The Aggregate Effects of Acquisitions on Innovation and Economic Growth*

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Abstract

Large incumbent firms routinely acquire startups. This may stimulate aggregate growth, as acquisitions provide an incentive for startup creation and could transfer ideas to more efficient users. However, large firms do not always implement the ideas of their acquisition targets. Moreover, frequent acquisitions create a rigid economy without turnover, where incumbents have little incentives to innovate themselves. To assess the net effect of these forces, we build a new endogenous growth model with heterogeneous firms and acquisitions. We discipline the model by matching aggregate moments and evidence from a rich micro dataset on acquisitions and patenting. Preliminary findings indicate that aggregate growth has an inverted U-shape in the frequency of acquisitions.

Keywords: Acquisitions, Innovation, Productivity growth, Firm dynamics.

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1 Introduction

Large incumbent firms routinely acquire startups. For instance, the Tech giants Amazon, Apple, Facebook, Google and Microsoft have acquired at least 770 firms since their foundation.[1] Moreover, even though tech acquisitions have often captured the headlines, startup acquisitions are common in many other industries as well.

Such acquisitions are viewed with increasing scepticism by regulators. In the United States, the Federal Trade Commission (FTC) announced an inquiry into several high-profile cases in 2020, and subsequently filed lawsuits against Facebook and Google.[2] While these inquiries mainly focus on competition and prices, regulators and academics have also become increasingly interested in the effects of startup acquisitions on innovation. These effects are not obvious a priori. On the one hand, acquirers may choose to sideline startup innovations that threaten their existing business, and this could slow down aggregate productivity growth. Frequent acquisitions may also create a rigid economy without turnover, where incumbents have little incentives to innovate themselves. On the other hand, the prospect of being acquired may stimulate startup creation, and actual acquisitions may improve the allocation of ideas between firms. These forces could accelerate aggregate productivity growth.

In this paper, we aim to assess the relative strength of these forces. To do so, we develop a general equilibrium endogenous growth model with heterogeneous firms and acquisitions. We discipline the model by matching empirical evidence from a new panel data set on acquisitions and patenting in the United States. In a preliminary calibration, we find that there is an inverted U-shape relationship between the frequency of acquisitions and the aggregate growth rate. The current frequency of acquisitions is somewhat below the growth-maximizing level.

Our model embeds the trade-offs implied by startup acquisition into a general equilibrium framework with endogenous growth and creative destruction. Production is carried out by a continuum of firms producing differentiated products. Firms operate under Bertrand competition, which implies that their profits are increasing in their productivity advantage over their closest competitor. Thus, firms have an incentive to invest into Research and Development (R&D) to increase this productivity advantage. Incumbent firms coexist with a continuum of startups which do not produce, but do invest into R&D. When a startup

[1] See https://www.cbinsights.com/research/tech-giants-billion-dollar-acquisitions-infographic/. Between 2015 and 2017 alone, these five firms did 175 acquisitions (Gautier and Lamesch, 2020).

[2] See https://www.ftc.gov/news-events/press-releases/2020/02/ftc-examine-past-acquisitions-large-technology-companies. On the basis of this inquiry, the FTC has filed lawsuits against Google in October 2020 and Facebook in December 2020.

successfully develops an innovation and implements it, it enters the market and thereby displaces an incumbent. However, if the incumbent "notices" the threatening startup, it can propose an acquisition to avoid displacement. The probability that an incumbent notices a threatening startup is endogenous, and depends on the amount of effort that incumbents spend on monitoring the startup scene.

The equilibrium of the model captures the different effects of acquisitions on innovation and economic growth discussed above. On the one hand, startup acquisitions could reduce aggregate innovation for two reasons. First, as the incumbent is already operating, it has an incentive to acquire the startup in order to protect its existing profits. However, existing profits also imply that its marginal benefit from implementing an innovation is smaller than the one of a startup (the classical Arrow (1962) replacement effect). Thus, when an acquisition occurs, the incumbent may not find it worthwhile to pay the implementation cost of the startup's innovation. In that case, the acquisition has effectively eliminated an innovation. Cunningham, Ederer and Ma (2020) label these events "killer acquisitions", and show that US pharmaceutical firms are likely to stop drug research projects of acquired companies when these overlap with their own drug portfolio. Their empirical results suggest (in line with our model) that killer acquisitions are more likely if incumbents have a dominant market position, as this simultaneously increases the profits that they want to protect and lowers their marginal benefit from innovation.

A second negative effect from acquisitions is that they slow down creative destruction, allowing incumbents to avoid displacement. All else equal, this creates an economy dominated by entrenched incumbents which have high productivity advantages over their competitors. In this environment, innovation incentives for incumbents are low.

However, startup acquisitions might also have positive effects. First, our model allows for differences in the implementation costs of innovations between incumbents and startups. When costs are lower for incumbents (due to economies of scale and scope, a larger customer base, greater business experience, etc.), acquisitions transfer innovations to more efficient users.[3] Second, the prospect of an acquisition provides an incentive for startups to invest in R&D in the first place. In our model, the acquisition price for a startup always exceeds its outside option of independent entry. Thus, all else equal, incentives for

[3] Indeed, there may be a beneficial division of labor between startups and incumbents, with startups specializing in generating ideas and incumbents specializing in implementing them. Time Magazine describes some of the issues involved in the case of the startup Nest and its founder Tony Fadell. "As Nest grew, so did Fadell's logistical headaches. By 2013, he says he was spending 90% of his time on [...] managing finances, talking to investors, wrestling with taxes and fending off patent lawsuits. [...] When Google came knocking again, offering a big payday and the chance to keep Nest's name brand intact—a key requirement for Fadell—an acquisition seemed more appealing. Now Fadell says he spends 95% of his time focused on product development and key relationships" (see https://time.com/3815612/silicon-valley-acquisition).

startup creation are higher in the presence of acquisitions. In the business world, many commentators see acquisitions as a natural outcome for startups, and numerous guides advise entrepreneurs how to position their startup in order to be acquired.[4]

Finally, acquisitions reallocate employees and researchers. The extent of reallocation varies widely: Cunningham *et al.* (2020) show that in the pharmaceutical industry, only 22% of researchers keep working for the acquirer, but this number is three times as high at Google.[5] Reallocation may change the productivity of the affected researchers (and their colleagues) through knowledge spillovers. To capture these effects, our model assumes that incumbents are endowed with knowledge capital, and that this capital changes after acquisitions, regardless of whether the startup's ideas are implemented by the acquirer.

We relate the patterns emerging from our model to empirical evidence from a rich micro-level dataset that combines information on acquisitions (from the ThomsonONE M&A database), patents (from the NBER patent data project) and publicly traded firms (from Compustat). In line with our model, our empirical analysis focuses on startup acquisitions, which we proxy as acquisitions of firms that are not publicly listed. In this sample, we document three stylized facts. First, on average, patenting of the acquiring firm increases after an acquisition. Second, we find that this effect is lower for acquirers that are already large in the patent space. Third, the positive effect on patenting increases in the technological proximity between acquirer and target, which we measure based on Bloom, Schankerman and Van Reenen (2013).

We discipline the model by matching aggregate moments, as well as some key moments from our micro-level data (including the coefficients from our empirical regressions). A preliminary calibration fits the data well. In our main quantitative exercise, we vary the cost of incumbents for noticing threatening startups. These costs can be seen as a reduced-form indicator of frictions in the acquisition market. When they are zero, incumbents may acquire any startup that threatens them, if they find it optimal to do so. When they are infinitely high, there are no acquisitions.

Our model suggests an inverted U-shape relationship between the frequency of acquisitions and economic growth. When acquisitions are very costly, entry is high, and the economy is mainly composed by low-profit firms with little market power. Lowering acquisition frictions lowers the entry rate, as some startups now get acquired. However,

[4]For some examples, see (1) https://www.forbes.com/sites/alejandrocremades/2019/08/02/how-to-get-your-business-acquired, (2) https://www.inc.com/john-boitnott/how-to-boost-your-businesss-odds-of-an-acquisition or (3) https://thinkgrowth.org/how-to-build-a-startup-that-gets-acquired-85ada592bfd7.

[5]See https://time.com/3815612/silicon-valley-acquisition. Tech companies have even coined the term "acqui-hire", with Facebook's CEO Marc Zuckerberg stating that “we have not once bought a company for the company. We buy companies to get excellent people” (https://www.youtube.com/watch?v=OlBDyItD0Ak).

this is more than compensated by a higher startup rate, because most incumbents choose to implement the ideas of acquired startups. Moreover, acquisitions increase the expected lifespan of incumbents. This reduces the rate at which these firms discount future profits and increases their own innovation incentives. Further decreases in acquisition frictions, however, lead to lower innovation and growth rates. Now, almost all startups are acquired. Thus, exit becomes unlikely, and the average incumbent has a high productivity advantage over its competitors. Such an entrenched firm has low incentives to innovate, but high incentives to acquire a threatening startup. Thus, all acquisitions become killer acquisitions and growth falls to close to zero. Note, however, that this economy has a high R&D share and a vibrant startup scene. In fact, startups have strong incentives to threaten high-profit incumbents and extract high acquisition prices, even though none of their ideas are actually implemented.

Our preliminary calibration suggests that the US economy is currently slightly below the growth-maximizing level of the meeting probability. However, a substantial increase in the frequency of acquisitions could drastically lower aggregate productivity growth.

Related literature There is a growing empirical literature on the effect of acquisitions on innovation. Most importantly, the influential work of Cunningham *et al.* (2020) for the US pharmaceutical industry provides evidence for several of the channels discussed above. The authors show that drug research projects of acquired companies are more likely to be shut down if the acquirer has overlapping projects. This finding is driven by drugs for which there is little competition, and there is no evidence for positive knowledge spillovers (e.g., through patent citations) from the acquired firm to the acquirer. We build on their results by developing a macroeconomic model to assess the aggregate effect of acquisitions. This allows us to take into account general equilibrium spillovers, as well as additional channels not considered in their paper (e.g., changing incentives for startup creation).

In earlier studies, Seru (2014) and Haucap, Rasch and Stiebale (2019) also provide evidence for a negative effect of mergers and acquisitions (M&As) on firm R&D. Phillips and Zhdanov (2013) instead argue that acquisitions stimulate innovation by small firms that want to be acquired in the future. Using data on publicly traded firms, they show that the R&D of small firms increases after an industry-level acquisition shock. Bena and Li (2014) provide evidence for positive knowledge spillovers after mergers, while Kim (2020) shows that employee mobility after acquisitions can be detrimental to the acquirer in the long run.

On the theoretical side, there has been an intense interest in the industrial organization literature on the effect of acquisitions (of startups or other competitors) on the level and direction of innovation (see Federico, Langus and Valletti, 2017; Cabral, 2018; Bourreau, Jullien and Lefouili, 2018; Bryan and Hovenkamp, 2020; Callander and Matouschek, 2020; Fumagalli, Motta and Tarantino, 2020; Kamepalli, Rajan and Zingales, 2020; Letina, Schmutzler and Seibel, 2020). These studies are based on partial equilibrium models, while we take an aggregate general equilibrium perspective.

There are also some recent quantitative studies on the macroeconomic effect of M&As. For instance, Dimopoulos and Sacchetto (2017) and David (2020) analyze the effects of M&As on static outcomes such as the allocation of capital, but do not consider innovation and productivity growth.[6] More closely related to us, Cavenaile, Celik and Tian (2020) develop a Schumpeterian endogenous growth model with mergers between incumbent firms, and analyze the effect of M&As on the innovation incentives of incumbents. Our focus is different, as we study the acquisition of startups by incumbents, leading us to consider novel issues such as killer acquisitions or knowledge spillovers.[7] Finally, Lentz and Mortensen (2016) and Akcigit, Celik and Greenwood (2016) incorporate different versions of a "market for ideas" (through buyouts or patent sales) in endogenous growth models, showing that such a market improves the allocation of ideas across firms. More broadly, we contribute to the literature on endogenous growth and firm dynamics (see Klette and Kortum, 2004; Akcigit and Kerr, 2018; Peters, 2020), by extending its standard framework to incorporate acquisitions and study their macroeconomic impact.

The remainder of the paper is organized as follows. Section 2 presents some stylized facts on M&A and innovation in the US, based on a new micro-level dataset. Section 3 presents our model environment and characterizes the Balanced Growth Path equilibrium. Section 4 discusses the calibration and our main counterfactual experiment. We conclude in Section 5.

2 Data and empirical evidence

In this section, we first introduce the datasets and the merging exercise between the firms involved in acquisitions and their patenting activity. We then present descriptive statistics on the number and value of transactions over time and show the evolution of patent-owning target firms over time. Finally, we will discuss the three main stylized facts.

[6]There is also an extensive literature on the microeconomic effects of M&As on investment, the allocation of capital, firm productivity and competition. Important studies include Jovanovic and Rousseau (2002), Rhodes-Kropf and Robinson (2008), Blonigen and Pierce (2016), and Wollmann (2019). Some studies have also considered startup acquisitions in particular. For instance, Andersson and Xiao (2016) document a number of stylized facts on startup acquisition in Sweden.

[7]Our paper is also related to Celik, Tian and Wang (2020), who study the effects of information frictions in the merger market on firm innovation and business dynamism.

2.1 Data

2.1.1 Acquisitions

The M&A data comes from ThomsonONE. It provides transaction-level merger data and includes practically all M&As deals between US firms during 1981 and 2014. This dataset provides several variables of interest like the company name, the industry to which the firms belong, the announced and effective dates of the deal, or the transaction value. Revenue and total assets are also sometimes available.

An important variable is whether the firm taking part in the deal is publicly traded or privately owned. For public firms, Thomson ONE includes a firm identifier that allows for an easy merge with Compustat and patents data. For private firms, this firm identifier is missing. Consequently, we need to conduct a multi-step cleaning process of private company names before merging it with the patents data.

2.1.2 NBER Patents Data Project

The patents dataset used in this paper is the NBER Patents Data Project (NBER-PDP), which includes US patent data for 1976-2006.[8] For publicly listed firms, it includes an assignee identifier for a straightforward match to Compustat. In addition to the patent owner, this dataset also provides us with the forward and backward citations, each patent's originality and generality, and IPC technology classes.

A challenge in matching firm-level data to patents is that firm names are inconsistently recorded on patent files, which leads to many false negative matches. There are two reasons for this: first, the lack of a unique firm identifier in the patent data; second, the lack of uniformity in how company names appear. To address this problem, the NBER-PDP standardizes commonly used words in firm names (Bessen, 2009).

2.1.3 Merging patents to acquisitions

The matching procedure between the acquisitions database and the patents database depends on the nature of the firm. For *public* firms, ThompsonONE provides a firm identifier that can be readily matched to NBER-PDP using Compustat as a bridge file in a fairly straightforward exercise. A bigger challenge is the match between patents and the *private* firms. We proceed as follows. ThompsonONE provides us with a company name which we standardize. We then employ a fuzzy name matching algorithm and a large scale manual

[8]Source: https://sites.google.com/site/patentdataproject/.

check. As our focus is on acquisitions of private firms, we have only implemented this matching for private target firms so far.

2.2 Descriptive Statistics

Table A1 in the Appendix displays the number of M&A transactions taking place between 1980 and 2006, decomposed by whether the acquirer or target firms are public entities or not. Approximately half of the transactions involve a public acquirer, and only around 15% of targets are public.

Figure A1 in the Appendix displays the number of M&A transactions per year and its average transaction value. The observed cyclicality is consistent with previous literature documenting the fact that certain periods experience a boom in acquisitions and sectoral restructurings. This is what we observe in the graph during the late 1980s and the dot-com bubble around the year 2000. The value of these transactions has steadily being rising except for a few years during the 2000s. By 2006, the average transaction value was 400 million US dollars. Figure A2 in the Appendix shows the fraction of acquirers and targets with a positive stock of patents at that time of the M&A during the years 1980-2006. For acquirers, this fraction has gone down from around 20% to 13%. The downward trend has been more pronounced for the target companies; while during the mid-1980s this fraction was around 20%, it has fallen to 4% by the mid-2000. Finally, Figure A3 in the Appendix shows the evolution in the number of patents granted to companies involved in M&A transactions, conditional on these firms having a positive stock of patents at the time of the transaction. We see a strong upward trend over the years reaching a point where both acquirer and target have an average stock of patents of 3000 by the year 2006.

2.3 Empirical Evidence

Stylized Fact 1: The acquirer's patenting activity rises after an acquisition.

Table 1 illustrates the impact of an acquisition on the acquirer's patenting activity. All regressions control for firm and year fixed effects. In Column (1), our regressor of interest, $TreatmentObs$, is a dummy that takes value 1 only in the precise years in which the acquisition takes place. In Column (2), we include all acquisitions and the dummy $Post(All)$ takes value 1 for the actual acquisition year (year = t) and the four subsequent years (until year = $t + 4$). This allows us to evaluate whether the impact of the acquisition is long-lasting. In Column (3) we reestimate the previous column after limiting our sample of acquisitions to the subset where the acquirer does not participate in any other acquisition between years $t-4$ and $t+4$. We do this to verify whether results go through in a cleaner sample that is not contaminated by multiple M&A in a short spell of time. As we only keep a subset of acquisitions, the sample shrinks to 64,682 observations. Finally, in Column (4) we run an event study specification. For each acquisition, we only consider the precise 9-year window that goes from 4 years prior to 4 years after the acquisition takes place. This explains the substantial reduction in sample size to 6,504 observations. Throughout this table we find that acquirers experience an increase in the cumulative number of patents granted in the years immediately after they acquire a private target firm.

Table 1: Stylized Fact 1 - Effects of Acquisitions on the Acquirer's Patenting Activity

	(1)	(2)	(3)	(4)
Treatment Obs	0.049***			
	(0.007)			
Post (All)		0.094***		
		(0.010)		
Post (Clean Subset)			0.049***	0.033*
			(0.014)	(0.017)
Constant	-0.006***	-0.028***	-0.058***	0.146***
	(0.001)	(0.003)	(0.001)	(0.009)
Observations	73,050	73,050	64,682	6,504
Year FE	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes
R-squared	0.918	0.918	0.915	0.960

Notes: The dependent variable is the cumulative number of patents (standardized). ***/**/* indicate significance at the 1%/5%/10% level. Standard errors are clustered at the firm and year level.

Table A2 in the Appendix repeats the same structure as Table 1 with a different dependent variable: the acquirer's cumulative *cite-weighted* patents obtained per year. We do this to test the concern that the additional patents granted could just be of marginal importance. Throughout the four columns, we find a quality-weighted increase in patenting, supporting the claim that these patents are indeed making a scientific impact.

Stylized Fact 2: The acquirer's rise in patenting activity diminishes with its share of patents in the industry.

Table 2 presents results with our preferred specification, i.e. the event study sample in which we keep a 9-year window for each acquisition. We test whether the increase in patenting activity by the acquirer after an acquisition diminishes with its market share in the innovation market. The regressor of interest is the interaction term between a dummy

Table 2: Stylized Fact 2 - The Role of the Acquirer's Share of Patents in its Industry

	(1)	(2)	(3)	(4)
Post	0.178***	0.029*	0.027	0.027
	(0.030)	(0.017)	(0.017)	(0.017)
Patent Share	0.458***	0.124***	0.124***	0.124***
	(0.044)	(0.028)	(0.028)	(0.028)
Post * Patent Share	-0.036	-0.021***	-0.021***	-0.022***
	(0.030)	(0.008)	(0.008)	(0.008)
Target log of Cum. Patents				0.012
				(0.008)
Target Dummy for Patenting			0.022	-0.001
			(0.015)	(0.019)
Constant	0.025	0.151***	0.152***	0.152***
	(0.037)	(0.011)	(0.011)	(0.011)
Observations	6,366	6,365	6,365	6,365
Year FE	Yes	Yes	Yes	Yes
Sector FE	Yes	No	No	No
Firm FE	No	Yes	Yes	Yes
R-squared	0.288	0.962	0.962	0.962

Notes: The dependent variable is the cumulative number of patents (standardized). The variable *Patent Share* has also been standardized. ***/**/* indicate significance at the 1%/5%/10% level. Standard errors are clustered at the firm and year level.

Post taking value 1 for the 4 years after the acquisition takes place and the share of patents owned by the acquirer in the 4-digit SIC industry.

In Column (1) we control for sector fixed effects and year fixed effects. In Column (2) we instead control for firm fixed effects in addition to year dummies. In Column (3) we also include a dummy taking value 1 if the target firm has a positive patent stock. Column (4) additionally includes the target's log of its accumulated patents until the time it is acquired. In all specifications we find a negative estimated coefficient for the interaction term introduced previously: the larger the market share of the acquirer, the smaller the increase in patenting activity by the acquirer.

Table A3 reruns Table 2 with the acquirer's cumulative *cite-weighted* patents obtained per year as a dependent variable. Results are very much aligned with the findings in the previous table as the acquirer's quality-adjusting patenting activity experiences larger increases when the acquirer does not have a large share in the sectoral innovation market.

Stylized Fact 3: The acquirer's rise in patenting activity rises with the technologi-

Table 3: Stylized Fact 3 - The Role of Technological Proximity Between Acquirer and Target

	(1)	(2)	(3)	(4)	(5)	(6)
Post	0.127***	0.192***	0.208***	0.051**	0.058***	0.032
	(0.033)	(0.032)	(0.027)	(0.019)	(0.019)	(0.023)
Post * TechProximity	0.143***	0.144***	0.140***	0.023***	0.022**	0.008
	(0.025)	(0.025)	(0.025)	(0.008)	(0.009)	(0.009)
Constant	-0.007	-0.078***	-0.169***	-0.002**	-0.069***	-0.063***
	(0.012)	(0.011)	(0.034)	(0.001)	(0.001)	(0.013)
Observations	69,576	59,742	6,106	69,494	59,635	6,106
R-squared	0.259	0.265	0.186	0.872	0.868	0.927
Sector FE	Yes	Yes	Yes	No	No	No
Firm FE	No	No	No	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes

Notes: The dependent variable is the cumulative number of patents (standardized). The variable *TechProximity* has also been standardized. ***/**/* indicate significance at the 1%/5%/10% level. Standard errors are clustered at the firm and year level.

cal proximity to the target firm acquired.

Table 3 estimates whether the acquirer increases its quality-adjusted patenting especially if it acquires target firms that are close in the technological space. With this we mean that both the acquirer and the target firm patent in similar technological classes, defined as in Bloom *et al.* (2013). Columns (1)-(3) include 2-digit sector fixed effects and year fixed effects. Columns (4)-(6) control for firm fixed effects and year fixed effects. Columns (1) and (4) are our full sample; Columns (2) and (5) limit our sample of acquisitions to the subset where the acquirer does not participate in any other acquisition between years $t-4$ and $t+4$; finally, Columns (3) and (6) are the 'event study' sample. The overall picture that emerges is that acquirers increase their quality-adjusted patenting especially if they acquire target firms with which they have strong technological linkages.

3 Model

The stylized facts presented above show that acquisitions have effects on the patenting of incumbent firms. We next present a model that systematically investigates these linkages, and which is calibrated to match some of the facts presented above. The basic structure of the model is close to the Schumpeterian growth model of Peters (2020), but we augment this framework by including startups, acquisitions, and knowledge capital spillovers.

3.1 Environment

Preferences and technology Time is continuous, runs forever and is indexed by $t \in \mathbb{R}_+$. A representative consumer maximizes lifetime utility, given by

$$U = \int_0^{+\infty} e^{-\rho t} \ln(C_t)\, dt, \tag{1}$$

where $\rho > 0$ is the time discount rate and C_t stands for the consumption of the unique final good at instant t. The price of the final good is normalized to one. The household is endowed with L units of time, which are supplied inelastically to producers at the market-clearing wage w_t. The household owns all firms in the economy and accumulates wealth $\mathcal{A}_t$, equal to the value of all corporate assets and evolving according to the budget constraint $\dot{\mathcal{A}}_t = r_t \mathcal{A}_t + w_t L - C_t$, where r_t is the rate of return on assets.

The final good is produced under perfect competition, and assembled from a continuum of intermediate goods with a Cobb-Douglas production function:

$$Y_t = \exp\left(\int_0^1 \ln(y_{it})\, di\right), \tag{2}$$

where y_{it} is the output of intermediate good i produced at instant t. Each intermediate can a priori be produced by a large number of firms j, with a linear production technology using only labor:

$$y_{ijt} = q_{ijt} l_{ijt}, \tag{3}$$

where y_{ijt} is the output of intermediate i by firm j at instant t, and q_{ijt} is the productivity of firm j for intermediate i. We assume that there is Bertrand competition on all intermediate markets. Therefore, in equilibrium, each intermediate is only produced by by the highest-productivity firm. We denote the productivity of this firm by q_{it}. This productivity can be improved through innovations, generated either by the current producer of intermediate i (the incumbent) or by startups.

Incumbent R&D and innovation Incumbents can generate innovations at a Poisson rate z when paying a cost of

$$R(z; k_{it}) = \xi_I (k_{it})^{-\beta} (z)^{\psi}\, Y_t \tag{4}$$

units of the final good, where ξ_I and β are positive parameters, and $\psi > 1$. The variable k_{it} stands for the knowledge capital of the incumbent. Knowledge capital represents, roughly speaking, the productivity of the incumbent's research team: higher knowledge capital shifts the R&D cost function down. The elasticity of R&D costs to knowledge capital is given by the parameter β. Knowledge capital can be accumulated through acquisitions, as we describe later.

When the incumbent firm makes an innovation, it can implement it by paying a one-off implementation cost $\kappa_I Y_t$, where $\kappa_I > 0$ is a parameter. An implemented innovation increases the incumbents productivity by a factor of $\lambda > 1$.

Startups At each moment in time, there is also a large mass of potential entrants, which we refer to as *startups*. Startups belong to different technology classes, indexed by $\tau_E \in \mathbb{T} \equiv \{1, 2, .., \tau_{max}\}$. In each technology class τ_E, startups can generate a Poisson arrival rate x of innovations by paying a cost of

$$R_E(x) = \xi_E x^{\psi} Y_t \tag{5}$$

units of the final good, where ξ_E is a positive constant. A startup's innovation applies to a randomly drawn good from the interval $[0, 1]$. Just like innovations by incumbents, innovations by startups increase the leading productivity of a given good by a factor of λ if they are implemented.

When a startup generates an innovation, it can carry it out at an implementation cost $\kappa_E Y_t$, where $\kappa_E > 0$. In that case, the startup enters, displaces the incumbent and becomes the new sole producer of the good. We assume that this new incumbent starts out with knowledge capital $k = 1$. However, the startup may also be acquired by the incumbent. We describe this option in greater detail next.

Acquisitions Incumbents must exert costly effort to find startups to acquire in the product market. In particular, an incumbent must choose the probability $\gamma \in [0, 1]$ with which, conditional on a startup appearing in its product market, the incumbent can make an offer to acquire it. This search effort carries a cost of

$$S(\gamma) = \chi \gamma^{\phi} Y_t \tag{6}$$

units of the final good, where χ is a positive parameter and $\phi > 1$. In the event of a meeting, the incumbent gets a chance to make the startup an acquisition offer. If incumbent and startup do not meet, the incumbent gets displaced.

When an acquisition takes place, the incumbent pays a price p_{it}^A to the startup, and the startup exits forever. The acquisition price is determined through Nash bargaining, where the incumbent's bargaining power is given by $\alpha \in (0,1)$. Acquisitions allow the incumbent to avoid displacement. However, this is not their only effect. First, an acquisition also gives the incumbent the property rights to the startup's innovation, which it can implement by paying the cost $\kappa_I Y_t$. When a startup's innovation is implemented, we speak of an "innovative acquisition". When it is not implemented, we instead speak of a "killer acquisition". Second, the acquisition change the incumbent's knowledge capital. The new knowledge capital of the incumbent is given by $k' = \mathcal{K}(k, \tau, \tau_E)$, where $\mathcal{K}$ is a function of the incumbent's previous knowledge capital k, and the technology classes of incumbent (τ) and startup (τ_E).

3.2 Equilibrium

Throughout, we consider a balanced growth path (BGP) equilibrium with entry, in which aggregate consumption and aggregate output grow at a constant rate g and entry rates in all technology classes are positive.

Household's and production decisions On the BGP, the representative consumer's optimal consumption choice satisfies the Euler equation

$$\frac{\dot{C}_t}{C_t} \equiv g = r - \rho. \tag{7}$$

The demand for each intermediate good is given by $y_{it} = \frac{Y_t}{p_{it}}$. Bertrand competition implies limit pricing behavior by firms. That is, the firm with the highest productivity for a given good i charges a price

$$p_{it} = \frac{w_t}{q_{it}^F}, \tag{8}$$

where q_{it}^F is the productivity of the firm with the second-highest productivity for the good. As productivity evolves on a ladder (being improved by fixed steps of size λ), we can summarize the distance between the current producer and its closest follower by the technology gap $n_{it} \in \mathbb{N}$, holding $\lambda^{n_{it}} \equiv \frac{q_{it}}{q_{it}^F}$. By equation (8), this means that the firm sets a markup λ^n over its marginal cost of production, $\frac{w_t}{q_{it}}$. This pricing choice implies that labor demand equals $l_{it} = \frac{Y_t}{w_t}\lambda^{-n_{it}}$ and profits are

$$\Pi_{it} = \left(1 - \lambda^{-n_{it}}\right) Y_t. \tag{9}$$

That is, profits are an increasing and concave function of the technology gap n. This implies that firms have an incentive to innovate (either through in-house R&D or through the acquisition of startups) in order to increase their technological advantage and their profits. However, this innovation incentive decreases as the firm gets further ahead of its follower, as the marginal gains from innovation become smaller and smaller.

R&D and acquisition decisions The incumbent producer of any intermediate i chooses its optimal R&D spending and optimal search effort, and decides whether or not to try to acquire a startup if it gets the opportunity to do so. The dynamic problem of the incumbent has three state variables: the current technology gap n, the current knowledge capital k, and the incumbent's technology class τ. We denote the incumbent's value function by $V_{\tau,t}(n,k)$.

On the BGP, the Hamilton-Jacobi-Bellman equation of the dynamic problem is

$$\begin{aligned} rV_{\tau,t}(n,k) - \dot{V}_{\tau,t}(n,k) = \max_{z_\tau(n,k),\gamma_\tau(n,k)} \Bigg\{ & \underbrace{\left(1-\lambda^{-n}\right) Y_t}_{\text{Profits}} - \underbrace{\xi_I k^{-\beta} \left(z_\tau(n,k)\right)^{\psi} Y_t}_{\text{R\&D spending}} - \underbrace{\chi \left(\gamma_\tau(n,k)\right)^{\phi} Y_t}_{\text{Search effort}} \\ & + \underbrace{z_\tau(n,k) \left[\max\Big(V_{\tau,t}(n+1,k) - \kappa_I Y_t - V_{\tau,t}(n,k), 0 \Big) \right]}_{\text{Own innovation}} \\ & + \underbrace{\sum_{\tau_E \in \mathbb{T}} x(\tau_E) \left[\gamma_\tau(n,k) \widehat{V}_{\tau,t}(n,k,\tau_E) - V_{\tau,t}(n,k) \right]}_{\text{Startup appears}} \Bigg\}. \end{aligned} \tag{10}$$

The first line on the right-hand side of this equation collects instant flows, namely the profits earned by the incumbent and its spending on R&D and search efforts. The second line refers to the incumbent's own innovations, which arrive at a rate $z_\tau(n,k)$. When an innovation is implemented, it increases the technology gap by one unit and entails an implementation cost $\kappa_I Y_t$. Otherwise, the incumbent's value remains unchanged. Finally, the third line refers to the arrival of a startup. For each technology class $\tau_E \in \mathbb{T}$, the incumbent faces an arrival rate $x(\tau_E)$ of startups that have developed an innovation on its product. With probability $\gamma_\tau(n,k)$, the incumbent gets the opportunity to acquire the startup, in which case its continuation value is given by $\widehat{V}_{\tau,t}(n,k,\tau_E)$. When the incumbent does not meet the startup, it is displaced forever and earns a value of zero.

To derive the continuation value $\widehat{V}_{\tau,t}(n,k,\tau_E)$, note that conditional on an acquisition, the incumbent implements the startup's innovation if, and only if:

$$V_{\tau,t}(n+1,k') - \kappa_I Y_t \geq V_{\tau,t}(n,k'), \text{ with } k' = \mathcal{K}(k,\tau,\tau_E). \tag{11}$$

Implementing an innovation increases the technology gap by one unit, but has an implementation cost $\kappa_I Y_t$. Thus, the incumbent implements the idea of the startup if its marginal benefit exceeds its cost. Given this, we can derive an expression for the total surplus generated by an acquisition, denoted $\Sigma_{\tau,t}(n,k,\tau_E)$ and given by:

$$\Sigma_{\tau,t}(n,k,\tau_E) = \underbrace{\max\Big(V_{\tau,t}(n+1,k') - \kappa_I Y_t, V_{\tau,t}(n,k')\Big)}_{\text{Joint value after acquisition}} - \underbrace{0 - \Big(V_{\tau_E,t}(1,1) - \kappa_E Y_t\Big)}_{\text{Outside options}} \tag{12}$$

After an acquisition, the incumbent has knowledge capital k' and can choose whether to implement the startup's innovation. The incumbent transfers this acquisition price to the startup (as the acquisition price is a transfer, it does not appear in the expression for the joint surplus). The incumbent's outside option is zero: if it does not buy the startup, it is displaced. The startup's outside option, instead, is to enter, in which case it becomes an incumbent with technology gap $n = 1$ and knowledge capital $k = 1$.[9]

The incumbent makes an offer to acquire the startup if, and only if, the surplus from the acquisition is positive. The price paid to the startup is then the sum of the startup's outside option and a fraction $(1-\alpha)$, the startup's Nash bargaining weight, of the surplus:

$$p_t^{\mathcal{A}}(n,k,\tau,\tau_E) = V_{\tau_E,t}(1,1) - \kappa_E Y_t + (1-\alpha)\,\Sigma_{\tau,t}(n,k,\tau_E). \tag{13}$$

Therefore, we can finally write

$$\widehat{V}_{\tau,t}(n,k,\tau_E) = \max\Big(\alpha\Sigma_{\tau,t}(n,k,\tau_E), 0\Big). \tag{14}$$

When the surplus is negative, the incumbent does not make an offer, and its continuation value is zero. When the surplus is positive instead, the incumbent makes an offer and obtains a share α of the total surplus.

On the BGP, we can guess and verify that firm value holds $V_{\tau,t}(n,k) = v_\tau(n,k)Y_t$. Using the Euler equation (7), we can then rewrite equation (10) as:

[9] These outside options apply if the startup would find it profitable to enter, i.e., if $V_{\tau_E,t}(1,1) > \kappa_E Y_t$. As we consider an equilibrium with positive entry in each technology class, this condition must always hold.

$$\rho v_\tau(n,k) = \max_{z_\tau(n,k),\gamma_\tau(n,k)} \left\{ \left(1-\lambda^{-n}\right) - \xi_I k^{-\beta}\left(z_\tau(n,k)\right)^\psi - \chi\left(\gamma_\tau(n,k)\right)^\phi \right.$$
$$+ z_\tau(n,k) \max\Big(v_\tau(n+1,k) - \kappa_I - v_\tau(n,k), 0\Big)$$
$$\left. + \sum_{\tau_E \in \mathbb{T}} x(\tau_E)\Big[\gamma_\tau(n,k) \max\Big(\alpha\sigma_\tau(n,k,\tau_E), 0\Big) - v_\tau(n,k)\Big] \right\}. \tag{15}$$

where $\sigma_\tau(n,k,\tau_E) \equiv \frac{\Sigma_{\tau,t}(n,k,\tau_E)}{Y_t}$. The optimal level of incumbent innovation holds:

$$z_\tau(n,k) = k^{\frac{\beta}{\psi-1}}\left[\frac{1}{\xi_I\psi}\max\Big(v_\tau(n+1,k) - \kappa_I - v_\tau(n,k), 0\Big)\right]^{\frac{1}{\psi-1}}. \tag{16}$$

whereas the optimal search effort holds:[10]

$$\gamma_\tau(n,k) = \left[\frac{1}{\chi\phi}\sum_{\tau_E\in\mathbb{T}} x(\tau_E)\max\Big(\alpha\sigma_\tau(n,k,\tau_E), 0\Big)\right]^{\frac{1}{\phi-1}}. \tag{17}$$

Acquisition and implementation decisions (conditional on the incumbent getting the opportunity to acquire the startup) can in turn be summarized by the following policy functions:

$$\begin{aligned} I_A(n,k,\tau,\tau_E) &= \mathbb{1}\left[\sigma_\tau(n,k,\tau_E) > 0\right] \\ I_I(n,k,\tau,\tau_E) &= \mathbb{1}\left[v_\tau(n+1,k') - \kappa_I > v_\tau(n,k')\right] \end{aligned} \tag{18}$$

where $\mathbb{1}[.]$ denotes an indicator variable, $I_A \in \{0,1\}$ describes the acquisition decision, and $I_I \in \{0,1\}$ describes the implementation decision. Thus, conditional on getting the opportunity to acquire a startup, the incumbent carries out an innovative acquisition if $I_{AI} \equiv I_A \cdot I_I$ is equal to 1, and a killer acquisition if $I_{AK} \equiv I_A \cdot (1 - I_I)$ is equal to 1.

Startup decisions In each technology class τ_E, the representative startup solves the problem

[10] As γ is a probability, in practice we will focus on those combinations of parameter values for which $\gamma_\tau(n,k) \leq 1$, $\forall(n,k,\tau)$.

$$\max_{x(\tau_E)} \Bigg\{ -\xi_E \left(x\left(\tau_E\right)\right)^{\psi} Y_t + x\left(\tau_E\right) \Bigg[V_{\tau_E,t}(1,1) - \kappa_E Y_t \tag{19}$$

$$+ \sum_{\tau}\sum_{n}\sum_{k} \varphi_\tau(n,k)\gamma_\tau(n,k) \max\Big(\left(1-\alpha\right) \Sigma_{\tau,t}\left(n,k,\tau_E\right), 0 \Big) \Bigg] \Bigg\}.$$

where $\varphi_\tau(n,k)$ is the mass of incumbents of technology class τ with technology gap n and knowledge capital k. When the startup generates an innovation, it always receives at least its outside option. However, with probability $\gamma_\tau(n,k)$ it meets the incumbent and may even get a higher value if the incumbent is willing to acquire it. This "acquisition premium" depends on the bargaining weight of the startup, and on the characteristics of the incumbent owning the product to which the startup's innovation applies. Thus, the expected value of the acquisition premium depends on the distribution of incumbents across the (n,k,τ) space. On the BGP, this distribution is invariant over time, and we derive it in Appendix A.1.

On the BGP, the optimal startup rate in technology class τ_E is:

$$x(\tau_E) = \left[\frac{1}{\xi_E \psi} \left(v_{\tau_E}(1,1) - \kappa_E + \sum_{\tau}\sum_{n}\sum_{k} \varphi_\tau(n,k)\gamma_\tau(n,k) \max\Big(\left(1-\alpha\right) \sigma_\tau\left(n,k,\tau_E\right), 0 \Big) \right) \right]^{\frac{1}{\psi-1}} \tag{20}$$

Growth rate In equilibrium, innovations implemented by incumbents or successful entrants may contribute to aggregate growth depending on whether startups are bought up or not. In particular, the growth rate of aggregate output is given by:

$$g = \ln(\lambda) \sum_{\tau}\sum_{n}\sum_{k} \varphi_\tau(n,k) i_\tau(n,k) \tag{21}$$

where

$$i_\tau(n,k) \equiv \underbrace{z_\tau(n,k)}_{\substack{\text{Incumbent} \\ \text{innovation}}} + \underbrace{\sum_{\tau_E} x(\tau_E)\gamma_\tau(n,k) I_{AI}(n,k,\tau,\tau_E)}_{\substack{\text{Innovations by startups} \\ \text{implemented by incumbents}}}$$

$$+\underbrace{\sum_{\tau_E} x(\tau_E)\Big[\big(1-\gamma_\tau(n,k)\big)+\gamma_\tau(n,k)\Big(1-I_A(n,k,\tau,\tau_E)\Big)\Big]}_{\substack{\text{Innovations created and} \\ \text{implemented by startups}}} \tag{22}$$

is the aggregate innovation rate in state (n,k,τ). We formally derive equation (21) in Appendix A.2. The growth rate of the economy is the product of the log innovation step size and the aggregate innovation rate. For each state (n,k,τ), total innovation is composed of innovation by incumbents, innovation by startups which are acquired and whose idea is implemented by the incumbent, and innovations by startups which actually enter the market (which happens either when incumbents do not get a possibility to acquire them, or when they do but do not wish to acquire them).

Market clearing To close the equilibrium characterization, we impose feasibility. The single final good of the economy is used for consumption, R&D, implementation and acquisition costs,

$$Y_t = C_t + R_t + I_t + A_t, \tag{23}$$

where R_t stands for aggregate R&D expenditures, given by

$$R_t = \sum_\tau \sum_n \sum_k \left[\varphi_\tau(n,k)\Big(\xi_I k^{-\beta}\big(z_\tau(n,k)\big)^\psi\Big)\right] Y_t + \sum_{\tau_E} \xi_E\big(x(\tau_E)\big)^\psi Y_t, \tag{24}$$

I_t stands for the resources used for implementation costs, given by

$$\begin{aligned} I_t = \sum_\tau \sum_n \sum_k \varphi_\tau(n,k)\Big\{ z_\tau(n,k)\kappa_I + \sum_{\tau_E} x(\tau_E)\Big[I_{IA}(n,k,\tau,\tau_E)\kappa_I \\ + \Big(\big(1-\gamma_\tau(n,k)\big)+\gamma_\tau(n,k)\big(1-I_A(n,k,\tau,\tau_E)\big)\Big)\kappa_E\Big]\Big\} Y_t, \end{aligned} \tag{25}$$

and A_t stands for the resources used for the search effort cost of acquisitions, given by

$$A_t = \sum_\tau \sum_n \sum_k \varphi_\tau(n,k)\chi\big(\gamma_\tau(n,k)\big)^\phi Y_t. \tag{26}$$

On the other hand, labor market clearing requires $L = \int_0^1 l_{it} di$. As labor demand is proportional to the inverse of the markup, labor market clearing gives us a formula for the aggregate labor share:

$$\frac{w_t L}{Y_t} = \sum_{\tau} \sum_{n} \sum_{k} \varphi_{\tau}(n,k) \lambda^{-n}, \tag{27}$$

implying that the relative wage $\frac{w_t}{Y_t}$ is constant over time in a BGP.

3.3 Equilibrium properties

In this section, we discuss some key properties of the BGP equilibrium and describe some of the key qualitative features of the model.[11] To implement the model numerically, we assume that the technology gap is bounded above by some $n_{max} < +\infty$, so that $n \in \{1, 2, \ldots, n_{max}\}$.[12] Likewise, we assume $k \in \mathbb{K} \equiv \{k_1, k_2, \ldots, k_{max}\} \subset \mathbb{R}_+$, and $\tau \in \mathbb{T} \equiv \{1, 2, \ldots, \tau_{max}\}$. Furthermore, we assume that technology classes are evenly distributed on the unit circle, and define the distance between any two (τ, τ') as the length in radians of the shortest arc between them:[13]

$$\mathcal{D}(\tau, \tau') = 2\pi \cdot \min \left\{ \frac{|\tau - \tau'|}{\tau_{max}}, 1 - \frac{|\tau - \tau'|}{\tau_{max}} \right\} \tag{28}$$

Using this definition of technological proximity, we then assume a technology spillovers function $\mathcal{K}(k, \tau, \tau_E)$ which is decreasing in $\mathcal{D}(\tau, \tau_E)$. Intuitively, when the acquirer and the target are in closer technological proximity, there is a larger transfer of knowledge capital from the latter to the former. The specific functional form for $\mathcal{K}$ is given in equation (A.2) in Appendix A.1.

Figure 1 plots the value function, $v_\tau(n,k)$, the innovation policy function, $z_\tau(n,k)$, and the search effort policy function, $\gamma_\tau(n,k)$, for an incumbent of a given technology class τ. The value function and is increasing and concave in both arguments. Innovation rates are decreasing in n, a direct consequence of the concavity of the value function, and increasing in k, as a higher knowledge capital stock lowers the cost of R&D, all else equal. The effort search policy function is increasing and concave in n and k, as incumbents in these states have more to gain from acquiring startups, so they choose to search more

[11]The parameters throughout this section are set to their calibrated values, discussed in detail in Section 4.1.

[12]This assumption comes with no loss of generality. As we shall see shortly, in equilibrium there is an endogenous maximum technology gap, beyond which there are no firms. In practice, we pick a n_{max} and make sure ex-post that it is above this endogenous limit.

[13]To derive equation (28), suppose with no loss of generality that $\tau_1 = 1$ is located on coordinate $(1, 0)$ of the circle. Then, as points are evenly spaced, point k forms an angle of $2\pi \frac{k-1}{\tau_{max}}$ radians about the horizontal axis. As the circle has unit radius, the arc length between two points is simply the difference between their angles. The $\mathcal{D}$ function has the following properties: (i) $\mathcal{D}(\tau, \tau') \geq 0$ with $\mathcal{D}(\tau, \tau) = 0$; (ii) $\mathcal{D}(\tau, \tau') \leq \pi$; and (ii) $\mathcal{D}(\tau, \tau') = \mathcal{D}(\tau', \tau)$.

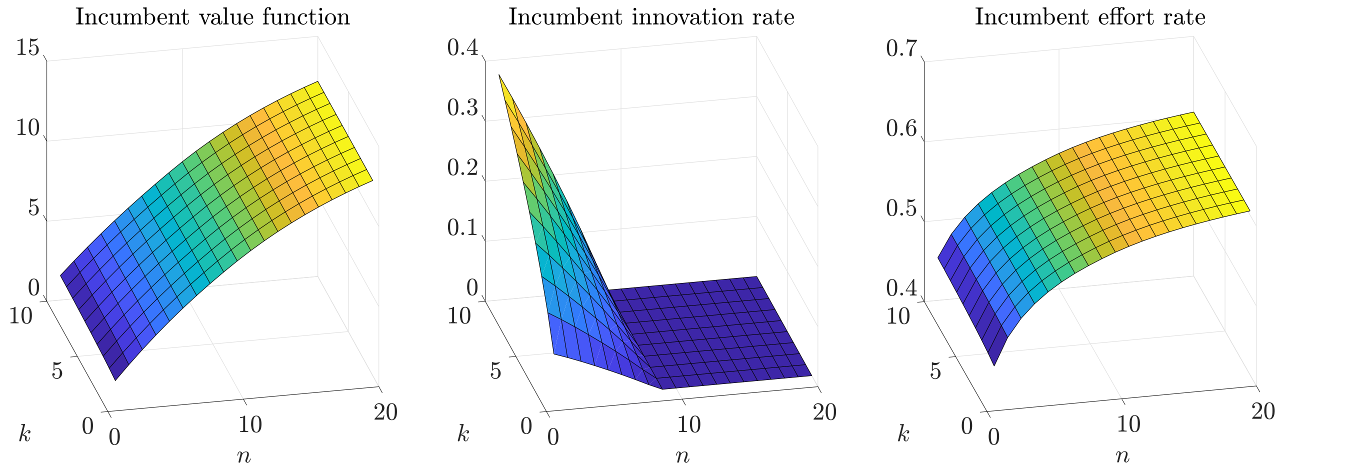

Figure 1: Value function and innovation and effort policy functions of an incumbent firm.

intensively for them. Note that because of implementation costs for innovation, incumbents stop innovating once they reach a certain technology gap (in this example, at $n = 9$). At this point, the low marginal benefit from an innovation does not justify paying the implementation cost any longer.

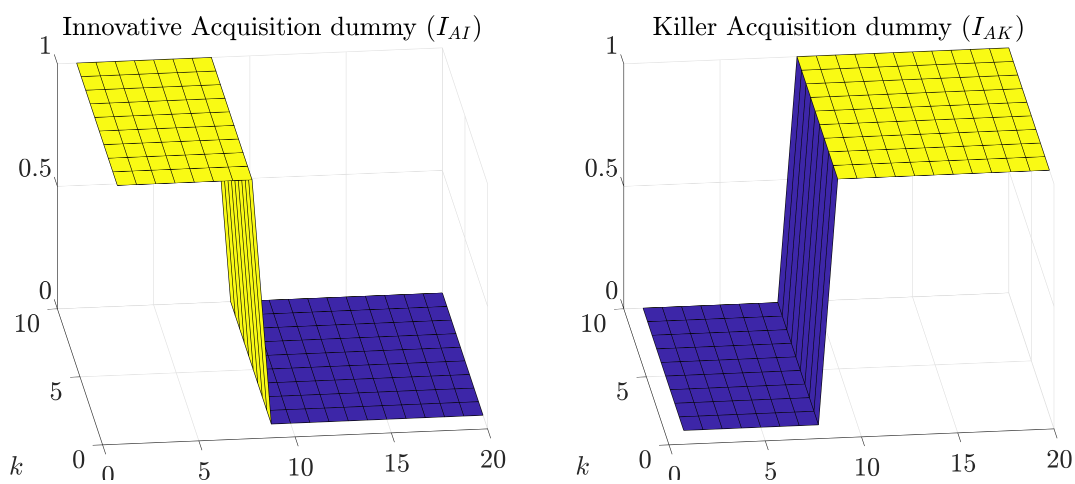

Figure 2: Acquisition decisions.

Figure 2 illustrates the acquisition decisions of an incumbent which is faced with the threat of entry of a startup of the same technology class. In this example, the incumbent decides to always acquire the startup, in order to protect itself from competition and displacement. The higher n, the more likely this is to be a killer acquisition (an acquisition in which the startup's idea is not implemented), because in this region the marginal benefit of an additional innovation is low. This is in line with the empirical results in Cunningham *et al.* (2020), who find that killer acquisitions are more likely if incumbents have a dominant market position.

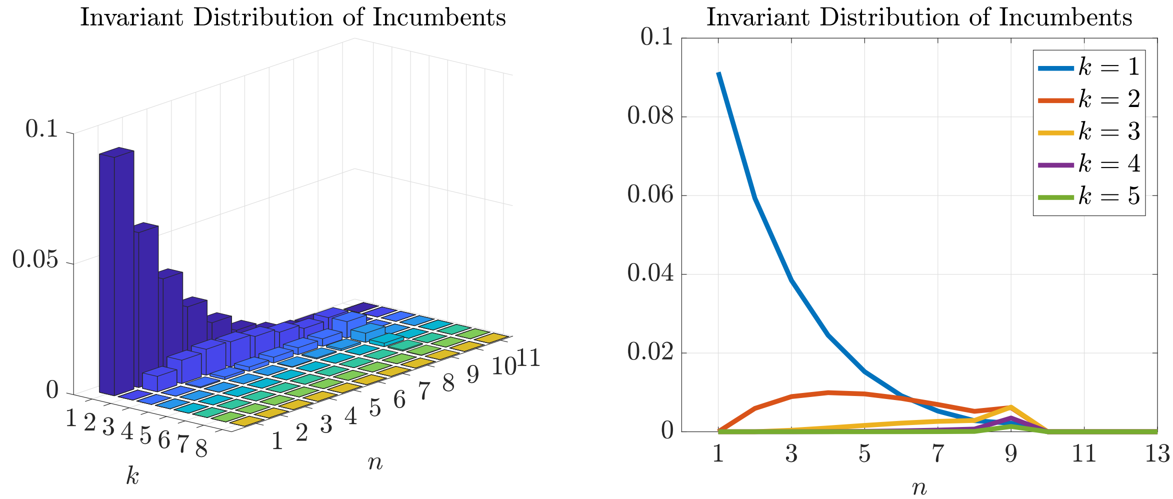

Figure 3: Invariant Distribution.

Finally, the left panel of Figure 3 shows, for a given technology class, the invariant distribution in equilibrium in the (n, k) space, whereas the right panel depicts the same distribution as cross-sections for different knowledge capital levels. For the lowest knowledge capital stock ($k = 1$), the distribution is right-skewed. When a startup appears, with an (endogenous) probability γ the incumbent notices it and has the possibility of acquiring it. Upon acquisition, the incumbent's knowledge capital increases to $k = 2$. This process continues into higher knowledge capital levels. Eventually, firms bunch up against the technology gap at which all in-house innovation stops ($n = 9$, in this example), at which point all acquisitions become killer acquisitions.

4 Quantitative Analysis

The purpose of our quantitative analysis is to understand the macroeconomic effects of acquisitions on aggregate innovative activity and growth. To this end, we first calibrate the model to match both aggregate and micro-level features of the data (Section 4.1), and then examine the relationship between the frequency of acquisitions and aggregate economic growth in the context of the calibrated model (Section 4.2).

4.1 Calibration

We calibrate the model at the annual frequency, and set the discount rate externally to $\rho = 0.02$. This leaves 10 parameters to be identified: the productivity step λ; the R&D cost shifters ξ_I and ξ_E; the elasticity of R&D costs to knowledge capital, β, and to innovation, ψ; the implementation costs for incumbents, κ_I, and entrants, κ_E; the Nash bargaining parameter for incumbents, α; and the scale and curvature parameters in the effort cost function, χ and ϕ. These parameters are calibrated internally using an indirect inference approach: we choose the set of parameter values that minimizes the distance between a set of model-generated moments and their empirical counterparts.[14]

Table 4: Set of calibrated parameters.

Parameter	**Value**	**Description**
λ	1.080	Innovation step size
ξ_I	4.770	R&D cost scale (incumbents)
ξ_E	20.636	R&D cost scale (entrants)
β	1.166	R&D cost elasticity to knowledge capital
ψ	2.430	R&D cost elasticity to innovation
κ_I	0.626	Implementation cost (incumbents)
κ_E	0.903	Implementation cost (entrants)
α	0.960	Nash bargaining parameter
χ	4.689	Search effort cost scale
ϕ	8.250	Search effort cost curvature
Period	1 year	

Table 4 lists the parameter values that result from this estimation exercise, and Table 5 shows the results in terms of moment matching. We target 10 moments related to innovation, acquisitions and the dynamics of firms in the United States. Of these moments, 7 are aggregate moments, and 3 are regression coefficients from our empirical analysis of Section 2. First, we target a growth rate of GDP of 2% which, together with a 2% discount rate, implies a real interest rate of 4% annually. Second, we target the average R&D share of U.S. GDP over the 1993-2013 period, equal to 2.58% according to OECD data.[15] Third, we target the average entry rate of firms over the same time period, computed as the ratio of

[14]Formally, the vector of parameters $\boldsymbol{\theta}$ is chosen to minimize $\sum_{m=1}^{M} \left| \frac{\text{Moment}_m(\text{Model},\boldsymbol{\theta}) - \text{Moment}_m(\text{Data})}{\text{Moment}_m(\text{Data})} \right|$.

[15]The OECD data can be accessed here: https://data.oecd.org/rd/gross-domestic-spending-on-r-d.htm.

the number of new firms between consecutive years to the number of total firms in the first year. Using data from the 2018 release of the Business Dynamics and Statistics database of the U.S. Census Bureau, we find this number to be 9.04%.[16] Fourth, to avoid overstating the contribution of entrants to productivity growth, we target the share of TFP growth that can be attributed to innovative activity by new firms. Over our sample period, this number is 21.1% according to the estimates in Garcia-Macia, Hsieh and Klenow (2019).

Table 5: Targeted moments: model versus data.

Moment	Model	Data	Data Source
	Aggregate moments		
Growth rate	2.0%	2.0%	Standard
R&D share of GDP	1.7%	2.6%	OECD
Firm entry rate	5.5%	9.0%	US Census Bureau, BDS
Contribution entrants to growth	17.0%	21.1%	Garcia-Macia *et al.* (2019)
Acquisition rate	4.4%	3.7%	David (2020)
Share of killer acquisitions	6.9%	6.0%	Cunningham *et al.* (2020)
Acquisition premium	47.1%	47.0%	David (2020)
	Regression coefficients		
Coefficient on *Post*	0.295	0.178	Col. (1), Table 2
Coefficient on *Patent Share*	0.793	0.458	Col. (1), Table 2
Coefficient on *Post * Patent Share*	–0.088	–0.036	Col. (1), Table 2

We target three aggregate moments related to the acquisitions of firms. First, we target a 3.7% yearly rate of acquisitions. We borrow this number from David (2020), who computes it as the average yearly number of acquisitions as a share of total firms in Compustat. In the model, we compute the arrival rate of acquisitions as follows:

$$AcqArrRate \equiv \sum_{\tau_E} x(\tau_E) \left(\sum_{\tau} \sum_{n} \sum_{k} \varphi_\tau(n,k) \gamma_\tau(n,k) I_A(n,k,\tau,\tau_E) \right) \tag{29}$$

Second, to discipline how common killer acquisitions are in the model, we target a 6% share of killer acquisitions, a number taken from Cunningham *et al.* (2020). In the model,

[16]The BDS data can be accessed here: https://www.census.gov/data/datasets/time-series/econ/bds/bds-datasets.html.

the share of killer acquisitions is computed as the ratio of the arrival rate of acquisitions, defined in equation (29), to the arrival rate of killer acquisitions, defined similarly but replacing I_A by I_{AK} in equation (29). Finally, we target the average acquisition premium, which we take it to be 47% in the US, following David (2020). In the model, the acquisition premium in a typical transaction is the net acquisition price markup over the outside option of the startup, $\frac{p^{\mathcal{A}}(n,k,\tau,\tau_E)}{v_{\tau_E}(1,1)-\kappa_E} - 1$. Using the formula for the acquisition price, the average acquisition premium can be written as follows:

$$AcqPremium = \sum_{\tau_E} \widetilde{x}(\tau_E) \left[\sum_{\tau} \sum_{n} \sum_{k} \varphi_{\tau}(n,k)(1-\alpha) \left(\frac{\sigma_{\tau}(n,k,\tau_E)}{v_{\tau_E}(1,1) - \kappa_E} \right) \right] \tag{30}$$

where $\widetilde{x}(\tau_E)$ is defined as the share of all startups that belong to technology class τ_E.

The above targets ensure that the quantitative model provides predictions that are in line with macroeconomic aggregates. Additionally, to ensure that our theory provides consistent predictions on the micro-level behavior of innovative activity, the model is calibrated to yield similar firm-level responses in patenting to acquisition events. To this end, we run some of the panel regressions presented in Section 2 on a firm-level panel of model-generated data using about 2,000 products, which we simulate over 70 years partitioned in time steps of size $dt = 1/25$. The initial distribution at time $t = 0$ is drawn from the invariant distribution, and we let the economy evolve thereafter through the dynamics implied by the BGP policy functions. After constructing a panel at the firm-year observation level, we then implement the event-study-type regressions from Table 2, restricting our attention to the effects of acquisitions on patenting within the sub-sample of firms that experience only one acquisition in the four years prior and four years after the year the acquisition event takes place.[17]

Table 5 shows that the estimated model fits the data well. The calibration exercise predicts that entrants' cost scale parameter is larger than that of incumbents. At the same time, incumbents hold most of the bargaining power, absorbing the majority of the joint surplus from an acquisition. This combination of parameters allows the model to deliver a relatively high acquisition premium without sacrificing the substantial contribution that entrants have to economic growth. It is also worth noting that the model predicts, from the simulated panel of firms, quantitatively similar effects of acquisitions on innovation as in the data: an acquisition event in the subsample of firms leads to an increase of 0.295

[17] As we do not have a theory of patents in the model, we assume that firms increase their patent stock by one when they innovate and implement on their own, or when they acquire a startup regardless of whether or not the target's idea is subsequently implemented by the incumbent. To construct the *Post* dummy variable, we exclude the same-year change in patenting activity from our simulated regressions. As in the data and for comparability, all variables (except *Post*) are standardized.

standard deviations in patenting and a 0.793 standard deviation increase in the patent share of firms. Moreover, as in the data, the coefficient on the interaction term is negative: the acquirer's rise in patenting activity is lower the higher is its share of overall patents.

4.2 The Aggregate Effects of Acquisitions

We are now ready to consider the quantitative implications of our model. As we are interested in the aggregate effects of acquisitions, we will explore variations in the frequency of acquisitions in equilibrium and study their implications on startup activity, actual entry and economic growth. To do so, we conduct a comparative statics exercise between BGP equilibria by shifting the value for the parameter χ, which governs how costly it is for incumbents to notice threatening startups (and therefore shifts the frequency at which acquisitions occur in equilibrium).

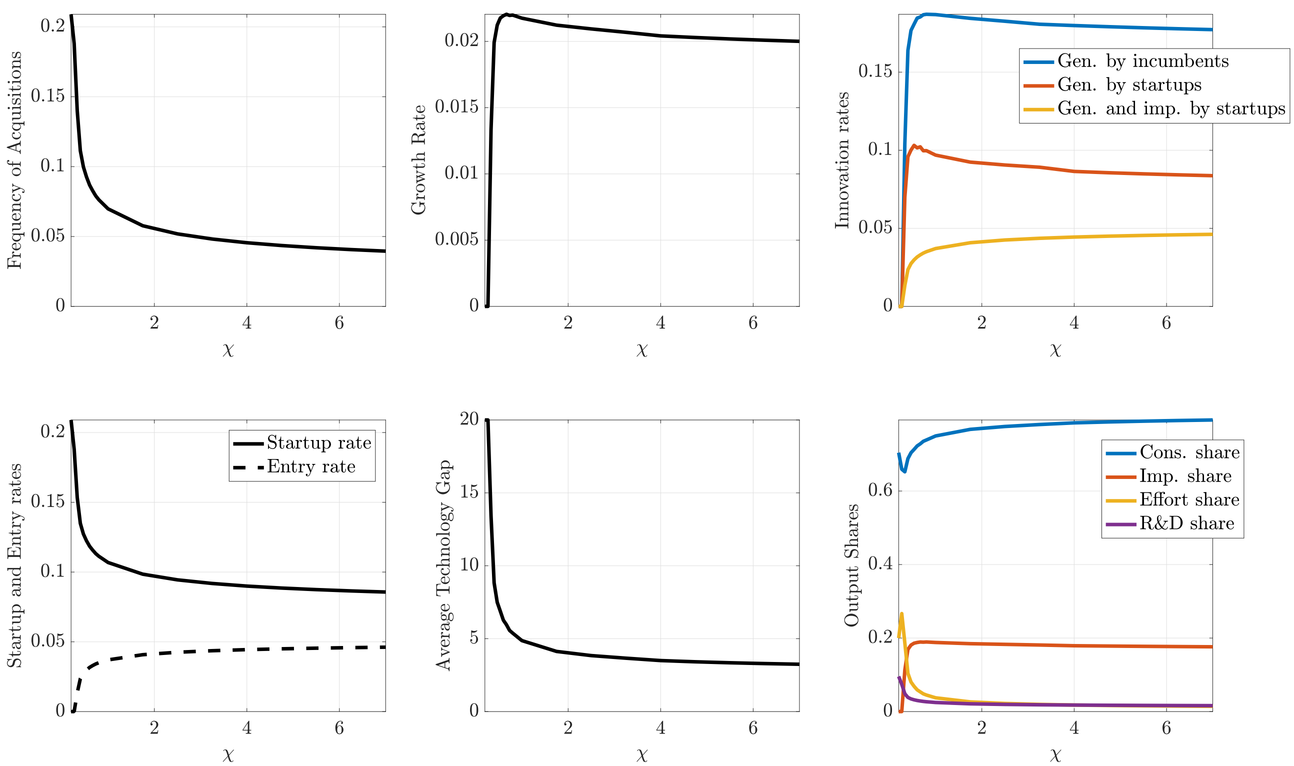

Figure 4: BGP equilibria for different values of effort cost, χ. The calibrated value is $\chi = 4.689$ (see Table 4).

Figure 4 plots the BGP values of several key endogenous variables as a function of χ, keeping all other parameters at their baseline levels throughout. The upper left-most plot shows that the frequency of acquisitions is monotonically decreasing in χ. At higher values of χ, acquisitions become rarer events. In such region, the aggregate entry rate is higher, implying a lot of creative destruction: incumbents are frequently replaced by startups, and as a result, the average technology gap is low. As χ decreases, more startups start to be acquired. This increases the wedge between the startup rate and the entry rate: while the startup rate increases (boosted by the prospect of acquisition), the entry rate actually falls. All else equal, a fall in the entry rate would lower aggregate productivity growth. However, as the top right-most panel of Figure 4 indicates, lower entry is more than compensated by an increase in incumbent innovation, both because incumbents innovate more on their own and because they implement some startup ideas.

In particular, the total contribution of startups to growth (indicated by the orange line in the top right panel) initially increases as χ falls, because the higher startup rate more than compensates for the fact that some startup ideas are not implemented. Indeed, for high enough levels of χ, the fraction of killer acquisitions is low: as entry rates are still high, creative destruction ensures that most incumbents have low technology gaps, and therefore high incentives to implement any ideas that they might acquire. Moreover, incumbents' own innovations increase as well (as indicated by the blue line in the top right panel). This is because acquisitions increase the value of incumbents, mainly because they allow them to survive longer. This higher value increases the marginal gain from innovation (as the firm can expect to enjoy higher profits for a longer time), and therefore causes incumbents to invest more into R&D. Finally, as shown in the bottom right panel, the increase in the rate of productivity growth is accompanied by a fall in the consumption share of output, as more resources are needed for R&D, implementation and effort costs.

For low values of the effort cost χ, however, these effects begin to change. In particular, productivity growth starts to decrease (as shown in the top middle panel of Figure 4). Thus, productivity growth has overall an inverted U-shape in the meeting probability, and therefore an inverted U-shape in the frequency of acquisitions. Indeed, at low values of χ, almost all startups are bought up by incumbents, so that the entry rate falls substantially (as shown in the bottom left panel). As a result, exit becomes unlikely for incumbents, and the average incumbent enjoys a high technology gap and high profits. This has two negative implications for innovation. First, these entrenched firms have little incentive to innovate themselves, as their marginal benefit from innovation is low. Thus, incumbent innovation starts to decrease (as shown by the blue line in the top right panel). Second, virtually all acquisitions by these firms are now killer acquisitions. Thus, not only are most startups acquired, but most of their innovations are actually discarded (as shown by the orange line in the top right panel).

In the extreme, with χ close to zero, productivity growth essentially falls to zero.

However, this economy may well appear highly innovative to an external observer, as both the startup rate and the R&D share of GDP are very high. This extreme case shows that unlimited acquisitions can imply socially undesirable outcomes, even though they generate large private benefits for incumbents and startups. In fact, acquisitions are now a way for startups to earn innovation profits without actually innovating.[18] Thus, a large share of resources is wasted on activities that only redistribute rents, but have no benefit for the consumer.[19]

Finally, note that in the current preliminary calibration, the growth-maximizing level of the effort cost is $\chi = 0.7$, delivering a growth rate of 2.1% per year. This is not far from the baseline growth rate of 2.0% achieved with the baseline value of χ. Thus, these preliminary results suggest that the US economy is not far from the "growth-maximizing" level of acquisitions. However, they also suggests that increases in the frequency of acquisitions could trigger a very sharp decrease in growth rates beyond a certain point.

5 Conclusion

In this paper, we assess the net effect of acquisitions on aggregate productivity growth, using a macroeconomic model that takes into accounts potential advantages (startup incentives, idea transfers) and disadvantages (killer acquisitions). We find that productivity growth has an inverted U-shape in the frequency of acquisitions: as long as the level of creative destruction remains sufficiently high, acquisitions have a positive effect, but once creative destruction gets too low, acquisitions become socially undesirable. A preliminary calibration shows that the current frequency of acquisitions in the United States is slightly below the growth-maximizing level.

[18] Note that the social benefit of innovation is independent of the technology gap n, because of the Cobb-Douglas aggregation structure. However, private incentives to implement are decreasing in n, and private incentives for killer acquisition are increasing in n. Thus, in a high acquisition - low entry equilibrium, private incentives and social benefits are far from being aligned.

[19] For lower levels of χ, the average knowledge capital in the economy increases. However, in the current calibration, this increase does not have a large effect on aggregate productivity growth.

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The Aggregate Effects of Acquisitions on Innovation and Economic Growth

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Appendix Materials

A Derivations and Proofs

A.1 Distribution Dynamics and the Invariant Distribution

We seek to write the evolution of the distribution of incumbents in our continuous-time setting using matrix notation. To do this, we must build an *intensity* (also known as *infinitesimal generator*) matrix. For a homogeneous continuous-time Markov chain z_t taking values in some discrete space $\{z_1, z_2, \dots, z_S\} \in \mathbb{R}^S$, a generator matrix $\boldsymbol{M}_z$ is defined by:

$$\boldsymbol{M}_z \equiv \begin{pmatrix} -\sum_{j\neq 1}\lambda_{1j} & \lambda_{12} & \dots & \lambda_{1S} \\ \lambda_{21} & -\sum_{j\neq 2}\lambda_{2j} & \dots & \lambda_{2S} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_{S1} & \lambda_{S2} & \dots & -\sum_{j\neq S}\lambda_{Sj} \end{pmatrix} \tag{A.1}$$

where $\lambda_{ij} \geq 0$ is the intensity rate for a z_i-to-z_j transition. Denoting the probability mass function (pmf) of z by the vector $\vec{f}_t \in [0,1]^S$, the evolution of the distribution can then be written as $\frac{\partial \vec{f}_t}{\partial t} = \boldsymbol{M}_z^\top \vec{f}_t$.[20] Note that the diagonal elements of $\boldsymbol{M}_z$ collect outflows, while the off-diagonal elements collect inflows. Thus, each row of an infinitesimal generator matrix must add up to zero.[21]

To build this matrix in our model, we assume $n \in \{1, 2, \dots, n_{max}\}$, i.e. that the technology gap is bounded above by $n_{max} < +\infty$. Likewise, we assume $k \in \mathbb{K} \equiv \{k_1, k_2, \dots, k_{max}\}$, and $\tau \in \mathbb{T} \equiv \{1, 2, \dots, \tau_{max}\}$. Moreover, we use the following knowledge spillover function:

$$\mathcal{K}(k, \tau, \tau_E) = k + \left\lfloor 1 - \frac{\mathcal{D}(\tau, \tau_E)}{\pi} \right\rceil, \tag{A.2}$$

where the notation "$\lfloor \cdot \rceil$" means rounding to the nearest element in the $\mathbb{K}$ grid. In words, when an incumbent from class τ acquires a startup from class τ_E, the new knowledge

[20] Our notation in this section is as follows: the symbol $\vec{x}$ denotes a vector; $\boldsymbol{X}$ denotes a matrix, and $\vec{x}^\top$ (or $\boldsymbol{X}^\top$) denotes transpose.

[21] For recent uses of Markov processes in continuous-time economic models, and how to implement them in practice, see e.g. Perla (2019) and Roldan-Blanco and Gilbukh (2020).

capital of the incumbent equals its old knowledge capital k plus, with a certain probability, the knowledge capital of the startup (which is normalized to one). This probability, given by the term $\left[1-\frac{\mathcal{D}(\tau,\tau_E)}{\pi}\right]$, is decreasing in the technological distance $\mathcal{D}(\tau,\tau_E)$ between incumbent and startup (defined in equation (28)), so that meetings between firms in closer technological proximity yield a transfer of knowledge with a higher probability.[22] Intuitively, this is meant to capture the idea that the incumbent is able to use some of the knowledge of the startup when the two firms perform their research activities in technology classes that are close to one another.

We denote by $\varphi_{\tau,t}(n,k)$ the share of firms in state (n,k,τ) at time t. Our assumptions imply that there are $S \equiv n_{max} \cdot k_{max} \cdot \tau_{max}$ possible states. The law of motion of $\varphi_{\tau,t}(n,k)$ can be written as follows:

$$\frac{\partial \vec{\varphi}_t}{\partial t} = \boldsymbol{M}_{\varphi}^{\top} \vec{\varphi}_t \tag{A.3}$$

where we denote $\vec{\varphi}_t \equiv \left[\vec{\varphi}_{1,t}(1,\bullet), \vec{\varphi}_{1,t}(2,\bullet), \ldots, \vec{\varphi}_{1,t}(n_{max},\bullet), \vec{\varphi}_{2,t}(1,\bullet), \ldots, \vec{\varphi}_{2,t}(n_{max},\bullet), \ldots, \vec{\varphi}_{\tau_{max},t}(1,\bullet), \ldots, \varphi_{\tau_{max},t}(n_{max},\bullet)\right]^{\top}$, where for each τ and n, we denote $\vec{\varphi}_{\tau,t}(n,\bullet) \equiv \left[\varphi_{\tau,t}(n,1), \varphi_{\tau,t}(n,2), \ldots, \varphi_{\tau,t}(n,k_{max})\right]^{\top}$.

The generator matrix is then given by the following $S \times S$ partitioned matrix:

$$\mathbf{M}_{\varphi} \equiv \begin{pmatrix} \mathbf{M}_{1,1} & \mathbf{M}_{1,2} & \cdots & \mathbf{M}_{1,\tau_{max}} \\ \mathbf{M}_{2,1} & \mathbf{M}_{2,2} & \cdots & \mathbf{M}_{2,\tau_{max}} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{M}_{\tau_{max},1} & \mathbf{M}_{\tau_{max},2} & \cdots & \mathbf{M}_{\tau_{max},\tau_{max}} \end{pmatrix}, \tag{A.4}$$

where each $\mathbf{M}_{\tau,\tau'}$ is an $n_{max} \cdot k_{max} \times n_{max} \cdot k_{max}$ matrix that summarizes the transitions from technology class τ into technology class τ'. For $\tau \neq \tau'$, we have

$$\mathbf{M}_{\tau,\tau'} \equiv \begin{pmatrix} \vec{e}_{\tau,\tau'}(1,\bullet) & 0 & \cdots & 0 \\ \vec{e}_{\tau,\tau'}(2,\bullet) & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \vec{e}_{\tau,\tau'}(n_{max},\bullet) & 0 & \cdots & 0 \end{pmatrix}, \tag{A.5}$$

[22] Here, we normalize $\mathcal{D}$ by π to translate radians into a number between zero and one.

where $\vec{e}_{\tau,\tau'}(1,\bullet) = \left[e_{\tau,\tau'}(n,1), e_{\tau,\tau'}(n,2), \cdots, e_{\tau,\tau'}(n,k_{max})\right]^\top$ and

$$e_{\tau,\tau'}(n,k) = x(\tau')\Big((1-\gamma) + \gamma\big(1 - I_A(n,k,\tau,\tau')\big)\Big). \tag{A.6}$$

That is, transitions between different technology classes happen only in case the entry of a startup of technology class τ' displaces an incumbent of technology class τ. In that case, there is a transition from whichever (n,k) state the incumbent was in into $(1,1)$.

Instead, the matrices on the block diagonal of $\mathbf{M}_\varphi$ are given by

$$\mathbf{M}_{\tau,\tau} \equiv \begin{pmatrix} \vec{m}_{\tau,1,1}(1,\bullet) & \vec{m}_{\tau,1,1}(2,\bullet) & \cdots & \vec{m}_{\tau,1,1}(n_{max},\bullet) \\ \vec{m}_{\tau,1,2}(1,\bullet) & \vec{m}_{\tau,1,2}(2,\bullet) & \cdots & \vec{m}_{\tau,1,2}(n_{max},\bullet) \\ \vdots & \vdots & \ddots & \vdots \\ \vec{m}_{\tau,1,k_{max}}(1,\bullet) & \vec{m}_{\tau,1,k_{max}}(2,\bullet) & \cdots & \vec{m}_{\tau,1,k_{max}}(n_{max},\bullet) \\ \vec{m}_{\tau,2,1}(1,\bullet) & \vec{m}_{\tau,2,1}(2,\bullet) & \cdots & \vec{m}_{\tau,2,1}(n_{max},\bullet) \\ \vdots & \vdots & \ddots & \vdots \\ \vec{m}_{\tau,2,k_{max}}(1,\bullet) & \vec{m}_{\tau,2,k_{max}}(2,\bullet) & \cdots & \vec{m}_{\tau,2,k_{max}}(n_{max},\bullet) \\ \vdots & \vdots & \ddots & \vdots \\ \vec{m}_{\tau,n_{max},k_{max}}(1,\bullet) & \vec{m}_{\tau,n_{max},k_{max}}(2,\bullet) & \cdots & \vec{m}_{\tau,n_{max},k_{max}}(n_{max},\bullet) \\ \vdots & \vdots & \ddots & \vdots \\ \vec{m}_{\tau,n_{max},k_{max}}(1,\bullet) & \vec{m}_{\tau,n_{max},k_{max}}(2,\bullet) & \cdots & \vec{m}_{\tau,n_{max},k_{max}}(n_{max},\bullet) \end{pmatrix}, \tag{A.7}$$

where $\vec{m}_{\tau,n,k}(n_d,\bullet) \equiv \left[m_{\tau,n,k}(n_d,1), m_{\tau,n,k}(n_d,2), \cdots, m_{\tau,n,k}(n_d,k_{max})\right]$. The elements of these vectors are given by the following formulas. For each $n < n_{max}$, we have

$$m_{\tau,n,k}(n+1,k) = z_\tau(n,k) + \sum_{\tau_E \neq \tau} x(\tau_E) \cdot \gamma \cdot I_{IA}(n,k,\tau,\tau_E). \tag{A.8}$$

That is, both an incumbents' own innovation and its innovative acquisition of a startup of a different technology class increase the technology gap by one unit, but leave knowledge capital unchanged. For each $k < k_{max}$, we have

$$m_{\tau,n,k}(n,k+1) = x(\tau) \cdot \gamma \cdot I_{KA}(n,k,\tau,\tau_E). \tag{A.9}$$

That is, a killer acquisition of a startup of the same technology class increases knowledge capital by one unit, but leaves the technology gap unchanged. For $n < n_{max}$ and $k < k_{max}$, we have as well

$$m_{\tau,n,k}(n+1,k+1) = x(\tau) \cdot \gamma \cdot I_{IA}(n,k,\tau,\tau_E). \tag{A.10}$$

That is, an innovative acquisition of a startup in the same technology class increases both the technology gap and knowledge capital by one unit. At the boundaries, we have $m_{\tau,n_{max},k}(n_{max},k+1) = x(\tau) \cdot \gamma \cdot I_{IA}(n_{max},k,\tau,\tau_E)$ for $k < k_{max}$ and $m_{\tau,n,k_{max}}(n+1,k_{max}) = x(\tau) \cdot \gamma \cdot I_{IA}(n,k_{max},\tau,\tau_E)$ for $n < n_{max}$. Finally, for each $(n,k) \neq (1,1)$,

$$m_{\tau,n,k}(1,1) = x(\tau)\Big((1-\gamma) + \gamma\big(1 - I_A(n,k,\tau,\tau)\big)\Big). \tag{A.11}$$

That is, entry of a start-up of the same technology class brings back both the technology gap and knowledge capital to 1. All other non-diagonal elements of $\mathbf{M}_{\tau,\tau}$ are equal to 0. Finally, all diagonal elements of the overall generator matrix $\mathbf{M}_{\varphi}$ are given by the definition of a generator matrix in equation (A.1).

To find the invariant distribution, we impose $\frac{\partial \vec{\varphi}_t}{\partial t} = \vec{0}$ in equation (A.3) and solve for the unique solution of the system of linear equations holding $\sum_\tau \sum_n \sum_k \varphi_\tau(n,k) = 1$.

A.2 Growth Rate

Aggregate output can be written in logs as follows:

$$\ln Y_t = \int_0^1 \ln(y_{it})di = \underbrace{\int_0^1 \ln(q_{it})di}_{\equiv \mathcal{Q}_t} + \int_0^1 \ln(l_{it})di$$

Recall that $l_{it} = \frac{Y_t}{w_t}\lambda^{-n}$. In a BGP, $\frac{Y_t}{w_t}$ is constant over time, as shown in equation (27). Therefore, the additive term $\int_0^1 \ln(l_{it})di = \sum_\tau \sum_n \sum_k \varphi_\tau(n,k) \ln(\lambda^{-n})$ is constant, and thus:

$$g = \frac{\partial \mathcal{Q}_t}{\partial t}$$

To find this derivative, partition time in steps of arbitrarily small length $\Delta > 0$. Within an instant, there are three sources for an innovation in a given state (n,k,τ,τ_E), where recall (τ,τ_E) denote the technology classes of the incumbent and the startup, respectively.

- An innovation by the incumbent, with probability $z_\tau(n,k)\Delta + o(\Delta)$.
- An innovation by the startup, if it is acquired by the incumbent and the incumbent then implements the idea, with probability $\sum_{\tau_E} x(\tau_E)\gamma_\tau(n,k)I_{AI}(n,k,\tau,\tau_E)\Delta + o(\Delta)$.

- An innovation by the startup that is implemented by the startup itself (either because no acquisition is allowed, or it is allowed but the incumbent chooses not to acquire), with probability $\sum_{\tau_E} x(\tau_E)\Big[\big(1-\gamma_\tau(n,k)\big)+\gamma_\tau(n,k)\Big(1-I_{AI}(n,k,\tau,\tau_E)\Big)\Big]\Delta+o(\Delta)$.

As all three types of innovation advance quality by a step λ, we have:

$$\mathcal{Q}_{t+\Delta}=\Delta\left(\sum_\tau\sum_n\sum_k \varphi_\tau(n,k)i_\tau(n,k)\right)\ln(\lambda)+\mathcal{Q}_t+o(\Delta) \tag{A.12}$$

where $i_\tau(n,k)$ is defined in equation (22), and $o(\Delta)$ collects higher-order terms with the property $\lim_{\Delta\to 0}\frac{o(\Delta)}{\Delta}=0$. To arrive at this expression, we have used the log-linearity of $\mathcal{Q}_t$, which implies that the effect of an innovation does not depend on the productivity level of the state (n,k,τ) to which it applies. Subtracting $\mathcal{Q}_t$ from both sides, dividing by Δ and letting $\Delta\to 0$, we obtain:

$$\frac{\partial\mathcal{Q}_t}{\partial t}=\ln(\lambda)\sum_\tau\sum_n\sum_k\varphi_\tau(n,k)i_\tau(n,k)$$

our desired result.

B Additional Tables and Figures

Table A1: Number of M&A transactions between 1980 and 2006

		Public Target 0	Public Target 1	
Public	0	28,138	5,467	33,605
acquirer	1	24,398	4,253	28,651
		52,536	9,720	62,256

Table A2: Stylized Fact 1 - Cumulative cites received per patent

	(1)	(2)	(3)	(4)
Treatment Obs	0.050***			
	(0.009)			
Post (All)		0.114***		
		(0.014)		
Post (Clean Subset)			0.066***	0.044**
			(0.019)	(0.016)
Constant	-0.006***	-0.034***	-0.059***	0.058***
	(0.001)	(0.004)	(0.001)	(0.009)
Observations	73,050	73,050	64,682	6,504
Year FE	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes
R-squared	0.872	0.873	0.870	0.937

Notes: ***/**/* indicate significance at the 1%/5%/10% level. Standard errors are clustered at the firm and year level. The dependent variable is standardized.

Table A3: Stylized Fact 2 - Cumulative cites received per patent

	(1)	(2)	(3)	(4)
Post	0.184***	0.040**	0.037**	0.037**
	(0.026)	(0.016)	(0.016)	(0.016)
Patent Share	0.390***	0.132***	0.132***	0.132***
	(0.034)	(0.029)	(0.029)	(0.029)
Post * Patent Share	-0.039	-0.017*	-0.017*	-0.017*
	(0.024)	(0.009)	(0.009)	(0.009)
Target log of Cum. Patents				0.020**
				(0.010)
Target Dummy for Patenting			0.027	-0.010
			(0.017)	(0.022)
Constant	-0.048	0.065***	0.066***	0.066***
	(0.034)	(0.010)	(0.010)	(0.010)
Observations	6,366	6,365	6,365	6,365
Year FE	Yes	Yes	Yes	Yes
Sector FE	Yes	No	No	No
Firm FE	No	Yes	Yes	Yes
R-squared	0.332	0.939	0.939	0.939

Notes: ***/**/* indicate significance at the 1%/5%/10% level. Standard errors are clustered at the firm and year level. The dependent variable is standardized.

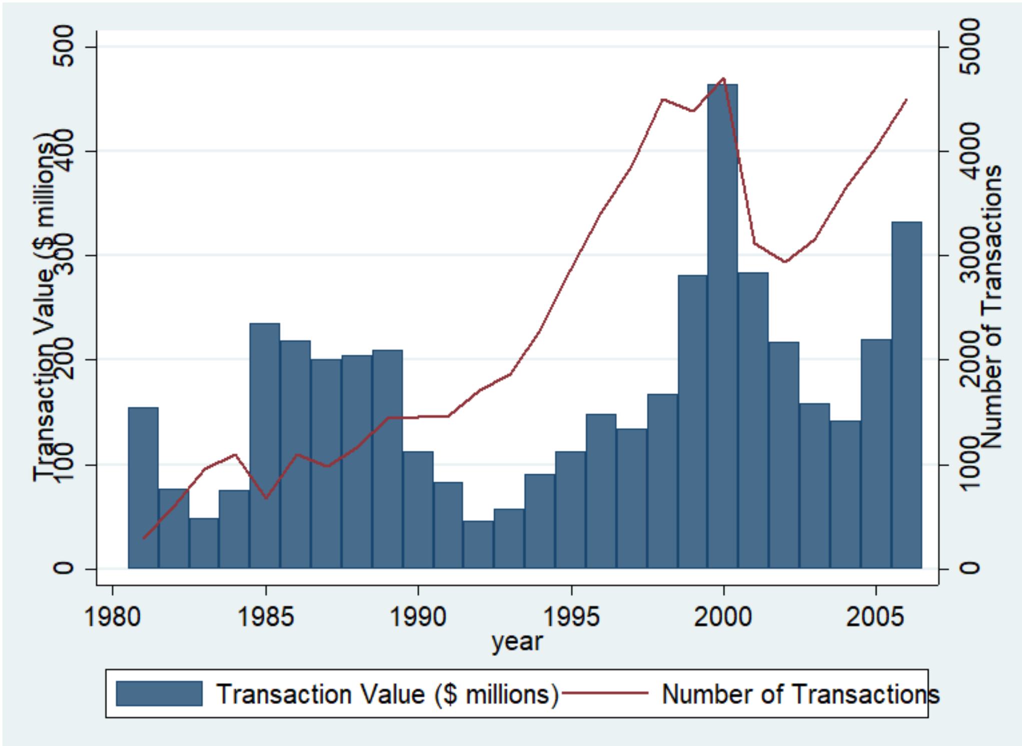

Figure A1: Number of Transactions and Value per Year

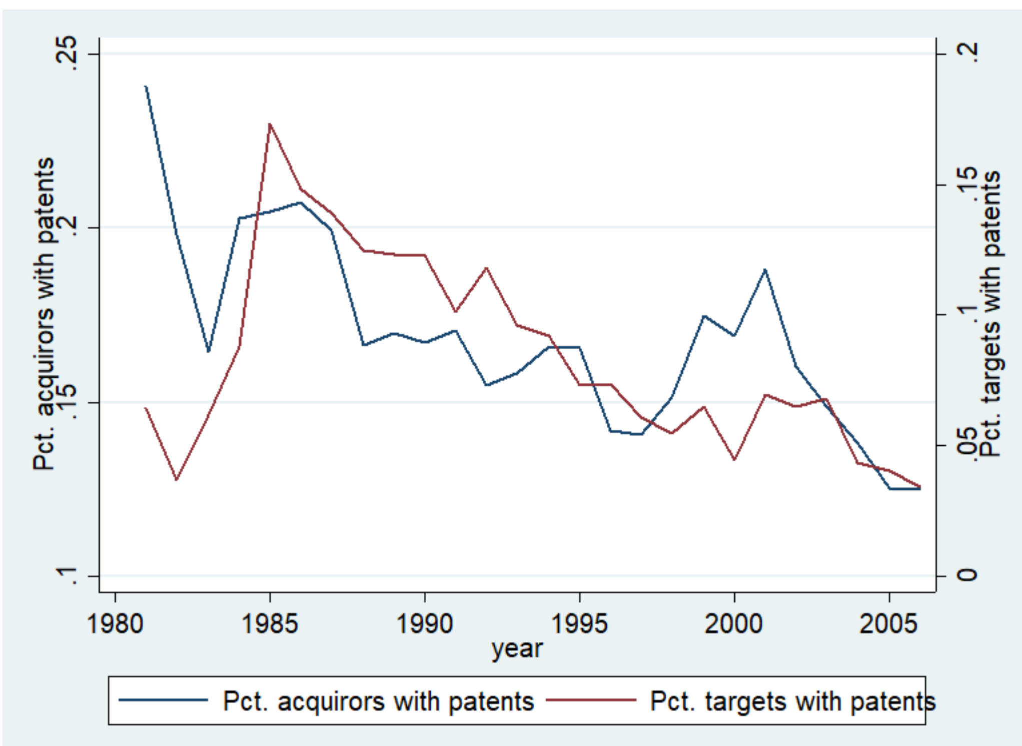

Figure A2: Fraction of firms holding patents

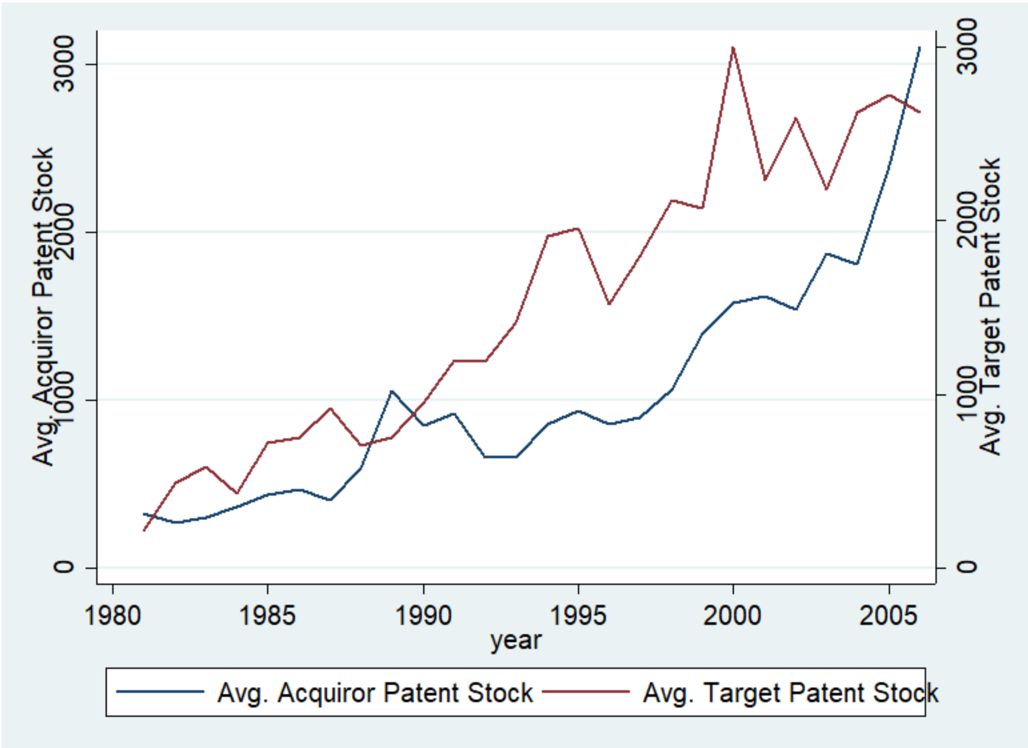

Figure A3: Stock of patents conditional on holding patents