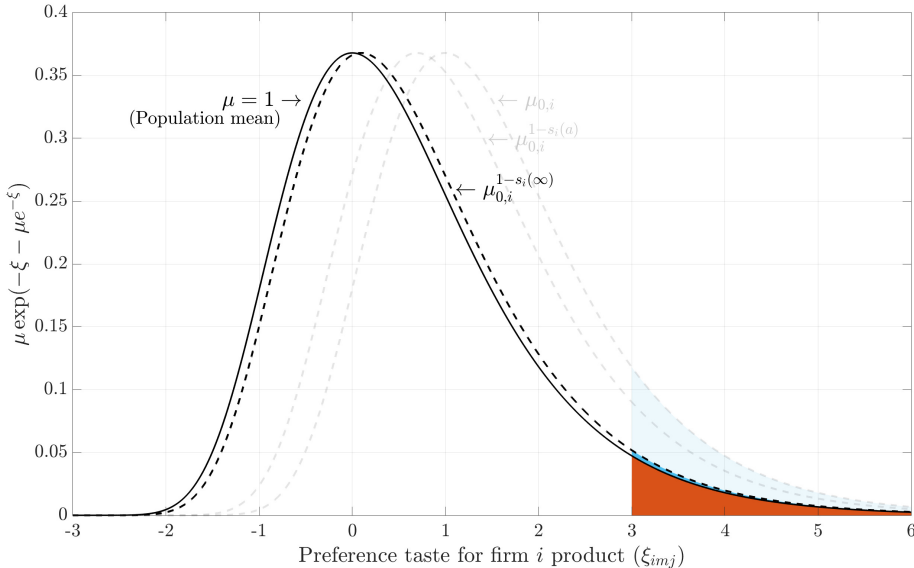
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- As product category ages ( $a \uparrow$ ), firm's network saturates ( $s(a) \uparrow$ ) → Becomes “harder to distort” ( $\mu_i(a) \downarrow$ ).



# Assumptions IV: Visualizing Targeting

■ As  $a \rightarrow +\infty$ , every firm is in every awareness set:  $f_N(a) \rightarrow 1$  and  $s(a) \rightarrow 1$ , so  $\mu_i(a) \rightarrow 1$ .





# Equilibrium

## Equilibrium I: Consumer Problem

Given real income  $\Omega_{jt}$ , price index  $P_{jt}$ , and nominal prices ( $\widehat{p}_{imt}$ ) for  $i \in A_{mjt}$ :

- 1 **Extensive-margin:** Consumer  $j$  purchases from firm  $i$  and from no other firm  $i' \in A_{mjt} \setminus \{i\}$  iff

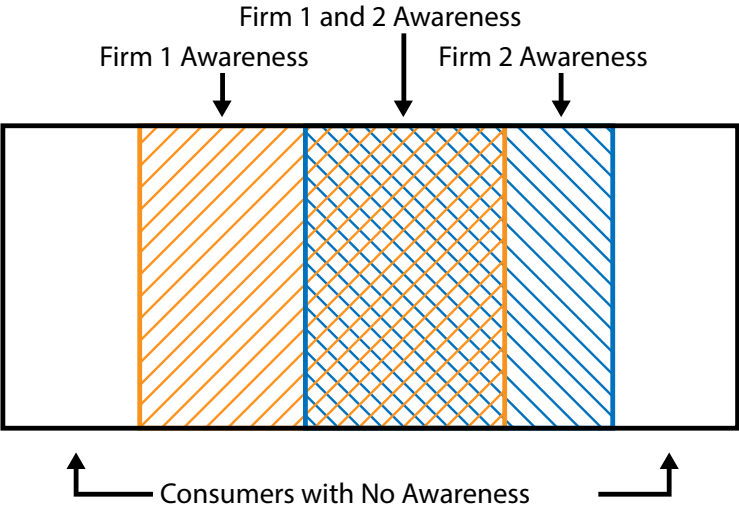
$$\ln \left( \frac{\widehat{p}_{i' mt}}{\widehat{p}_{imt}} \right) > \sigma (\xi_{i' mj} - \xi_{imj}).$$

- 2 **Intensive-margin:** If consumer  $j$  chooses firm  $i \in A_{mjt}$ , her demand is:

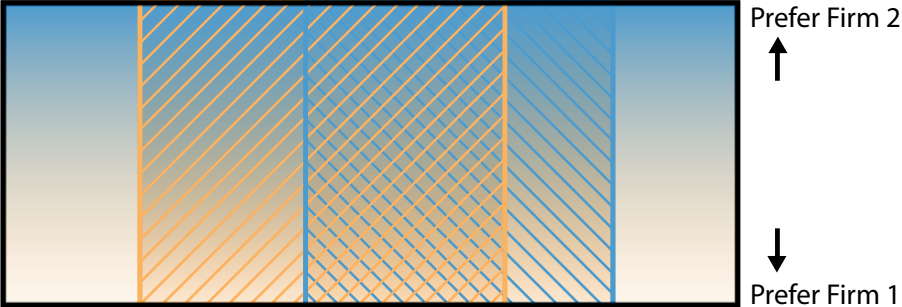
$$y_{imjt}^d = \bar{\Gamma}^{\kappa-1} e^{\sigma(\kappa-1)\xi_{imj}} \left( \frac{\widehat{p}_{imt}}{P_{jt}} \right)^{-\kappa} \Omega_{jt},$$

- In eq'm, each consumer purchases from **only one** firm in each product category (almost surely).
- For this one firm, intensive demand is downward-sloping,  $\left( \frac{\widehat{p}_{imt}}{P_{jt}} \right)^{-\kappa} \Omega_{jt}$ .

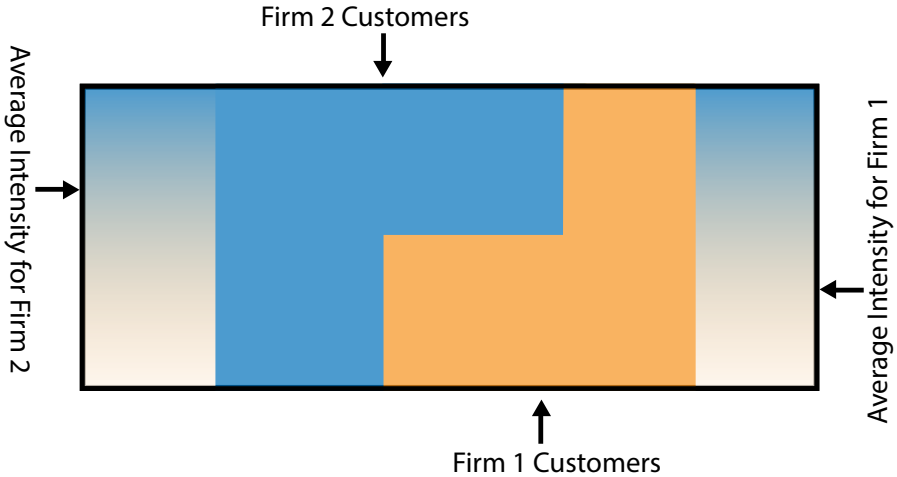
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## Equilibrium II: Firm Problem and Markups

- Focus on a *symmetric equilibrium* (in prices and targeting).
- Targeting** is a demand shifter, **sorting** matters only through *size* of (non-empty) sets,  $\hat{n} \equiv |A|$ .

► Details

Firm demand:

$$y_t(a) = \underbrace{(1 - f_0(a))}_{\text{Overall awareness}} \underbrace{\mu(a)^{\sigma(\kappa-1)}}_{\text{Targeting}} \underbrace{p(a)^{-\kappa} \frac{\Omega_t}{N}}_{\text{Downward-sloping demand}} \underbrace{q(a)}_{\text{Sorting}}$$

where  $q(a) \equiv \mathbb{E}_a [\hat{n}^{\sigma(\kappa-1)}]$

**Markups.** If a symmetric pure-strategy Nash equilibrium exists for  $N$  firms, then:

$$\Lambda(a) \equiv \frac{p(a)}{mc_t} = \frac{\varepsilon(a)}{\varepsilon(a) - 1}, \quad \text{where} \quad \underbrace{\varepsilon(a)}_{\text{Total demand elasticity}} = \underbrace{\kappa}_{\text{Intensive-margin elasticity}} + \underbrace{\left( -\frac{\partial \ln(q(a))}{\partial \ln(p)} \right)}_{\text{Extensive-margin elasticity}}$$

### ■ Mechanism:

- When product is young ( $a \approx 0$ ), awareness sets are sparse  $\rightarrow$  EM price-elasticity low.
- As  $a \uparrow$ , consumers sort into better options  $\rightarrow$  EM elasticity increases  $\rightarrow$  Competition intensifies  $\rightarrow \Lambda(a) \downarrow$

## Equilibrium II: Firm Problem and Markups

- Focus on a *symmetric equilibrium* (in prices and targeting).
- Targeting** is a demand shifter, **sorting** matters only through *size* of (non-empty) sets,  $\hat{n} \equiv |A|$ . ▶ Details

Firm demand:

$$y_t(a) = \underbrace{(1 - f_0(a))}_{\text{Overall awareness}} \underbrace{\mu(a)^{\sigma(\kappa-1)}}_{\text{Targeting}} \underbrace{p(a)^{-\kappa} \frac{\Omega_t}{N}}_{\text{Downward-sloping demand}} \underbrace{q(a)}_{\text{Sorting}}$$

where  $q(a) \equiv \mathbb{E}_a [\hat{n}^{\sigma(\kappa-1)}]$

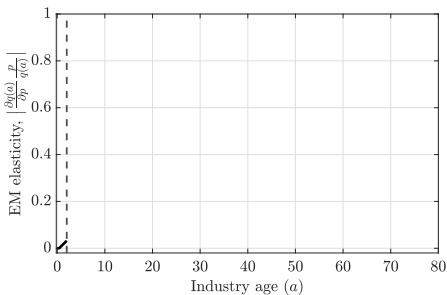
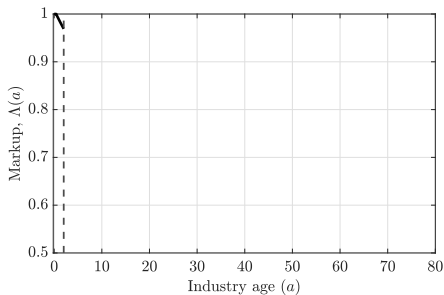
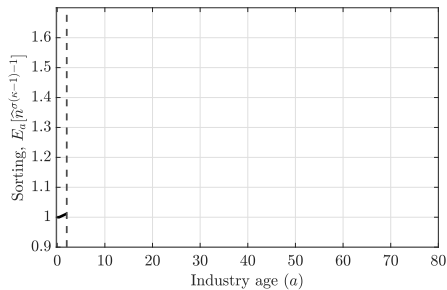
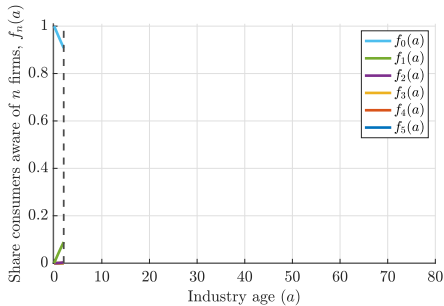
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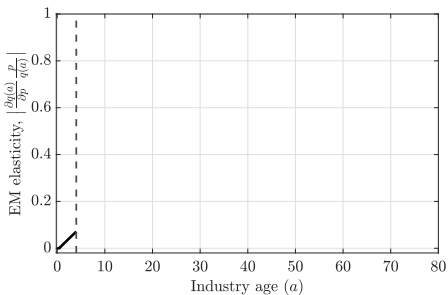
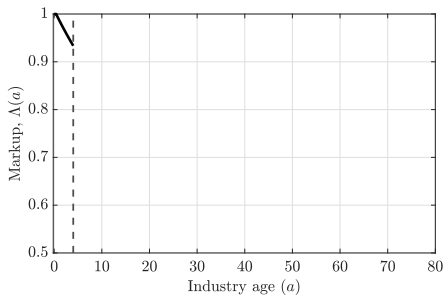
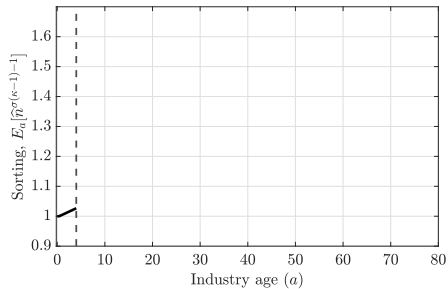
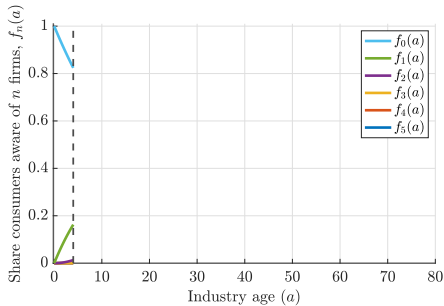
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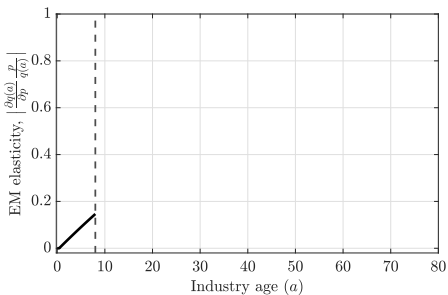
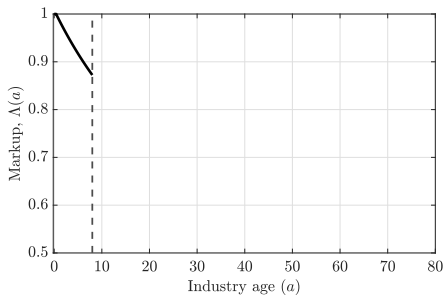
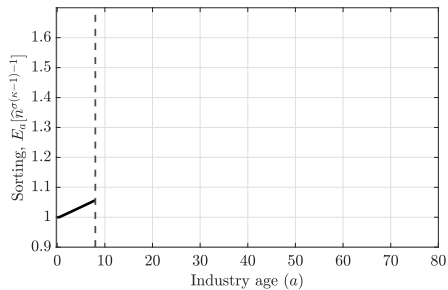
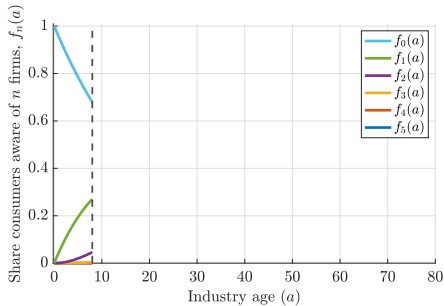




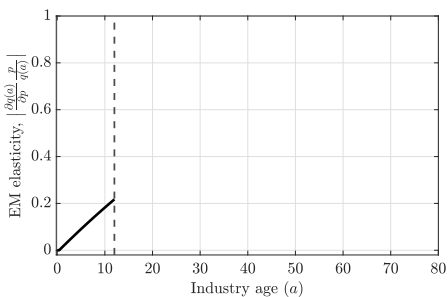
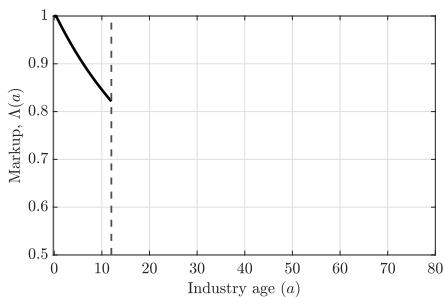
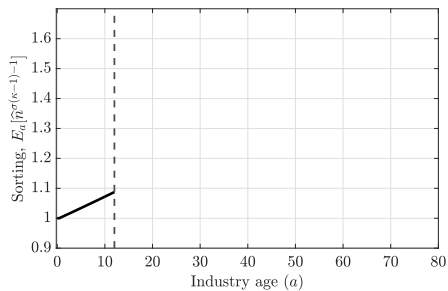
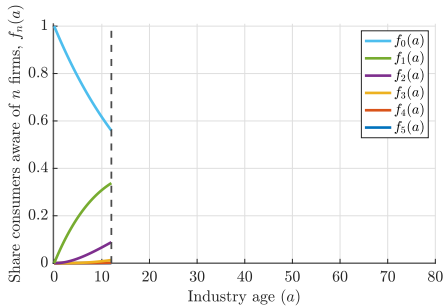
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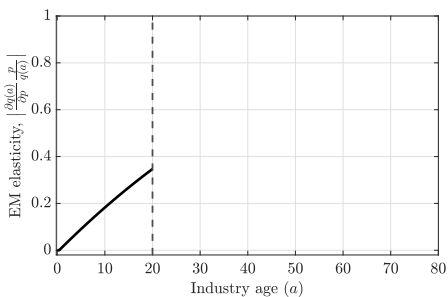
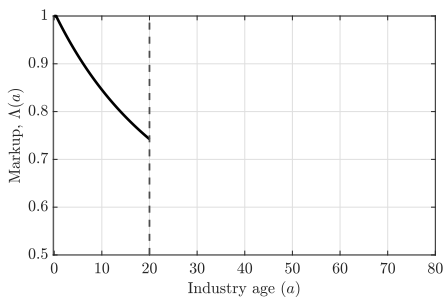
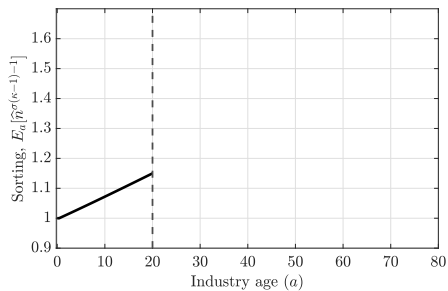
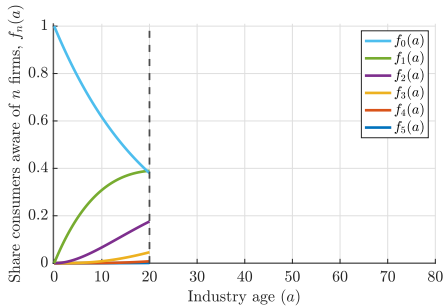
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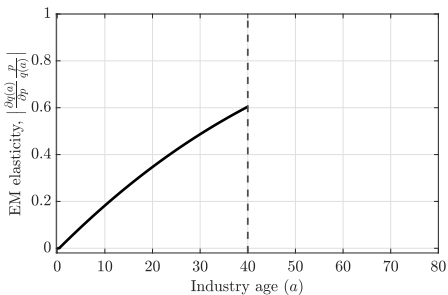
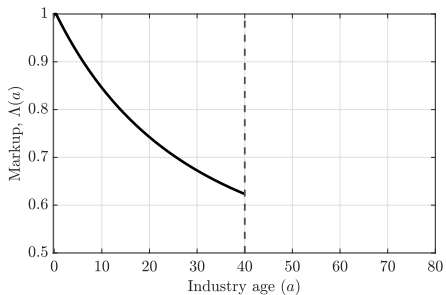
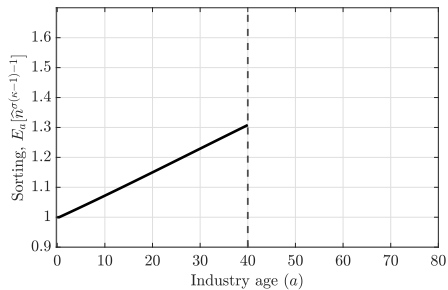
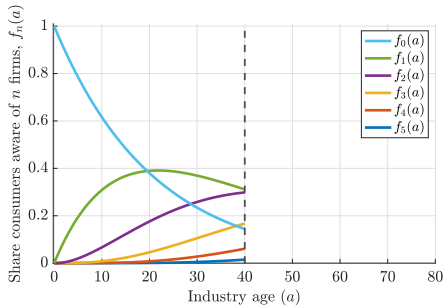
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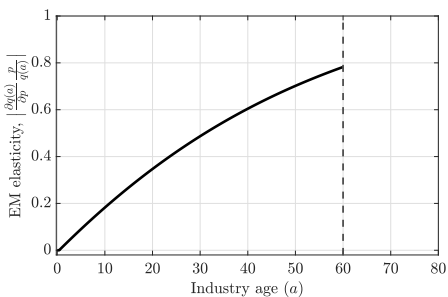
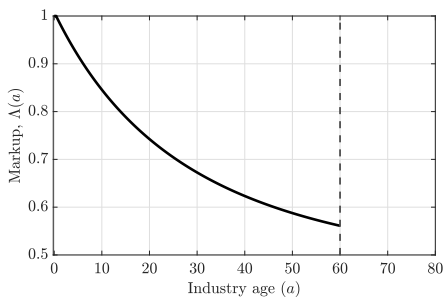
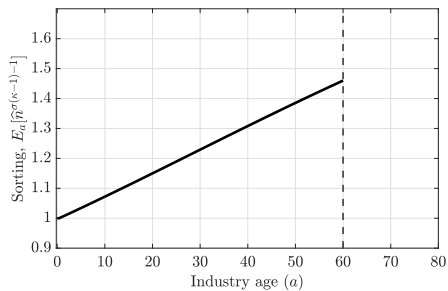
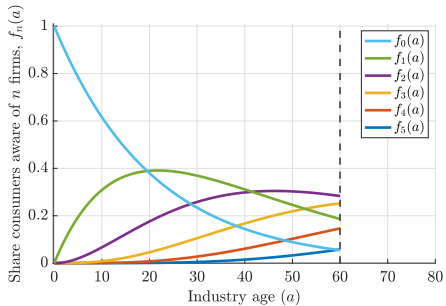
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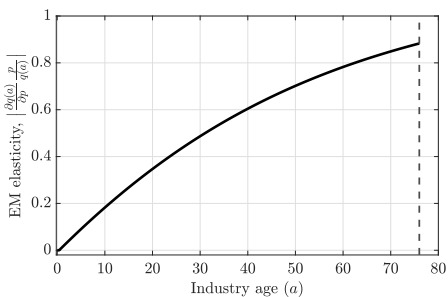
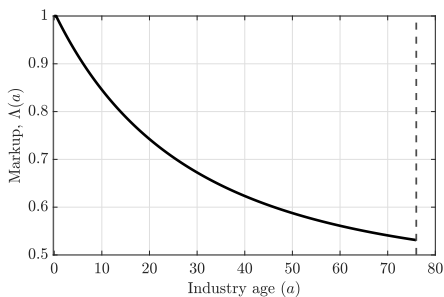
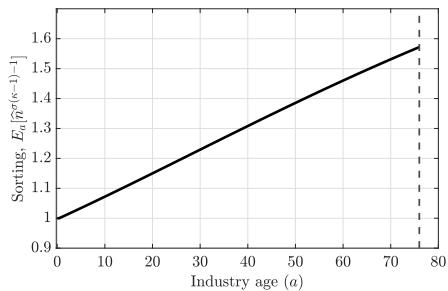
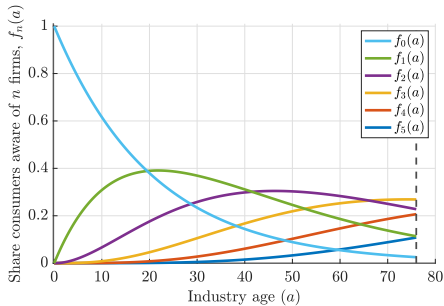
# Equilibrium III: Illustrative Example



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- Stationary economy aggregates to a **Neoclassical Growth Model** with endogenous TFP:

## 1 Aggregate markup:

$$\Lambda \equiv \left( \int_0^{+\infty} \varphi(a) (\Lambda(a))^{-1} d\Phi(a) \right)^{-1} \geq 1, \quad \text{where } \varphi(a) \equiv \frac{p(a)y(a)}{Y}$$

## 2 Income shares:

$$\frac{wL}{Y} = (1 - \alpha)\Lambda^{-1}; \quad \frac{(r + \delta_K)K}{Y} = \alpha\Lambda^{-1}; \quad \frac{\Pi}{Y} = 1 - \Lambda^{-1}.$$

## 3 Aggregate output:

$$Y = \underbrace{Z}_{\text{Physical TFP}} \underbrace{M^{\frac{1}{\kappa-1}}}_{\text{Love-of-variety}} \underbrace{Q}_{\text{Aggregate quality}} \underbrace{\Lambda}_{\text{Markup wedge}} K^\alpha L^{1-\alpha}, \quad \text{with } Q \equiv \left[ \int_0^{+\infty} \underbrace{(1 - f_0(a))}_{\text{Awareness}} \underbrace{\mu(a)^{\sigma(\kappa-1)}}_{\text{Targeting}} \underbrace{q(a)}_{\text{Sorting}} \Lambda(a)^{1-\kappa} d\Phi(a) \right]^{\frac{1}{\kappa-1}}$$

$Z \equiv \text{Endogenous TFP}$

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**Application:**  
The Rise of Targeted Advertising

## Two Calibrations

■ **Goal:** Quantify effects of ↑ targeting (rise in digital ADV in the 2000s) → 2 calibrations: [Details](#)

1 **“Early” calibration (2005)** → *Low return* of ADV targeting.

2 **“Late” calibration (2014)** → *High return* of ADV targeting (5 times higher, [Farahat and Bailey \(2012\)](#)).

| Parameter                     |          | Value  | Moment                          | Data   | Model  |
|-------------------------------|----------|--------|---------------------------------|--------|--------|
| A. “Early” calibration (2005) |          |        |                                 |        |        |
| Product differentiation       | $\sigma$ | 0.4183 | Average markup (sales-weighted) | 0.4674 | 0.4658 |
| Category creation efficiency  | $z_M$    | 0.1059 | Mass of categories ( $M$ )      | 1.0000 | 1.0000 |
| Contact rate cost             | $\nu$    | 0.0267 | Advertising share of GDP        | 0.0220 | 0.0220 |
| Targeting cost                | $\eta$   | 0.2527 | Return to targeting             | 0.0482 | 0.0482 |
| B. “Late” calibration (2014)  |          |        |                                 |        |        |
| Product differentiation       | $\sigma$ | 0.4099 | Average markup (sales-weighted) | 0.4850 | 0.4603 |
| Category creation efficiency  | $z_M$    | 0.0999 | Mass of categories ( $M$ )      | 1.0000 | 1.0000 |
| Contact rate cost             | $\nu$    | 0.0229 | Advertising share of GDP        | 0.0224 | 0.0224 |
| Targeting cost                | $\eta$   | 0.0352 | Return to targeting             | 0.2129 | 0.2129 |

■ Digital ADV rises → Both contacting ( $\nu$ ) and targeting ( $\eta$ ) become cheaper, but targeting relatively more so:

$$\frac{\eta_{\text{early}}}{\nu_{\text{early}}} = 9.5 \gg 1.5 = \frac{\eta_{\text{late}}}{\nu_{\text{late}}}$$

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# Implications of the Rise of Targeted Advertising

|                             |   | 2005  | 2014  | % change |
|-----------------------------|---|-------|-------|----------|
| Contact rate                | $\theta$                                  | 1.924 | 1.853 | -3.7%    |
| Targeting rate              | $\mu_0$                                   | 1.230 | 2.088 | +69.8%   |
| Average targeting           | $\int \mu(a)^{\sigma(\kappa-1)} d\Phi(a)$ | 0.023 | 0.101 | +339.1%  |
| Wage                        | $w$                                       | 2.087 | 2.205 | +5.7%    |
| Aggregate consumption       | $C$                                       | 2.977 | 3.123 | +4.9%    |
| Match quality               | $Q$                                       | 1.474 | 1.529 | +3.7%    |
| Aggregate markup            | $\Lambda$                                 | 1.466 | 1.460 | -0.4%    |
| Distortion-adjusted quality | $Q\Lambda$                                | 2.161 | 2.233 | +3.3%    |
| Output level                | $Y$                                       | 4.589 | 4.829 | +5.2%    |
| Aggregate TFP               | $Z$                                       | 2.161 | 2.233 | +3.3%    |

## ■ Since targeting is now relatively cheaper than contacting:

- Targeting  $\mu_0$  goes up strongly → Firms get better at finding customers with higher taste (match quality ↑).
- Contact rate  $\theta$  decreases slightly → Firms find fewer new customers per unit of time (sorting ↓).
- *In net...* Strong increase in welfare → Consumption ↑ by 4.9%, coming from...
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▶ [More details within product category](#)

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▶ More – Effects within product category

# Counterfactual Experiments I

- How did improvement in ADV technologies impact **markups**, **product market dynamics** and **welfare**?
- **Exercise:** Starting from 2014 economy ...
  - ... Both ADV cost parameters,  $\nu$  (contacting) and  $\eta$  (targeting), are **set back** to their 2005 values.
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**Results:** If there had been no reduction in ADV costs  $\rightarrow$  Less targeting ( $\downarrow$  40.9%) and faster contact ( $\uparrow$  5.2%).

1) **Negative effects:**

2) **Positive effects:**



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## 1 Negative effects:

- ... aggregate consumer-firm match quality  $Q$  would have worsened.

## 2 Positive effects:

- ... awareness would have accumulated faster → Stronger competition for customers → Markups ↓
- ... markup distortions  $\Lambda$  would have been lower → Profit share lower,  $(1 - \Lambda^{-1})$  ↓
- ... GE effects:
  - 1 ... There would have been more product varieties ( $M$  ↑).
  - 2 ... Welfare (per-product consumption) would have been higher ( $C/M$  ↑).

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|                             |            | Early<br>(base) | Late<br>(base) | Late<br>(cf) | Change<br>(wrt late base) |
|-----------------------------|------------|-----------------|----------------|--------------|---------------------------|
| Contact rate                | $\theta$   | 1.924           | 1.853          | 1.949        | +5.16%                    |
| Targeting rate              | $\mu_0$    | 1.230           | 2.088          | 1.236        | -40.81%                   |
| Consumption share           | $C/Y$      | 0.649           | 0.647          | 0.650        | +0.56%                    |
| Advertising share           | $D/Y$      | 0.022           | 0.022          | 0.021        | -5.44%                    |
| Profit share                | $\Pi/Y$    | 0.318           | 0.315          | 0.314        | -0.46%                    |
| Mass of categories          | $M$        | 1.000           | 1.000          | 1.183        | +18.32%                   |
| Aggregate consumption       | $C$        | 2.977           | 3.123          | 3.770        | +20.72%                   |
| Normalized consumption      | $C/M$      | 2.977           | 3.123          | 3.186        | +2.03%                    |
| Match quality               | $Q$        | 1.474           | 1.529          | 1.462        | -4.40%                    |
| Aggregate markup            | $\Lambda$  | 1.466           | 1.460          | 1.457        | -0.24%                    |
| Distortion-adjusted quality | $Q\Lambda$ | 2.161           | 2.233          | 2.130        | -4.60%                    |
| Output level                | $Y$        | 4.589           | 4.829          | 5.798        | +20.05%                   |
| Aggregate TFP               | $Z$        | 2.161           | 2.233          | 2.521        | +12.88%                   |

# Conclusion

- Study implications of targeted ADV for **product market dynamics** and **macroeconomic aggregates**.
- **Theory:**
  - Consumers are unaware of some products.
  - ADV affects: (i) speed at which new consumers are contacted; (ii) prob. of contacting high-valuation consumers.
- **Application:**
  - Rise in targeting driven by a **decrease** in both cost of targeting and cost of contacting.
  - Generates an **increase** in welfare (through better matches).
  - **Counterfactual:** Had rise in targeting not been accompanied by technological change...
    - ... match quality would have been lower.
    - ... still, welfare would have been higher, due to lower markup distortions.

**Thank you!**

# Appendix

- In eq'm, it is sufficient to keep track of the **size of awareness sets** as a function of product category age.
  - Let  $f_n(a)$  be the **proportion of consumers aware** of  $n \in \{0, 1, \dots, N\}$  firms at age  $a > 0$ .
  - We assume  $\vec{f}(a) \equiv [f_0(a), f_1(a), \dots, f_N(a)]^\top \in [0, 1]^{N+1}$  evolves according to:

$$\frac{\partial \vec{f}(a)}{\partial a} = \vec{f}(a) \cdot Q$$

with

$$Q = \begin{bmatrix} -\theta & \theta & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & -\frac{N-1}{N}\theta & \frac{N-1}{N}\theta & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & -\frac{N-2}{N}\theta & \frac{N-2}{N}\theta & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 0 & -\frac{1}{N}\theta & \frac{1}{N}\theta \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 \end{bmatrix}.$$

■ *Intuitively:*

- Each consumer has an intensity  $\theta > 0$  (the **contact rate**) of becoming aware of a particular firm in the category.
- When the consumer is aware of  $n \leq N$  firms, the intensity with which she becomes aware of a new firm is  $\frac{N-n}{N}\theta$ .

**Firm demand.** In a symmetric equilibrium with  $p(a) = p_{-i}$  and  $\mu(a) = \mu_{-i}$ , firm  $i$ 's demand function is:

$$y_t(a) = \underbrace{(1 - f_n(a))}_{\text{Awareness}} \underbrace{\mu(a)^{\alpha(n-1)}}_{\text{Targeting}} \underbrace{p(a)^{-\alpha} \frac{\Omega_i}{N}}_{\text{Downward-sloping demand}} \underbrace{\sum_{m \in \tilde{n}} \left[ \frac{p_m(a)}{p(a)} \right]^{\alpha}}_{\text{Sorting}}.$$

### 1 Awareness:

- $f_n(a) \equiv$  Share of consumers aware of  $n = 0, 1, \dots, N$  firms at age  $a$ .
- Thus, the more consumers are aware of the existence of product category  $m$ , the higher is demand (for all firms).

### 2 Targeting:

- Targeting shifts demand, until network eventually becomes saturated ( $f_N(a) \rightarrow 1$ , so  $s(a) \rightarrow 1$  and  $\mu(a) \rightarrow 1$ ).

### 3 Downward-sloping demand:

- *Intensive-margin* component  $\rightarrow$  Current customers of firm demand more intensively if  $p \downarrow$  and/or  $\Omega \uparrow$ .

### 4 Sorting:

- *Extensive-margin* component  $\rightarrow$  *Consumer sorting*, where  $\tilde{n}$  denotes size of (non-empty) awareness set.
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- In an equilibrium with symmetric strategies,  $\vec{p}_{-i} = \{p_{-i}, \dots, p_{-i}\}$  and  $\vec{\mu}_{-i} = \{\mu_{-i}, \dots, \mu_{-i}\}$ , demand is:

$$y(a, p) = (1 - f_0(a)) \mu(a)^{\sigma(\kappa-1)} p^{-\kappa} \frac{\Omega_t}{N} \mathbb{E}_a \underbrace{\left[ \hat{n} \left( 1 + (\hat{n} - 1) \frac{\mu_{-i}}{\mu(a)} \left( \frac{p_{-i}}{p} \right)^{-\frac{1}{\sigma}} \right)^{\sigma(\kappa-1)-1} \right]}_{\text{Sorting component of demand}}$$

- Targeting  $\mu(a) = \mu_0^{1-s(a)}$  affects demand in two ways:

- 1 Shifts idiosyncratic demand conditional on purchasing,  $\mu(a)^{\sigma(\kappa-1)-1}$ .
- 2 Shifts market power relative to competitors,  $\frac{\mu_{-i}}{\mu(a)}$ .

- In a symmetric equilibrium,  $\mu(a) = \mu_{-i}$ , only the first effect exists.

- Thanks to aggregation → Can solve for **consumption-savings** problem as if it came from **representative HH**.

$$\max_{c_t, I_t^K, I_t^M} \int_0^{+\infty} e^{-\rho t} \frac{C_t^{1-\gamma}}{1-\gamma} dt \quad \text{s.t.} \quad \left\{ \begin{array}{l} \dot{K}_t = I_t^K - \delta_K K_t \\ \dot{A}_t = \underbrace{r_t A_t}_{\text{Financial income}} + \underbrace{w_t}_{\text{Labor income}} + \underbrace{(r_t + \delta_K) K_t}_{\text{Returns from renting } K \text{ to firms}} - \underbrace{(C_t + I_t^K + I_t^M)}_{\text{Consumption and investment expenditures}} + \underbrace{Z_M I_t^M V_t^0}_{\text{Returns from category creation}} \end{array} \right.$$

- Household trades in firm shares:

$$A_t = M_t \int_0^{+\infty} V_t(a) d\Phi_t(a), \quad \text{where } V_t(a) \equiv \underbrace{\int_t^{+\infty} e^{-\int_t^s (r_\tau + \delta_M) d\tau} N \pi_s(a + s - t) ds}_{\text{Value of a product category at age } a \geq 0}.$$

- Age-zero advertising choices:**

- When new product category is created ( $a = 0$ ), blueprint owner chooses  $(\theta, \mu_0)$ , common to all firms:

$$V_t^0 \equiv \max_{\theta, \mu_0} \left\{ V_t(0) - N \left( \nu \theta^2 + \eta (\mu_0 - 1)^2 \right) \right\}$$

- Optimal choices →  $\theta_t^* = \frac{1}{2N\nu} \frac{\partial V_t(0)}{\partial \theta}$  and  $\mu_{0,t}^* = 1 + \frac{1}{2N\eta} \frac{\partial V_t(0)}{\partial \mu_0}$ .

- A few more equations to close the model:

- 1 Euler equation:

$$\frac{\dot{C}_t}{C_t} = \frac{r_t - \rho}{\gamma}$$

- 2 Resource constraint:

$$Y_t = C_t + I_t^K + I_t^M + z_M I_t^M N (\nu \theta^2 + \eta (\mu_0 - 1)^2)$$

- 3 Product category free entry condition:

$$z_M V_t^0 \leq 1 \quad \text{with equality if, and only if, } I_t^M > 0$$

- 4 Invariant distribution of product categories:

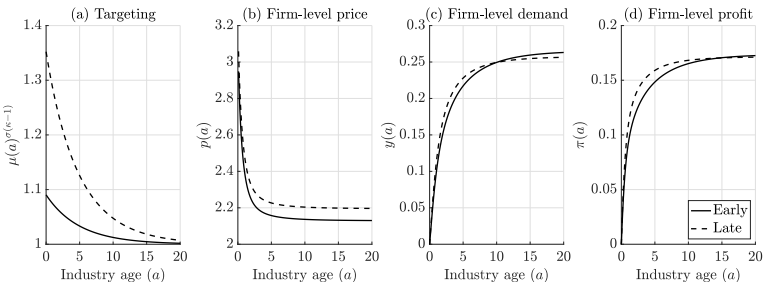
$$\frac{d\Phi(a)}{da} = 1 - e^{-\delta_M a}$$

- Parameters ( $N, z, \rho, \kappa, \alpha, \gamma, \delta_K, \delta_M$ ) are set externally.

| Parameter                             |            | Value | Source/Target  |
|---------------------------------------|------------|-------|--|
| Number of firms per product category  | $N$        | 10    |  |
| Firm-level productivity               | $z$        | 1     |  |
| Connection destruction rate           | $\zeta$    | 0     |  |
| Time discount rate                    | $\rho$     | 0.04  | 4% annual interest rate                                  |
| Cross elasticity of substitution      | $\kappa$   | 2     | <a href="#">Oberfield and Raval (2021)</a>               |
| Capital share of non-profit income    | $\alpha$   | 0.33  | Capital share of non-profit income                       |
| Coefficient of relative risk aversion | $\gamma$   | 2     | <a href="#">Havranek et al. (2015)</a>                   |
| Capital depreciation                  | $\delta_K$ | 0.069 | <a href="#">Celik et al. (2022)</a> and U.S. NIPA tables |
| Product destruction rate              | $\delta_M$ | 0.09  | <a href="#">Broda and Weinstein (2010)</a>               |

- Parameters ( $\sigma, z_M, \nu, \eta$ ) are calibrated internally → 4 targets for each calibration (“early” and “late”):

- Mass of categories → Normalized to  $M_t = 1$  in both calibrations (pins down  $z_M$ ).
- Average markup → From 46.7% (2005) to 48.5% (2014), using estimates from [De Loecker et al. \(2020\)](#).
- Advertising share of GDP → From 2.20% (2005) to 2.13% (2014), using data from [Greenwood et al. \(2021\)](#).
- Return to targeting:
  - [Farahat and Bailey \(2012\)](#) → Targeting ↑ click-through rate for brands by 79%.
  - Share of digital in total ADV rose from 6% (2005) to 30% (2014).
  - Adjusting for this, return to targeting → From 0.048 (2005) to 0.213 (2014) → A nearly 5-fold increase.



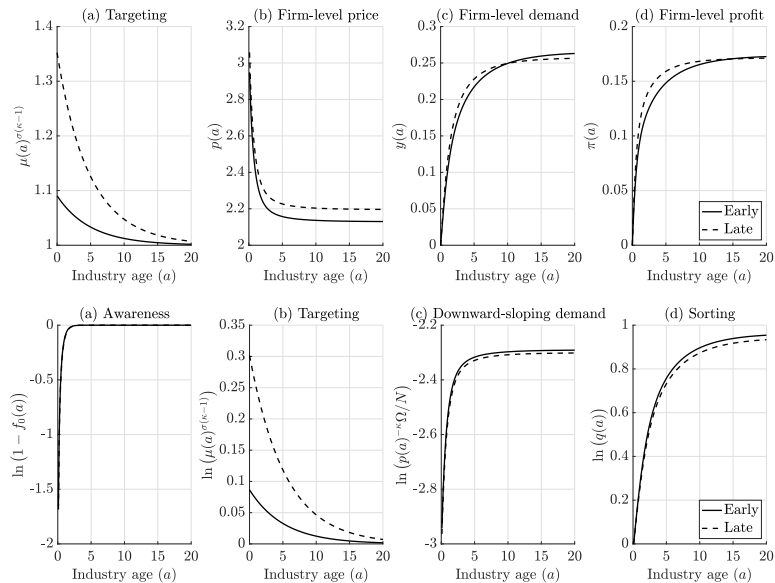
- Awareness spreads **more slowly**.
- Targeting is **much higher**, especially at early stages of the product category.
- Firms charge higher prices throughout.
- Demand and profits  $\uparrow$  early.

■ Components of demand (in logs):

$$\ln(y) = \ln(1 - f_0) + (\sigma(\kappa - 1) - 1)(1 - s) \ln(\mu_0) + \ln(\Omega/N) - \kappa \ln(p(a)) + \ln[\mathbb{E}_a(\hat{n}^{\sigma(\kappa-1)} - 1)]$$

where  $s = \frac{1}{N} \sum_{n=1}^N n f_n(a)$  [saturation].



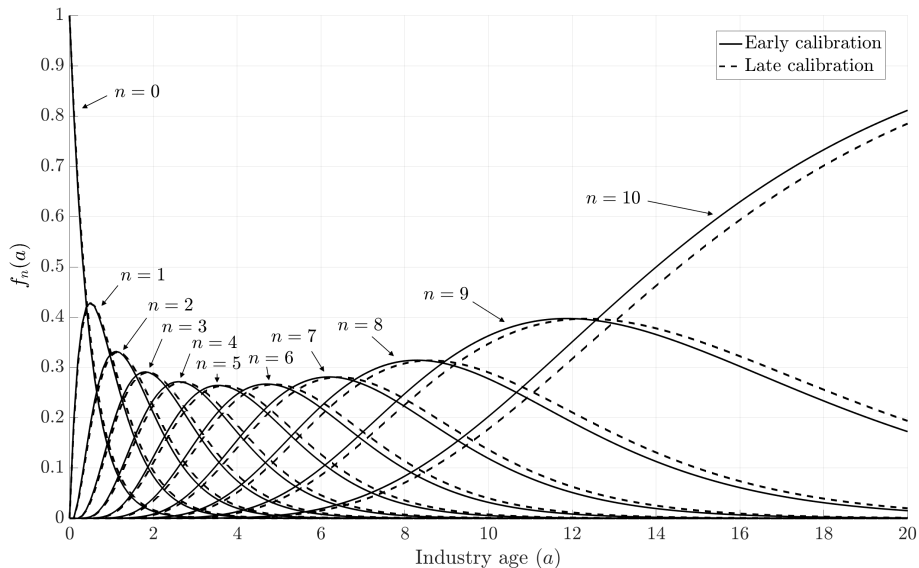


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where  $s = \frac{1}{N} \sum_{n=1}^N n f_n(a)$  [saturation].



# Appendix: Counterfactual Experiments (Full Table)

|                                   |            | Early<br>(base) | Late<br>(base) | Late<br>(cf) | Change<br>(wrt late base) |
|-----------------------------------|------------|-----------------|----------------|--------------|---------------------------|
| <i>A. Advertising and Markups</i> |            |                 |                |              |                           |
| Contact rate                      | $\theta$   | 1.924           | 1.853          | 1.949        | +5.16%                    |
| Targeting rate                    | $\mu_0$    | 1.230           | 2.088          | 1.236        | -40.81%                   |
| <i>B. GDP Shares</i>              |            |                 |                |              |                           |
| Consumption share                 | $C/Y$      | 0.649           | 0.647          | 0.650        | +0.56%                    |
| Advertising share                 | $D/Y$      | 0.022           | 0.022          | 0.021        | -5.44%                    |
| Category creation inv. share      | $I^M/Y$    | 0.185           | 0.186          | 0.184        | -1.44%                    |
| Capital investment share          | $I^K/Y$    | 0.144           | 0.144          | 0.145        | +0.21%                    |
| Profit share                      | $\Pi/Y$    | 0.318           | 0.315          | 0.314        | -0.46%                    |
| <i>C. Aggregates</i>              |            |                 |                |              |                           |
| Wage                              | $w$        | 2.087           | 2.205          | 2.653        | +20.31%                   |
| Mass of categories                | $M$        | 1.000           | 1.000          | 1.183        | +18.32%                   |
| Aggregate consumption             | $C$        | 2.977           | 3.123          | 3.770        | +20.72%                   |
| Normalized consumption            | $C/M$      | 2.977           | 3.123          | 3.186        | +2.03%                    |
| Match quality                     | $Q$        | 1.474           | 1.529          | 1.462        | -4.40%                    |
| Aggregate markup                  | $\Lambda$  | 1.466           | 1.460          | 1.457        | -0.24%                    |
| Distortion-adjusted quality       | $Q\Lambda$ | 2.161           | 2.233          | 2.130        | -4.60%                    |
| Output level                      | $Y$        | 4.589           | 4.829          | 5.798        | +20.05%                   |
| Aggregate TFP                     | $Z$        | 2.161           | 2.233          | 2.521        | +12.88%                   |

# Appendix: Other Counterfactual Experiments

[▶ Back](#)

|                                   |            | (1)             | (2)            | (3)                                   | (4)               | (5)                      | (6)               | (7)                       | (8)               |
|-----------------------------------|------------|-----------------|----------------|---------------------------------------|-------------------|--------------------------|-------------------|---------------------------|-------------------|
|                                   |            | Early<br>(base) | Late<br>(base) | Late<br>( $\nu_{2005}, \eta_{2005}$ ) | Change<br>wrt (2) | Late<br>( $\nu_{2005}$ ) | Change<br>wrt (2) | Late<br>( $\eta_{2005}$ ) | Change<br>wrt (2) |
| <i>A. Advertising and markups</i> |            |                 |                |                                       |                   |                          |                   |                           |                   |
| Contact rate                      | $\theta$   | 1.924           | 1.853          | 1.949                                 | +5.16%            | 1.748                    | -5.66%            | 2.066                     | +11.47%           |
| Targeting rate                    | $\mu_0$    | 1.230           | 2.088          | 1.236                                 | -40.81%           | 2.118                    | +1.46%            | 1.227                     | -41.22%           |
| <i>B. Shares of GDP</i>           |            |                 |                |                                       |                   |                          |                   |                           |                   |
| Consumption share                 | $C/Y$      | 0.649           | 0.647          | 0.650                                 | +0.56%            | 0.647                    | +0.00%            | 0.650                     | +0.53%            |
| Advertising share                 | $D/Y$      | 0.022           | 0.022          | 0.021                                 | -5.44%            | 0.023                    | +3.97%            | 0.020                     | -8.84%            |
| Category creation inv. share      | $I^M/Y$    | 0.185           | 0.186          | 0.184                                 | -1.44%            | 0.186                    | -0.42%            | 0.185                     | -1.00%            |
| Capital investment share          | $I^K/Y$    | 0.144           | 0.144          | 0.145                                 | +0.21%            | 0.144                    | -0.09%            | 0.145                     | +0.29%            |
| Profit share                      | $\Pi/Y$    | 0.318           | 0.315          | 0.314                                 | -0.46%            | 0.316                    | +0.21%            | 0.313                     | -0.64%            |
| <i>C. Economic aggregates</i>     |            |                 |                |                                       |                   |                          |                   |                           |                   |
| Wage                              | $w$        | 2.087           | 2.205          | 2.653                                 | +20.31%           | 2.272                    | +3.04%            | 2.552                     | +15.74%           |
| Mass of categories                | $M$        | 1.000           | 1.000          | 1.183                                 | +18.32%           | 1.027                    | +2.70%            | 1.143                     | +14.25%           |
| Aggregate consumption             | $C$        | 2.977           | 3.123          | 3.770                                 | +20.72%           | 3.221                    | +3.14%            | 3.623                     | +16.02%           |
| Normalized consumption            | $C/M$      | 2.977           | 3.123          | 3.186                                 | +2.03%            | 3.136                    | +0.42%            | 3.171                     | +1.55%            |
| Match quality                     | $Q$        | 1.474           | 1.529          | 1.462                                 | -4.40%            | 1.519                    | -0.67%            | 1.476                     | -3.51%            |
| Distortion-adjusted quality       | $Q\Lambda$ | 2.161           | 2.233          | 2.130                                 | -4.60%            | 2.220                    | -0.58%            | 2.148                     | -3.79%            |
| Output level                      | $Y$        | 4.589           | 4.829          | 5.798                                 | +20.05%           | 4.981                    | +3.14%            | 5.574                     | +15.41%           |
| Aggregate TFP                     | $Z$        | 2.161           | 2.233          | 2.521                                 | +12.88%           | 2.280                    | +2.11%            | 2.455                     | +9.92%            |